C3 Trigonometry

1. **June 2010 qu.3**
   (i) Express the equation \( \csc \theta (3 \cos 2\theta + 7) + 11 = 0 \) in the form \( a \sin^2 \theta + b \sin \theta + c = 0 \), where \( a, b \) and \( c \) are constants. [3]
   (ii) Hence solve, for \(-180^\circ < \theta < 180^\circ\), the equation \( \csc \theta (3 \cos 2\theta + 7) + 11 = 0 \). [3]

2. **June 2010 qu.8**
   (i) Express \( 3 \cos x + 3 \sin x \) in the form \( R \cos(x - \alpha) \), where \( R > 0 \) and \( 0 < \alpha < \frac{1}{2} \pi \). [3]
   (ii) The expression \( T(x) \) is defined by \( T(x) = \frac{8}{3 \cos x + 3 \sin x} \).
      (a) Determine a value of \( x \) for which \( T(x) \) is not defined. [2]
      (b) Find the smallest positive value of \( x \) satisfying \( T(3x) = \frac{8}{9} \sqrt{6} \), giving your answer in an exact form. [4]

3. **Jan 2010 qu.2**
   The angle \( \theta \) is such that \( 0^\circ < \theta < 90^\circ \).
   (i) Given that \( \theta \) satisfies the equation \( 6 \sin 2\theta = 5 \cos \theta \), find the exact value of \( \sin \theta \). [3]
   (ii) Given instead that \( \theta \) satisfies the equation \( 8 \cos \theta \csc^2 \theta = 3 \), find the exact value of \( \cos \theta \). [5]

4. **Jan 2010 qu.9**
   The value of \( \tan 10^\circ \) is denoted by \( p \). Find, in terms of \( p \), the value of
   (i) \( \tan 55^\circ \), [3]
   (ii) \( \tan 5^\circ \), [4]
   (iii) \( \tan \theta \), where \( \theta \) satisfies the equation \( 3 \sin(\theta + 10^\circ) = 7 \cos(\theta - 10^\circ) \). [5]

5. **June 2009 qu.1**
   - **Fig. 1**
   - **Fig. 2**
   - **Fig. 3**

   Each diagram above shows part of a curve, the equation of which is one of the following:
   \( y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \csc x, \quad y = \cot x \).

   State which equation corresponds to
   (i) Fig. 1, [1]
   (ii) Fig. 2, [1]
   (iii) Fig. 3. [1]

6. **June 2009 qu.3**
   The angles \( \alpha \) and \( \beta \) are such that \( \tan \alpha = m + 2 \) and \( \tan \beta = m \), where \( m \) is a constant.
   (i) Given that \( \sec^2 \alpha - \sec^2 \beta = 16 \), find the value of \( m \). [3]
   (ii) Hence find the exact value of \( \tan(\alpha + \beta) \). [3]
7. **June 2009 qu.7**
   (i) Express $8 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. 
   (ii) Hence
       (a) solve, for $0^\circ < \theta < 360^\circ$, the equation $8 \sin \theta - 6 \cos \theta = 9$,
       (b) find the greatest possible value of $32 \sin x - 24 \cos x - (16 \sin y - 12 \cos y)$ as the angles $x$ and $y$ vary.

8. **Jan 2009 qu.3**
   (i) Express $2 \tan^2 \theta - \frac{1}{\cos \theta}$ in terms of $\sec \theta$.
   (ii) Hence solve, for $0^\circ < \theta < 360^\circ$, the equation $2 \tan^2 \theta - \frac{1}{\cos \theta} = 4$.

9. **Jan 2009 qu.9**
   (i) By first expanding $\cos(2\theta + \theta)$, prove that $\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta$.
   (ii) Hence prove that $\cos 6\theta \equiv 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1$.
   (iii) Show that the only solutions of the equation $1 + \cos 6\theta = 18 \cos^2 \theta$ are odd multiples of $90^\circ$.

10. **June 2008 qu.5**
    (a) Express $\tan 2\alpha$ in terms of $\tan \alpha$ and hence solve, for $0^\circ < \alpha < 180^\circ$, the equation $\tan 2\alpha \tan \alpha = 8$.
    (b) Given that $\beta$ is the acute angle such that $\sin \beta = \frac{6}{7}$, find the exact value of
        (i) $\cosec \beta$,
        (ii) $\cot^2 \beta$.

11. **June 2008 qu.8**
    The expression $T(\theta)$ is defined for $\theta$ in degrees by $T(\theta) = 3\cos(\theta - 60^\circ) + 2\cos(\theta + 60^\circ)$.
    (i) Express $T(\theta)$ in the form $A \sin \theta + B \cos \theta$, giving the exact values of the constants $A$ and $B$.
    (ii) Hence express $T(\theta)$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$.
    (iii) Find the smallest positive value of $\theta$ such that $T(\theta) + 1 = 0$.

12. **Jan 2008 qu.3**
    (a) Solve, for $0^\circ < \alpha < 180^\circ$, the equation $\sec \frac{1}{2}\alpha = 4$.
    (b) Solve, for $0^\circ < \beta < 180^\circ$, the equation $\tan \beta = 7\cot \beta$.

