Core 3 Numerical Methods Questions

2 Use Simpson’s rule with 5 ordinates (4 strips) to find an approximation to

\[ \int_{1}^{3} \frac{1}{\sqrt{1 + x^3}} \, dx \]

giving your answer to three significant figures. \(4\) marks

6 [Figure 1, printed on the insert, is provided for use in this question.]

The curve \( y = x^3 + 4x - 3 \) intersects the \( x \)-axis at the point \( A \) where \( x = a \).

(a) Show that \( a \) lies between 0.5 and 1.0. \(2\) marks

(b) Show that the equation \( x^3 + 4x - 3 = 0 \) can be rearranged into the form \( x = \frac{3 - x^3}{4} \). \(1\) mark

(c) (i) Use the iteration \( x_{n+1} = \frac{3 - x_n^3}{4} \) with \( x_1 = 0.5 \) to find \( x_3 \), giving your answer to two decimal places. \(3\) marks

(ii) The sketch on Figure 1 shows parts of the graphs of \( y = \frac{3 - x^3}{4} \) and \( y = x \), and the position of \( x_1 \).

On Figure 1, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of \( x_2 \) and \( x_3 \) on the \( x \)-axis. \(3\) marks

1 The curve \( y = x^3 - x - 7 \) intersects the \( x \)-axis at the point where \( x = a \).

(a) Show that \( a \) lies between 2.0 and 2.1. \(2\) marks

(b) Show that the equation \( x^3 - x - 7 = 0 \) can be rearranged in the form \( x = \sqrt[3]{x + 7} \). \(1\) mark

(c) Use the iteration \( x_{n+1} = \sqrt[3]{x_n + 7} \) with \( x_1 = 2 \) to find the values of \( x_2, x_3 \) and \( x_4 \), giving your answers to three significant figures. \(3\) marks
6  (a) Use the mid-ordinate rule with four strips to find an estimate for \( \int_{1}^{5} \ln x \, dx \), giving your answer to three significant figures. \( (3 \text{ marks}) \)

(c) The region \( R \) is bounded by the curve \( y = \sec x \), the \( x \)-axis and the lines \( x = 0 \) and \( x = 1 \).

Find the volume of the solid formed when \( R \) is rotated through \( 2\pi \) radians about the \( x \)-axis, giving your answer to three significant figures. \( (3 \text{ marks}) \)

1  Use the mid-ordinate rule with four strips of equal width to find an estimate for \( \int_{1}^{5} \frac{1}{1 + \ln x} \, dx \), giving your answer to three significant figures. \( (4 \text{ marks}) \)

(b) The diagram shows the curve with equation \( y = 2\sqrt{(x - 1)^3} \) for \( x \geq 1 \).

The shaded region \( R \) is bounded by the curve \( y = 2\sqrt{(x - 1)^3} \), the lines \( x = 2 \) and \( x = 4 \), and the \( x \)-axis.
Find the exact value of the volume of the solid formed when the region \( R \) is rotated through 360° about the \( x \)-axis. \( \quad (4 \text{ marks}) \)

(c) Describe a sequence of two geometrical transformations that maps the graph of \( y = \sqrt{x^3} \) onto the graph of \( y = 2\sqrt{(x - 1)^3} \). \( \quad (4 \text{ marks}) \)

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4 [Figure 1, printed on the insert, is provided for use in this question.]

(a) Use Simpson’s rule with 5 ordinates (4 strips) to find an approximation to \( \int_{1}^{2} 3^x \, dx \), giving your answer to three significant figures. \( \quad (4 \text{ marks}) \)

(b) The curve \( y = 3^x \) intersects the line \( y = x + 3 \) at the point where \( x = \alpha \).

(i) Show that \( \alpha \) lies between 0.5 and 1.5. \( \quad (2 \text{ marks}) \)

(ii) Show that the equation \( 3^x = x + 3 \) can be rearranged into the form \[ x = \frac{\ln(x + 3)}{\ln 3} \] \( \quad (2 \text{ marks}) \)

(iii) Use the iteration \( x_{n+1} = \frac{\ln(x_n + 3)}{\ln 3} \) with \( x_1 = 0.5 \) to find \( x_3 \) to two significant figures. \( \quad (2 \text{ marks}) \)

(iv) The sketch on Figure 1 shows part of the graphs of \( y = \frac{\ln(x + 3)}{\ln 3} \) and \( y = x \), and the position of \( x_1 \).

