The function \( f(x) \) is defined by \( f(x) = \sqrt{4 - x^2} \) for \(-2 \leq x \leq 2\).

(i) Show that the curve \( y = \sqrt{4 - x^2} \) is a semicircle of radius 2, and explain why it is not the whole of this circle. \[3\]

Fig. 9 shows a point \( P(a, b) \) on the semicircle. The tangent at \( P \) is shown.

(ii) (A) Use the gradient of \( OP \) to find the gradient of the tangent at \( P \) in terms of \( a \) and \( b \).

(B) Differentiate \( \sqrt{4 - x^2} \) and deduce the value of \( f'(a) \).

(C) Show that your answers to parts (A) and (B) are equivalent. \[6\]

The function \( g(x) \) is defined by \( g(x) = 3f(x - 2) \), for \( 0 \leq x \leq 4 \).

(iii) Describe a sequence of two transformations that would map the curve \( y = f(x) \) onto the curve \( y = g(x) \).

Hence sketch the curve \( y = g(x) \). \[6\]

(iv) Show that if \( y = g(x) \) then \( 9x^2 + y^2 = 36x \). \[3\]
Fig. 7 shows part of the curve \( y = f(x) \), where \( f(x) = x\sqrt{1+x} \). The curve meets the \( x \)-axis at the origin and at the point \( P \).

\[ \int_{-1}^{0} x\sqrt{1+x} \, dx = \int_{0}^{1} \left( u^2 - u^3 \right) \, du. \]

Hence find the area of the region enclosed by the curve and the \( x \)-axis. [8]

(i) Verify that the point \( P \) has coordinates \((-1, 0)\). Hence state the domain of the function \( f(x) \). [2]

(ii) Show that \( \frac{dy}{dx} = \frac{2 + 3x}{2\sqrt{1+x}} \). [4]

(iii) Find the exact coordinates of the turning point of the curve. Hence write down the range of the function. [4]

(iv) Use the substitution \( u = 1 + x \) to show that

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3 Fig. 7 shows the curve \( y = \frac{x^2}{1 + 2x^3} \). It is undefined at \( x = a \); the line \( x = a \) is a vertical asymptote.

(i) Calculate the value of \( a \), giving your answer correct to 3 significant figures. \[3\]

(ii) Show that \( \frac{dy}{dx} = \frac{2x - 2x^4}{(1 + 2x^3)^2} \). Hence determine the coordinates of the turning points of the curve. \[8\]

(iii) Show that the area of the region between the curve and the \( x \)-axis from \( x = 0 \) to \( x = 1 \) is \( \frac{1}{6} \ln 3 \). \[5\]