13. **Jan 2008 qu.6**
    The diagram shows the graph of $y = -\sin^{-1}(x - 1)$.
14. \textit{Jan 2008 qu.9}
(i) Give details of the pair of geometrical transformations which transforms the graph of \( y = -\sin^{-1}(x - 1) \) to the graph of \( y = \sin^{-1} x \). [3]  
(ii) Sketch the graph of \( y = -\sin^{-1}(x - 1) \). [2]  
(iii) Find the exact solutions of the equation \( -\sin^{-1}(x - 1) = \frac{1}{2} \pi \). [3]  

15. \textit{June 2007 qu.7}
(i) Sketch the graph of \( y = \sec x \) for \( 0 \leq x \leq 2\pi \). [2]  
(ii) Solve the equation \( \sec x = 3 \) for \( 0 \leq x \leq 2\pi \), giving the roots correct to 3 significant figures. [3]  
(iii) Solve the equation \( \sec \theta = 5 \csc \theta \) for \( 0 \leq \theta \leq 2\pi \), giving the roots correct to 3 s.f. [4]  

16. \textit{June 2007 qu.9}
(i) Prove the identity \( \tan(\theta + 60^\circ) \tan(\theta - 60^\circ) \equiv \frac{\tan^2 \theta - 3}{1 - 3 \tan^2 \theta} \). [4]  
(ii) Solve, for \( 0^\circ < \theta < 180^\circ \), the equation \( \tan(\theta + 60^\circ) \tan(\theta - 60^\circ) = 4 \sec^2 \theta - 3 \), giving your answers correct to the nearest 0.1°. [5]  
(iii) Show that, for all values of the constant \( k \), the equation \( \tan(\theta + 60^\circ) \tan(\theta - 60^\circ) = k \) has two roots in the interval \( 0^\circ < \theta < 180^\circ \). [3]  

17. \textit{Jan 2007 qu.2}
It is given that \( \theta \) is the acute angle such that \( \sin \theta = \frac{12}{13} \). Find the exact value of
(i) \( \cot \theta \). [2]  
(ii) \( \cos 2\theta \). [3]  

18. \textit{Jan 2007 qu.5}
(i) Express \( 4 \cos \theta - \sin \theta \) in the form \( R \cos(\theta + \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). [3]  
(ii) Hence solve the equation \( 4 \cos \theta - \sin \theta = 2 \), giving all solutions for which \( -180^\circ < \theta < 180^\circ \). [5]  

19. \textit{June 2006 qu.5}
(i) Write down the identity expressing \( \sin 2\theta \) in terms of \( \sin \theta \) and \( \cos \theta \). [1]  
(ii) Given that \( \sin \alpha = \frac{1}{4} \) and \( \alpha \) is acute, show that \( \sin 2\alpha = \frac{1}{8} \sqrt{15} \). [3]  
(iii) Solve, for \( 0^\circ < \beta < 90^\circ \), the equation \( 5 \sin 2\beta \sec \beta = 3 \). [3]  

20. \textit{June 2006 qu.8}
(i) Express \( 5 \cos x + 12 \sin x \) in the form \( R \cos(x - \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \). [3]  
(ii) Hence give details of a pair of transformations which transforms the curve \( y = \cos x \) to the curve \( y = 5 \cos x + 12 \sin x \). [3]  
(iii) Solve, for \( 0^\circ < x < 360^\circ \), the equation \( 5 \cos x + 12 \sin x = 2 \), giving your answers correct to the nearest 0.1°. [5]  

21. \textit{Jan 2006 qu.2}
Solve, for \( 0^\circ < \theta < 360^\circ \), the equation \( \sec^2 \theta = 4 \tan \theta - 2 \). [5]
22. **Jan 2006 qu.9**
   (i) By first writing \(\sin 3\theta\) as \(\sin(2\theta + \theta)\), show that \(\sin 3\theta = 3\sin \theta - 4\sin^3 \theta\). \([4]\]
   (ii) Determine the greatest possible value of \(9\sin \left(\frac{10}{3}a\right) - 12\sin^3 \left(\frac{10}{3}a\right)\), and find the smallest positive value of \(a\) (in degrees) for which that greatest value occurs. \([3]\]
   (iii) Solve, for \(0^\circ < \beta < 90^\circ\), the equation \(3\sin 6\beta \csc 2\beta = 4\). \([6]\]

23. **June 2005 qu.5**
   (i) Express \(3\sin \theta + 2\cos \theta\) in the form \(R\sin(\theta + \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). \([3]\]
   (ii) Hence solve the equation \(3\sin \theta + 2\cos \theta = \frac{7}{2}\), giving all solutions for which \(0^\circ < \theta < 360^\circ\). \([5]\]

24. **June 2005 qu.7**
   (i) Write down the formula for \(\cos 2x\) in terms of \(\cos x\). \([1]\]
   (ii) Prove the identity \(\frac{4\cos 2x}{1 + \cos 2x} = 4 - 2\sec^2 x\). \([3]\]
   (iii) Solve, for \(0 < x < 2\pi\), the equation \(\frac{4\cos 2x}{1 + \cos 2x} = 3\tan x - 7\). \([5]\]