C2 Trigonometry

1. **June 2010 qu. 5**

   The diagram shows two congruent triangles, \( BCD \) and \( BAE \), where \( ABC \) is a straight line. In triangle \( BCD \), \( BD = 8 \text{ cm}, CD = 11 \text{ cm} \) and angle \( CBD = 65^\circ \). The points \( E \) and \( D \) are joined by an arc of a circle with centre \( B \) and radius 8 cm.

   (i) Find angle \( BCD \). \[2\]

   (ii) (a) Show that angle \( EBD \) is 0.873 radians, correct to 3 significant figures. \[2\]

   (b) Hence find the area of the shaded segment bounded by the chord \( ED \) and the arc \( ED \), giving your answer correct to 3 significant figures. \[4\]

2. **June 2010 qu. 7**

   (i) Show that \[2\]

   (ii) Hence solve the equation \( \frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = 5 - \tan x \), for \( 0^\circ \leq x \leq 360^\circ \). \[6\]

3. **Jan 2010 qu. 1**

   (i) Show that the equation \( 2 \sin^2 x = 5 \cos x - 1 \)

   can be expressed in the form \( 2 \cos^2 x + 5 \cos x - 3 = 0 \). \[2\]

   (ii) Hence solve the equation \( 2 \sin^2 x = 5 \cos x - 1 \),

   giving all values of \( x \) between \( 0^\circ \) and \( 360^\circ \). \[4\]

4. **Jan 2010 qu. 7**
The diagram shows triangle $ABC$, with $AB = 10$ cm, $BC = 13$ cm and $CA = 14$ cm. $E$ and $F$ are points on $AB$ and $AC$ respectively such that $AE = AF = 4$ cm. The sector $AEF$ of a circle with centre $A$ is removed to leave the shaded region $EBCF$.

(i) Show that angle $CAB$ is 1.10 radians, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region $EBCF$. [3]

(iii) Find the area of the shaded region $EBCF$. [5]

5. \textbf{June 2009 qu.1}

The lengths of the three sides of a triangle are 6.4 cm, 7.0 cm and 11.3 cm.

(i) Find the largest angle in the triangle. [3]

(ii) Find the area of the triangle. [2]

6. \textbf{June 2009 qu.5}

Solve each of the following equations for $0^\circ \leq x \leq 180^\circ$.

(i) $\sin 2x = 0.5$ [3]

(ii) $2 \sin^2 x = 2 - \sqrt{3} \cos x$ [5]

7. \textbf{June 2009 qu.8}

Fig. 1 shows a sector $AOB$ of a circle, centre $O$ and radius $OA$. The angle $AOB$ is 1.2 radians and the area of the sector is 60 cm$^2$.

(i) Find the perimeter of the sector. [4]

A pattern on a T-shirt, the start of which is shown in Fig. 2, consists of a sequence of similar sectors. The first sector in the pattern is sector $AOB$ from Fig. 1, and the area of each successive sector is $\frac{3}{5}$ of the area of the previous one.
(ii) (a) Find the area of the fifth sector in the pattern. [2]
(b) Find the total area of the first ten sectors in the pattern. [2]
(c) Explain why the total area will never exceed a certain limit, no matter how many sectors are used, and state the value of this limit. [3]

8. Jan 2009 qu.2

The diagram shows a sector $OAB$ of a circle, centre $O$ and radius 7 cm. The angle $AOB$ is $140^\circ$.

(i) Express $140^\circ$ in radians, giving your answer in an exact form as simply as possible. [2]
(ii) Find the perimeter of the segment shaded in the diagram, giving your answer correct to 3 significant figures. [4]

9. Jan 2009 qu.5

Some walkers see a tower, $T$, in the distance and want to know how far away it is. They take a bearing from a point $A$ and then walk for 50m in a straight line before taking another bearing from a point $B$.
They find that angle $TAB$ is $70^\circ$ and angle $TBA$ is $107^\circ$ (see diagram).
(i) Find the distance of the tower from $A$. [2]

(ii) They continue walking in the same direction for another 100m to a point $C$, so that $AC$ is 150 m. What is the distance of the tower from $C$? [3]

(iii) Find the shortest distance of the walkers from the tower as they walk from $A$ to $C$. [2]
10.  
(i) The polynomial \( f(x) \) is defined by \( f(x) = x^3 - x^2 - 3x + 3 \).

Show that \( x = 1 \) is a root of the equation \( f(x) = 0 \), and hence find the other two roots.  

(ii) Hence solve the equation \( \tan^3 x - \tan^2 x - 3 \tan x + 3 = 0 \)

for \( 0 \leq x \leq 2\pi \). Give each solution for \( x \) in an exact form.  

