1 An arithmetic progression has tenth term 11.1 and fiftieth term 7.1. Find the first term and the common difference. Find also the sum of the first fifty terms of the progression.

2 Jill has 3 daughters and no sons. They are generation 1 of Jill’s descendants.

Each of her daughters has 3 daughters and no sons. Jill’s 9 granddaughters are generation 2 of her descendants. Each of her granddaughter has 3 daughters and no sons; they are descendant generation 3.

Jill decides to investigate what would happen if this pattern continues, with each descendant having 3 daughters and no sons.

(i) How many of Jill’s descendants would there be in generation 8?

(ii) How many of Jill’s descendants would there be altogether in the first 15 generations?

(iii) After $n$ generations, Jill would have over a million descendants altogether. Show that $n$ satisfies the inequality

$$n > \frac{\log_{10} 2000003}{\log_{10} 3} - 1.$$ 

Hence find the least possible value of $n$.

(iv) How many fewer descendants would Jill have altogether in 15 generations if instead of having 3 daughters, she and each subsequent descendant has 2 daughters?

3 (i) Find $\sum_{r=1}^{5} \frac{21}{r + 2}$.

(ii) A sequence is defined by

$$u_1 = a, \text{ where } a \text{ is an unknown constant},$$

$$u_{n+1} = u_n + 5.$$ 

Find, in terms of $a$, the tenth term and the sum of the first ten terms of this sequence.
4. The second term of a geometric progression is 24. The sum to infinity of this progression is 150. Write down two equations in \( a \) and \( r \), where \( a \) is the first term and \( r \) is the common ratio. Solve your equations to find the possible values of \( a \) and \( r \).

5. \( S \) is the sum to infinity of a geometric progression with first term \( a \) and common ratio \( r \).

(i) Another geometric progression has first term 2\( a \) and common ratio \( r \). Express the sum to infinity of this progression in terms of \( S \).

(ii) A third geometric progression has first term \( a \) and common ratio \( r^2 \). Express, in its simplest form, the sum to infinity of this progression in terms of \( S \) and \( r \).

6. Find the second and third terms in the sequence given by
\[
\begin{align*}
    u_1 &= 5, \\
    u_{n+1} &= u_n + 3.
\end{align*}
\]
Find also the sum of the first 50 terms of this sequence.

7. A geometric progression has first term \( a \) and common ratio \( r \). The second term is 6 and the sum to infinity is 25.

(i) Write down two equations in \( a \) and \( r \). Show that one possible value of \( a \) is 10 and find the other possible value of \( a \). Write down the corresponding values of \( r \).

(ii) Show that the ratio of the \( n \)th terms of the two geometric progressions found in part (i) can be written as \( 2^{n-2} : 3^{n-2} \). 

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