Fig. 8 shows part of the curve $y = f(x)$, where
\[ f(x) = (e^x - 1)^2 \] for $x \geq 0$.

(i) Find $f'(x)$, and hence calculate the gradient of the curve $y = f(x)$ at the origin and at the point $(\ln 2, 1)$.

The function $g(x)$ is defined by $\sqrt{x}$ for $x \geq 0$.

(ii) Show that $f(x)$ and $g(x)$ are inverse functions. Hence sketch the graph of $y = g(x)$.

Write down the gradient of the curve $y = g(x)$ at the point $(1, \ln 2)$.

(iii) Show that $\int (e^x - 1)^2 \, dx = \frac{1}{2} e^{2x} - 2e^x + x + c$.

Hence evaluate $\int_{0}^{\ln 2} (e^x - 1)^2 \, dx$, giving your answer in an exact form.

(iv) Using your answer to part (iii), calculate the area of the region enclosed by the curve $y = g(x)$, the $x$-axis and the line $x = 1$. 

[5] [5] [5] [3]
Fig. 6 shows the curve \( y = f(x) \), where \( f(x) = \frac{1}{2} \arctan x \).

\[
\begin{align*}
\text{Fig. 6} \\
\end{align*}
\]

(i) Find the range of the function \( f(x) \), giving your answer in terms of \( \pi \). \hspace{1cm} [2]

(ii) Find the inverse function \( f^{-1}(x) \). Find the gradient of the curve \( y = f^{-1}(x) \) at the origin. \hspace{1cm} [5]

(iii) Hence write down the gradient of \( y = \frac{1}{2} \arctan x \) at the origin. \hspace{1cm} [1]
3 The function \( f(x) = \ln (1 + x^2) \) has domain \(-3 \leq x \leq 3\).

Fig. 9 shows the graph of \( y = f(x) \).

![Graph of y = f(x)](image)

Fig. 9

(i) Show algebraically that the function is even. State how this property relates to the shape of the curve. [3]

(ii) Find the gradient of the curve at the point \( P(2, \ln 5) \). [4]

(iii) Explain why the function does not have an inverse for the domain \(-3 \leq x \leq 3\). [1]

The domain of \( f(x) \) is now restricted to \( 0 \leq x \leq 3 \). The inverse of \( f(x) \) is the function \( g(x) \).

(iv) Sketch the curves \( y = f(x) \) and \( y = g(x) \) on the same axes.

State the domain of the function \( g(x) \).

Show that \( g(x) = \sqrt{e^x - 1} \). [6]

(v) Differentiate \( g(x) \). Hence verify that \( g'(\ln 5) = \frac{1}{2} \). Explain the connection between this result and your answer to part (ii). [5]