11.  
The diagram shows a sector \( AOB \) of a circle with centre \( O \) and radius 8 cm. The area of the sector is 48 cm\(^2\).

(i) Find angle \( AOB \), giving your answer in radians.  

(ii) Find the area of the segment bounded by the arc \( AB \) and the chord \( AB \).  

12.  
In the diagram, a lifeboat station is at point \( A \). A distress call is received and the lifeboat travels 15 km on a bearing of 030\(^\circ\) to point \( B \). A second call is received and the lifeboat then travels 27 km on a bearing of 110\(^\circ\) to arrive at point \( C \). The lifeboat then travels back to the station at \( A \).

(i) Show that angle \( ABC \) is 100\(^\circ\).  

(ii) Find the distance that the lifeboat has to travel to get from \( C \) back to \( A \).  

(iii) Find the bearing on which the lifeboat has to travel to get from \( C \) to \( A \).  

13.  
(a) (i) Show that the equation \( 2 \sin x \tan x - 5 = \cos x \)

   can be expressed in the form \( 3 \cos^2 x + 5 \cos x - 2 = 0 \).
(ii) Hence solve the equation \(2 \sin x \tan x - 5 = \cos x\), giving all values of \(x\), in radians, for \(0 \leq x \leq 2\pi\). [4]

(b) Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for 
\[\int_0^1 \cos x \, dx, \quad \text{where } x \text{ is in radians.} \]
Give your answer correct to 3 significant figures. [4]

14. Jan 2008 qu.1

The diagram shows a sector \(AOB\) of a circle with centre \(O\) and radius 11 cm. The angle \(AOB\) is 0.7 radians. Find the area of the segment shaded in the diagram. [4]

15. Jan 2008 qu.4

In the diagram, angle \(BDC = 50^\circ\) and angle \(BCD = 62^\circ\). It is given that \(AB = 10\) cm, \(AD = 20\) cm and \(BC = 16\) cm.

(i) Find the length of \(BD\). [2]  
(ii) Find angle \(BAD\). [3]

16. Jan 2008 qu.9

(i) Fig. 1 shows the curve \(y = 2 \sin x\) for values of \(x\) such that \(-180^\circ \leq x \leq 180^\circ\). State the coordinates of the maximum and minimum points on this part of the curve. [2]
(ii)

![Graph showing the curve $y = 2 \sin x$ and the line $y = k$.](image)

Fig. 2 shows the curve $y = 2 \sin x$ and the line $y = k$. The smallest positive solution of the equation $2 \sin x = k$ is denoted by $\alpha$. State, in terms of $\alpha$, and in the range $-180^\circ \leq x \leq 180^\circ$,

(a) another solution of the equation $2 \sin x = k$, \hspace{1cm} [1]
(b) one solution of the equation $2 \sin x = -k$. \hspace{1cm} [1]

(iii) Find the $x$-coordinates of the points where the curve $y = 2 \sin x$ intersects the curve $y = 2 - 3\cos^2 x$, for values of $x$ such that $-180^\circ \leq x \leq 180^\circ$. \hspace{1cm} [6]

17. **June 2007 qu.5**

(i) Show that the equation $3\cos^2 \theta = \sin \theta + 1$ can be expressed in the form $3\sin^2 \theta + \sin \theta - 2 = 0$. \hspace{1cm} [2]

(ii) Hence solve the equation $3\cos^2 \theta = \sin \theta + 1$, giving all values of $\theta$ between $0^\circ$ and $360^\circ$. \hspace{1cm} [5]

18. **June 2007 qu.8**

The diagram shows a triangle $ABC$, where angle $BAC$ is $0.9$ radians. $BAD$ is a sector of the circle with centre $A$ and radius $AB$.

(i) The area of the sector $BAD$ is $16.2$ cm$^2$. Show that the length of $AB$ is $6$ cm. \hspace{1cm} [2]

(ii) The area of triangle $ABC$ is twice the area of sector $BAD$. Find the length of $AC$. \hspace{1cm} [3]

(iii) Find the perimeter of the region $BCD$. \hspace{1cm} [6]
19. **Jan 2007 qu.2**

The diagram shows a sector $OAB$ of a circle, centre $O$ and radius 8 cm. The angle $AOB$ is 46°.

(i) Express 46° in radians, correct to 3 significant figures. [2]

(ii) Find the length of the arc $AB$. [1]

(iii) Find the area of the sector $OAB$. [2]

20. **Jan 2007 qu.4**

In a triangle $ABC$, $AB = 5\sqrt{2}$ cm, $BC = 8$ cm and angle $B = 60°$.

