1  (i) Show algebraically that the function \( f(x) = \frac{2x}{1 - x^2} \) is odd. \[2\]

Fig. 7 shows the curve \( y = f(x) \) for \( 0 \leq x \leq 4 \), together with the asymptote \( x = 1 \).

(ii) Use the copy of Fig. 7 to complete the curve for \( -4 \leq x \leq 4 \). \[2\]

2  The functions \( f(x) \) and \( g(x) \) are defined as follows.

\[
\begin{align*}
  f(x) &= \ln x, \quad x > 0 \\
  g(x) &= 1 + x^2, \quad x \in \mathbb{R}
\end{align*}
\]

Write down the functions \( fg(x) \) and \( gf(x) \), and state whether these functions are odd, even or neither. \[4\]
3 Each of the graphs of \( y = f(x) \) and \( y = g(x) \) below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for \( f(x) \) and \( g(x) \).

(i)

(ii)
Fig. 4 shows the curve \( y = f(x) \), where \( f(x) = \sqrt{1 - 9x^2}, -a \leq x \leq a \).

(i) Find the value of \( a \).

(ii) Write down the range of \( f(x) \).

(iii) Sketch the curve \( y = f(\frac{1}{3}x) - 1 \).

You are given that \( f(x) \) and \( g(x) \) are odd functions, defined for \( x \in \mathbb{R} \).

(i) Given that \( s(x) = f(x) + g(x) \), prove that \( s(x) \) is an odd function.

(ii) Given that \( p(x) = f(x)g(x) \), determine whether \( p(x) \) is odd, even or neither.

(i) State the algebraic condition for the function \( f(x) \) to be an even function.

What geometrical property does the graph of an even function have?

(ii) State whether the following functions are odd, even or neither.

(A) \( f(x) = x^2 - 3 \)

(B) \( g(x) = \sin x + \cos x \)

(C) \( h(x) = \frac{1}{x + x^3} \)
Fig. 8 shows part of the curve \( y = f(x) \), where \( f(x) = e^{-\frac{1}{3}x} \sin x \), for all \( x \).

(i) Sketch the graphs of

(A) \( y = f(2x) \),

(B) \( y = f(x + \pi) \). [4]

(ii) Show that the \( x \)-coordinate of the turning point \( P \) satisfies the equation \( \tan x = 5 \).

Hence find the coordinates of \( P \). [6]

(iii) Show that \( f(x + \pi) = e^{-\frac{1}{3}\pi} f(x) \). Hence, using the substitution \( u = x - \pi \), show that

\[
\int_{\pi}^{2\pi} f(x) \, dx = e^{-\frac{1}{3}\pi} \int_{0}^{\pi} f(u) \, du.
\]

Interpret this result graphically. [You should not attempt to integrate \( f(x) \).] [8]