Fig. 8 shows the curve $y = f(x)$, where $f(x) = (1 - x)e^{2x}$, with its turning point P.

(i) Write down the coordinates of the intercepts of $y = f(x)$ with the $x$- and $y$-axes. [2]

(ii) Find the exact coordinates of the turning point P. [6]

(iii) Show that the exact area of the region enclosed by the curve and the $x$- and $y$-axes is $\frac{1}{2}(e^2 - 3)$. [5]

The function $g(x)$ is defined by $g(x) = 3f\left(\frac{1}{2}x\right)$.

(iv) Express $g(x)$ in terms of $x$.

Sketch the curve $y = g(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the $x$- and $y$-axes and of its turning point. [4]

(v) Write down the exact area of the region enclosed by the curve $y = g(x)$ and the $x$- and $y$-axes. [1]
2 Fig. 9 shows the curve with equation \( y^3 = \frac{x^3}{2x - 1} \). It has an asymptote \( x = a \) and turning point P.

(i) Write down the value of \( a \). 

(ii) Show that \( \frac{dy}{dx} = \frac{4x^3 - 3x^2}{3y^2(2x - 1)^2} \).

Hence find the coordinates of the turning point P, giving the \( y \)-coordinate to 3 significant figures. 

(iii) Show that the substitution \( u = 2x - 1 \) transforms \( \int \frac{x}{\sqrt[3]{2x - 1}} \, dx \) to \( \frac{1}{4} \int (u^2 + u^{-\frac{3}{2}}) \, du \).

Hence find the exact area of the region enclosed by the curve \( y^3 = \frac{x^3}{2x - 1} \), the \( x \)-axis and the lines \( x = 1 \) and \( x = 4.5 \).
Fig. 9 shows the curves $y = f(x)$ and $y = g(x)$. The function $y = f(x)$ is given by

$$f(x) = \ln \left( \frac{2x}{1 + x} \right), \quad x > 0.$$ 

The curve $y = f(x)$ crosses the $x$-axis at P, and the line $x = 2$ at Q.

(i) Verify that the $x$-coordinate of P is 1.

Find the exact $y$-coordinate of Q. [2]

(ii) Find the gradient of the curve at P. [Hint: use $\ln a - \ln b = \ln \frac{a}{b}$.] [4]

The function $g(x)$ is given by

$$g(x) = \frac{e^x}{2 - e^x}, \quad x < \ln 2.$$ 

The curve $y = g(x)$ crosses the $y$-axis at the point R.

(iii) Show that $g(x)$ is the inverse function of $f(x)$.

Write down the gradient of $y = g(x)$ at R. [5]

(iv) Show, using the substitution $u = 2 - e^x$ or otherwise, that

$$\int_0^{\ln \frac{1}{2}} g(x) \, dx = \ln \frac{3}{2}.$$ 

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln \frac{32}{27}$. [Hint: consider its reflection in $y = x$.] [7]