1. Fig. 9 shows the curve \( y = \frac{x^2}{3x-1} \).

P is a turning point, and the curve has a vertical asymptote \( x = a \).

(i) Write down the value of \( a \). 

(ii) Show that \(\frac{dy}{dx} = \frac{x(3x - 2)}{(3x - 1)^2} \).

(iii) Find the exact coordinates of the turning point P.

Calculate the gradient of the curve when \( x = 0.6 \) and \( x = 0.8 \), and hence verify that P is a minimum point.

(iv) Using the substitution \( u = 3x - 1 \), show that \( \int \frac{x^2}{3x-1} \, dx = \frac{1}{27} \int \left( u + 2 + \frac{1}{u} \right) \, du \).

Hence find the exact area of the region enclosed by the curve, the x-axis and the lines \( x = \frac{2}{3} \) and \( x = 1 \).
2 Differentiate $\sqrt{1 + 6x^2}$. [4]

3 Show that the curve $y = x^2 \ln x$ has a stationary point when $x = \frac{1}{\sqrt{e}}$. [6]

4 The equation of a curve is $y = \frac{x^2}{2x + 1}$.

   (i) Show that $\frac{dy}{dx} = \frac{2x(x + 1)}{(2x + 1)^2}$. [4]

   (ii) Find the coordinates of the stationary points of the curve. You need not determine their nature. [4]

5 (i) Differentiate $\sqrt{1 + 2x}$.

   (ii) Show that the derivative of $\ln (1 - e^{-x})$ is $\frac{1}{e^x - 1}$. [4]
6 The function \( f(x) = \frac{\sin x}{2 - \cos x} \) has domain \(-\pi \leq x \leq \pi\). 

Fig. 8 shows the graph of \( y = f(x) \) for \( 0 \leq x \leq \pi \).

**Fig. 8**

(i) Find \( f(-x) \) in terms of \( f(x) \). Hence sketch the graph of \( y = f(x) \) for the complete domain \(-\pi \leq x \leq \pi\). [3]

(ii) Show that \( f'(x) = \frac{2\cos x - 1}{(2 - \cos x)^2} \). Hence find the exact coordinates of the turning point P.

State the range of the function \( f(x) \), giving your answer exactly. [8]

(iii) Using the substitution \( u = 2 - \cos x \) or otherwise, find the exact value of \( \int_{0}^{\pi} \frac{\sin x}{2 - \cos x} \, dx \). [4]

(iv) Sketch the graph of \( y = f(2x) \). [1]

(v) Using your answers to parts (iii) and (iv), write down the exact value of \( \int_{0}^{\frac{\pi}{2}} \frac{\sin 2x}{2 - \cos 2x} \, dx \). [2]
Fig. 3 shows the curve defined by the equation $y = \arcsin(x - 1)$, for $0 \leq x \leq 2$.

(i) Find $x$ in terms of $y$, and show that \( \frac{dx}{dy} = \cos y \). \[3\]

(ii) Hence find the exact gradient of the curve at the point where $x = 1.5$. \[4\]

A curve has equation $y = \frac{x}{2 + 3 \ln x}$. Find \( \frac{dy}{dx} \). Hence find the exact coordinates of the stationary point of the curve. \[7\]