On Figure 1, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of \( x_2 \) and \( x_3 \) on the \( x \)-axis. \( \quad (2 \text{ marks}) \)
Core 3 Numerical Methods Answers

2 \[
\int_{1}^{3} \frac{1}{\sqrt{1+x^3}} \, dx
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.707(1)</td>
</tr>
<tr>
<td>1.5</td>
<td>0.478(1)</td>
</tr>
<tr>
<td>2</td>
<td>0.333(3)</td>
</tr>
<tr>
<td>2.5</td>
<td>0.245(3)</td>
</tr>
<tr>
<td>3</td>
<td>0.189(0)</td>
</tr>
</tbody>
</table>

\[ A = \frac{1}{3} \times 0.5 \left[ y(1) + y(3) + \frac{4}{3} (y(1.5) + y(2.5)) + 2(y(2)) \right] \]

\[ = 0.743 \]

Total 4

6(a) \[ f(0.5) = -0.875 \]
\[ f(1) = 2 \]
Change of sign \( \Rightarrow \) root

Total 2

(b) \[ x^3 + 4x - 3 = 0 \]
\[ 4x = 3 - x^3 \]
\[ x = \frac{3 - x^3}{4} \]

Total 1

AG

(c)(i) \[ x_1 = 0.5 \]
\[ x_2 = 0.71875 \]
\[ x_3 = 0.72 \] AWRT
\[ x_4 = 0.66 \]

Total 3

(ii) For cobweb, \( x_1 \) to curve
For \( x_2 \)
For all correct

Total 9
1(a) \[
\begin{align*}
\text{f}(2) &= -1 \\
\text{f}(2.1) &= +0.161 \\
\text{change of sign} \implies 2 < \alpha < 2.1
\end{align*}
\]

\begin{tabular}{|l|c|c|}
\hline
\text{M1} & \text{both attempted} & \\
\hline
\text{A1} & 2 & \\
\hline
\end{tabular}

(b) \[x^3 - x - 7 = 0\]
\[x^3 = x + 7\]
\[x = \sqrt[3]{x + 7}\]

\begin{tabular}{|l|c|c|}
\hline
\text{B1} & \text{AG} & \\
\hline
\end{tabular}

(c) \[x_1 = 2\]
\[x_2 = 2.0801...\]
\[x_3 = 2.0862...\]
\[x_4 = 2.09\]

\begin{tabular}{|l|c|c|}
\hline
\text{M1} & \\
\text{A1} & \text{AWRT 2.08} \\
\text{A1} & \text{AWRT 2.09} \\
\hline
\text{A1} & 3 & \\
\hline
\end{tabular}

\text{Total} \quad 6

6(a) \[\therefore \ln x = 1(\ln 1.5 + \ln 2.5 + \ln 3.5 + \ln 4.5)\]
\[= 4.08\]

\begin{tabular}{|l|c|c|}
\hline
\text{M1} & \text{use of 1.5, 2.5,... ; 3 or 4 correct } x \text{ values} & \\
\text{A1} & \text{AWFW 4 to 4.2} & \\
\text{A1} & 3 & \text{CAO} \\
\hline
\end{tabular}

(c) \[V = (k) \int \sec^2 x \, dx\]
\[= (k) \left[\tan x\right]_0^1\]
\[= 4.89\]

\begin{tabular}{|l|c|c|}
\hline
\text{M1} & \\
\text{A1} & \text{CAO} & \\
\hline
\text{A1} & 3 & \\
\hline
\end{tabular}

1 \[x = 1.5, 2.5, 3.5, 4.5\]
\[\begin{align*}
y_1 &= 0.7115 \\
y_2 &= 0.5218 \\
y_3 &= 0.4439 \\
y_4 &= 0.3993
\end{align*}\]

\[A = 1 \times (y_1 + y_2 + y_3 + y_4) = 2.08\]

\begin{tabular}{|l|c|c|}
\hline
\text{AWRT} & \text{3 correct } y \text{'s} & \\
\text{A1} & \\
\hline
\text{A1} & 4 & \\
\hline
\end{tabular}

