C2 Logs and Indices

1. **June 2010 qu.8**
   (a) Use logarithms to solve the equation $5^{3w-1} = 4^{250}$, giving the value of $w$ correct to 3 s.f. [5]
   (b) Given that $\log_a(5y + 1) - \log_a 3 = 4$, express $y$ in terms of $x$. [4]

2. **Jan 2010 qu.9**
   (i) Sketch the curve $y = 6 \times 5^x$, stating the coordinates of any points of intersection with the axes. [3]
   (ii) The point $P$ on the curve $y = 9^x$ has $y$-coordinate equal to 150. Use logarithms to find the $x$-coordinate of $P$, correct to 3 significant figures. [3]
   (iii) The curves $y = 6 \times 5^x$ and $y = 9^x$ intersect at the point $Q$. Show that the $x$-coordinate of $Q$ can be written as $x = 2 + \log_3 \frac{2}{5}$. [5]

3. **June 2009 qu.3**
   Use logarithms to solve the equation $7^x = 2^{x+1}$, giving the value of $x$ correct to 3 s.f. [5]

4. **June 2009 qu.9**
   (i) Sketch the graph of $y = 4k^x$, where $k$ is a constant such that $k > 1$. State the coordinates of any points of intersection with the axes. [2]
   (ii) The point $P$ on the curve $y = 4k^x$ has its $y$-coordinate equal to $20k^2$. Show that the $x$-coordinate of $P$ may be written as $2 + \log_6 5$. [4]
   (iii) (a) Use the trapezium rule, with two strips each of width $\frac{1}{2}$, to find an expression for the approximate value of $\int_0^1 4k^x \, dx$. [3]
      (b) Given that this approximate value is equal to 16, find the value of $k$. [3]

5. **Jan 2009 qu.8**
   (a) Given that $\log_a x = p$ and $\log_a y = q$, express the following in terms of $p$ and $q$.
      (i) $\log_a (xy)$ [1] (ii) $\log_a \left( \frac{a^2 x^3}{y} \right)$ [3]
   (b) (i) Express $\log_{10}(x^2 - 10) - \log_{10}x$ as a single logarithm. [1]
      (ii) Hence solve the equation $\log_{10}(x^2 - 10) - \log_{10}x = 2 \log_{10}3$. [5]

6. **June 2008 qu.8**
   (i) Sketch the curve $y = 2 \times 3^x$, stating the coordinates of any intersections with the axes. [3]
   (ii) The curve $y = 2 \times 3^x$ intersects the curve $y = 8^x$ at the point $P$. Show that the $x$-coordinate of $P$ may be written as $\frac{1}{3 - \log_2 3}$. [5]
7. **Jan 2008 qu.3**
Express each of the following as a single logarithm:

(i) \( \log_a 2 + \log_a 3 \), \hspace{1cm} [1]
(ii) \( 2 \log_{10} x - 3 \log_{10} y \). \hspace{1cm} [3]

8. **June 2007 qu.3**
Use logarithms to solve the equation \( 3^{2x+1} = 5^{200} \), giving the value of \( x \) correct to 3 significant figures. \hspace{1cm} [5]

8. **June 2007 qu.9**
The polynomial \( f(x) \) is given by \( f(x) = x^3 + 6x^2 + x - 4 \).

(i) (a) Show that \( (x + 1) \) is a factor of \( f(x) \). \hspace{1cm} [1]
(b) Hence find the exact roots of the equation \( f(x) = 0 \). \hspace{1cm} [6]

(ii) (a) Show that the equation \( 2\log_2(x + 3) + \log_2 x - \log_2 (4x + 2) = 1 \)
can be written in the form \( f(x) = 0 \). \hspace{1cm} [5]
(b) Explain why the equation \( 2\log_2(x + 3) + \log_2 x - \log_2 (4x + 2) = 1 \)
has only one real root and state the exact value of this root. \hspace{1cm} [2]

9. **Jan 2007 qu.5**

(a) (i) Express \( \log_3(4x + 7) - \log_3 x \) as a single logarithm. \hspace{1cm} [1]
(ii) Hence solve the equation \( \log_3(4x + 7) - \log_3 x = 2 \). \hspace{1cm} [3]

(b) Use the trapezium rule, with two strips of width 3, to find an approximate value for
\[ \int_3^9 \log_{10} x \, dx \]
giving your answer correct to 3 significant figures. \hspace{1cm} [4]

10. **Jan 2006 qu.7**

(i) Express each of the following in terms of \( \log_{10} x \) and \( \log_{10} y \).

(a) \( \log_{10} \left( \frac{x}{y} \right) \) \hspace{1cm} [1]
(b) \( \log_{10}(10x^2y) \) \hspace{1cm} [3]

(ii) Given that \( 2 \log_{10} \left( \frac{x}{y} \right) = 1 + \log_{10}(10x^2y) \), find the value of \( y \) correct to 3 decimal places. \hspace{1cm} [4]

11. **June 2005 qu.7**

(i) Evaluate \( \log_5 15 + \log_5 20 - \log_5 12 \). \hspace{1cm} [3]

(ii) Given that \( y = 3 \times 10^2x \), show that \( x = a \log_{10}(by) \), where the values of the constants \( a \) and \( b \) are to be found. \hspace{1cm} [4]