(i) Find the exact area of the triangle, giving your answer as simply as possible. [3]

(ii) Find the length of $AC$, correct to 3 significant figures. [3]

21. **Jan 2007 qu.7**

(i) (a) Sketch the graph of $y = 2 \cos x$ for values of $x$ such that $0° \leq x \leq 360°$, indicating the coordinates of any points where the curve meets the axes. [2]

(b) Solve the equation $2 \cos x = 0.8$, giving all values of $x$ between $0°$ and $360°$. [3]

(ii) Solve the equation $2 \cos x = \sin x$, giving all values of $x$ between $-180°$ and $180°$. [3]

22. **June 2006 qu.5**

Solve each of the following equations, for $0° \leq x \leq 180°$.

(i) $2 \sin^2 x = 1 + \cos x$. [4]

(ii) $\sin 2x = -\cos 2x$. [4]

23. **June 2006 qu.7**

The diagram shows a triangle $ABC$, and a sector $ACD$ of a circle with centre $A$.

It is given that $AB = 11$ cm, $BC = 8$ cm, angle $ABC = 0.8$ radians and angle $DAC = 1.7$ radians.

The shaded segment is bounded by the line $DC$ and the arc $DC$.

(i) Show that the length of $AC$ is 7.90 cm, correct to 3 significant figures. [3]

(ii) Find the area of the shaded segment. [3]

(iii) Find the perimeter of the shaded segment. [4]

24. **Jan 2006 qu.2**

Triangle $ABC$ has $AB = 10$ cm, $BC = 7$ cm and angle $B = 80°$. Calculate

(i) the area of the triangle, [2]

(ii) the length of $CA$, [2]

(iii) the size of angle $C$. [2]
25. **Jan 2006 qu.4**

The diagram shows a sector $OAB$ of a circle with centre $O$. The angle $AOB$ is 1.8 radians. The points $C$ and $D$ lie on $OA$ and $OB$ respectively.
It is given that $OA = OB = 20$ cm and $OC = OD = 15$ cm.
The shaded region is bounded by the arcs $AB$ and $CD$ and by the lines $CA$ and $DB$.

(i) Find the perimeter of the shaded region. [3]

(ii) Find the area of the shaded region. [3]

26. **Jan 2006 qu.9**

(i) Sketch, on a single diagram showing values of $x$ from $-180^\circ$ to $+180^\circ$, the graphs of $y = \tan x$ and $y = 4 \cos x$. [3]

The equation $\tan x = 4 \cos x$

has two roots in the interval $-180^\circ \leq x \leq 180^\circ$. These are denoted by $\alpha$ and $\beta$, where $\alpha < \beta$.

(ii) Show $\alpha$ and $\beta$ on your sketch, and express $\beta$ in terms of $\alpha$. [3]

(iii) Show that the equation $\tan x = 4 \cos x$ may be written as $4 \sin^2 x + \sin x - 4 = 0$.

Hence find the value of $\beta - \alpha$, correct to the nearest degree. [6]

27. **June 2005 qu.2**

A sector $OAB$ of a circle of radius $r$ cm has angle $\theta$ radians. The length of the arc of the sector is 12 cm and the area of the sector is 36 cm$^2$ (see diagram).

(i) Write down two equations involving $r$ and $\theta$. [2]

(ii) Hence show that $r = 6$, and state the value of $\theta$. [2]

(iii) Find the area of the segment bounded by the arc $AB$ and the chord $AB$. [3]
28. \textit{June 2005 qu.4}

In the diagram, $ABCD$ is a quadrilateral in which $AD$ is parallel to $BC$. It is given that $AB = 9$, $BC = 6$, $CA = 5$ and $CD = 15$.

(i) Show that $\cos BCA = -\frac{1}{3}$, and hence find the value of $\sin BCA$. \hfill [4]

(ii) Find the angle $ADC$ correct to the nearest 0.1°. \hfill [4]

29. \textit{June 2005 qu.9}

(a) (i) Write down the exact values of $\cos \frac{1}{6} \pi$ and $\tan \frac{1}{3} \pi$ (where the angles are in radians). Hence verify that $x = \frac{1}{6} \pi$ a solution of the equation $2 \cos x = \tan 2x$. \hfill [3]

(ii) Sketch, on a single diagram, the graphs of $y = 2 \cos x$ and $y = \tan 2x$, for $x$ (radians) such that $0 \leq x \leq \pi$. Hence state, in terms of $\pi$, the other values of $x$ between 0 and $\pi$ satisfying the equation $2 \cos x = \tan 2x$. \hfill [4]

(b) (i) Use the trapezium rule, with 3 strips, to find an approximate value for the area of the region bounded by the curve $y = \tan x$, the $x$-axis, and the lines $x = 0.1$ and $x = 0.4$. (Values of $x$ are in radians.) \hfill [4]

(ii) State with a reason whether this approximation is an underestimate or an overestimate. \hfill [1]