\text{Total} \quad 4
8(a) \[ A(-1, \pi) \quad B \left( \frac{\pi}{2} \right) \]

<table>
<thead>
<tr>
<th>B1</th>
<th>2</th>
</tr>
</thead>
</table>

(b) \[ \cos^{-1} x - 3x - 1 = 0 \]
\[ f(0.1) = 0.17 \quad \text{allow 0.2, 0.1} \]
\[ f(0.2) = -0.23 \quad \text{allow -0.2} \]
Change of sign : root

<table>
<thead>
<tr>
<th>B1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>Or comparing ‘sides’</td>
</tr>
<tr>
<td>A1</td>
<td>2</td>
</tr>
</tbody>
</table>

(c) \[ x_1 = 0.1 \]
\[ x_2 = 0.1569 = 0.157 \]
\[ x_3 = 0.1378 = 0.138 \]
\[ x_4 = 0.144 \]

<table>
<thead>
<tr>
<th>M1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>3</td>
</tr>
</tbody>
</table>

Total 7

(b) \[ V = 4 (\pi) \int_2^4 (x-1)^3 \, dx \]
\[ = 4 \pi \left[ \frac{(x-1)^4}{4} \right]_2^4 \]
\[ = \pi (81-1) = 80\pi \]

<table>
<thead>
<tr>
<th>M1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>(\pi) \int y^2 , dx</td>
</tr>
<tr>
<td>M1</td>
<td>[ k(x-1)^4 ] (\pi) or in expanded form</td>
</tr>
<tr>
<td>M1</td>
<td>correct substitution of limits into</td>
</tr>
<tr>
<td>M1</td>
<td>[ k(x-1)^4 ]</td>
</tr>
<tr>
<td>A1</td>
<td>CAO</td>
</tr>
<tr>
<td>OE</td>
<td></td>
</tr>
</tbody>
</table>

(c) Translate
\[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]
Stretch (I) SF 2 (II)
// y axis (III)

<table>
<thead>
<tr>
<th>M1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>4</td>
</tr>
<tr>
<td>B1</td>
<td>OE</td>
</tr>
<tr>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>E1</td>
<td>for I and (II or III)</td>
</tr>
<tr>
<td>M1</td>
<td>for I and II and III</td>
</tr>
</tbody>
</table>

4(a)
\[ \begin{array}{|c|c|c|}
0 & 1 & 3 \\
1 & 1.25 & 3948(2) \\
2 & 1.5 & 6196(2) \\
3 & 1.75 & 838(5) \\
4 & 2 & 9 \\
\end{array} \]
\[ A = \frac{1}{3} \times \frac{1}{4} (3 + 4 \times 3.9482 + 2 \times 5.1962 + 9) \\
= 5.46 \]

<table>
<thead>
<tr>
<th>B1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>x values PI</td>
</tr>
<tr>
<td>M1</td>
<td>(4 + ) y values correct</td>
</tr>
<tr>
<td>A1</td>
<td>CAO</td>
</tr>
</tbody>
</table>

(b)(i) \[ f(x) = 3^x - x - 3 \]
\[ f(0.5) = -1.77 \]
\[ f(1.5) = 0.696 \]
change of sign : root

| M1A1 | 2 |
(ii) \[ 3^x = x + 3 \]
\[ \ln 3^x = \ln (x + 3) \]
\[ x \ln 3 = \ln (x + 3) \]
\[ x = \frac{\ln (x + 3)}{\ln 3} \]
M1 correct use of logs
A1 2 correct with no mistakes; AG

(iii) \( x_1 = 0.5 \)
\( (x_2 = 1.14) \)
\( x_3 = 1.29 = 1.3 \)
M1
A1 2 CAO

(iv) ![Graph](image)
M1 staircase
A1 2 \( x_2, x_3 \) correct and labelled on x-axis

| Total | 12 |