Fig. 4 shows the curve $y = f(x)$, where

$$f(x) = a + \cos bx, \ 0 \leq x \leq 2\pi,$$

and $a$ and $b$ are positive constants. The curve has stationary points at $(0, 3)$ and $(2\pi, 1)$.

(i) Find $a$ and $b$. [2]

(ii) Find $f^{-1}(x)$, and state its domain and range. [5]
Fig. 9 shows the line \( y = x \) and the curve \( y = f(x) \), where \( f(x) = \frac{1}{2}(e^x - 1) \). The line and the curve intersect at the origin and at the point \( P(a, a) \).

(i) Show that \( e^a = 1 + 2a \). [1]

(ii) Show that the area of the region enclosed by the curve, the \( x \)-axis and the line \( x = a \) is \( \frac{1}{2}a \). Hence find, in terms of \( a \), the area enclosed by the curve and the line \( y = x \). [6]

(iii) Show that the inverse function of \( f(x) \) is \( g(x) \), where \( g(x) = \ln(1 + 2x) \). Add a sketch of \( y = g(x) \) to the copy of Fig. 9. [5]

(iv) Find the derivatives of \( f(x) \) and \( g(x) \). Hence verify that \( g'(a) = \frac{1}{f'(a)} \).

Give a geometrical interpretation of this result. [7]
The function $f(x)$ is defined by $f(x) = 1 - 2\sin x$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.

(i) Write down the range of the function $f(x)$. [2]

(ii) Find the inverse function $f^{-1}(x)$. [3]

(iii) Find $f'(0)$. Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. [3]
Fig. 6 shows the curve $y = f(x)$, where $f(x) = 2\arcsin x$, $-1 \leq x \leq 1$.

Fig. 6 also shows the curve $y = g(x)$, where $g(x)$ is the inverse function of $f(x)$.

P is the point on the curve $y = f(x)$ with $x$-coordinate $\frac{1}{2}$.

(i) Find the $y$-coordinate of P, giving your answer in terms of $\pi$. [2]

The point Q is the reflection of P in $y = x$.

(ii) Find $g(x)$ and its derivative $g'(x)$. Hence determine the exact gradient of the curve $y = g(x)$ at the point Q.

Write down the exact gradient of $y = f(x)$ at the point P. [6]
Fig. 9 shows the curve \( y = f(x) \). The endpoints of the curve are \( P(-\pi, 1) \) and \( Q(\pi, 3) \), and
\[ f(x) = a + \sin bx, \]
where \( a \) and \( b \) are constants.

(i) Using Fig. 9, show that \( a = 2 \) and \( b = \frac{1}{2} \).  

(ii) Find the gradient of the curve \( y = f(x) \) at the point \((0, 2)\).

Show that there is no point on the curve at which the gradient is greater than this.

(iii) Find \( f^{-1}(x) \), and state its domain and range.

Write down the gradient of \( y = f^{-1}(x) \) at the point \((2, 0)\).

(iv) Find the area enclosed by the curve \( y = f(x) \), the \( x \)-axis, the \( y \)-axis and the line \( x = \pi \).