1 Given that $a > 0$, state the values of

(i) $\log_a 1$, 

(ii) $\log_a (a^3)^6$, 

(iii) $\log_a \sqrt{a}$. 

2 Use logarithms to solve the equation $235 \times 5^x = 987$, giving your answer correct to 3 decimal places. 

3 Given that $y = a + x^b$, find $\log_{10} x$ in terms of $y$, $a$ and $b$. 

4 Simplify

(i) $\log_{10} x^5 + 3 \log_{10} x^4$, 

(ii) $\log_a 1 - \log_a a^b$. 

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5. The table shows the size of a population of house sparrows from 1980 to 2005.

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</thead>
<tbody>
<tr>
<td>Population</td>
<td>25000</td>
<td>22000</td>
<td>18750</td>
<td>16250</td>
<td>13500</td>
<td>12000</td>
</tr>
</tbody>
</table>

The ‘red alert’ category for birds is used when a population has decreased by at least 50% in the previous 25 years.

(i) Show that the information for this population is consistent with the house sparrow being on red alert in 2005. [1]

The size of the population may be modelled by a function of the form \( P = a \times 10^{-kt} \), where \( P \) is the population, \( t \) is the number of years after 1980, and \( a \) and \( k \) are constants.

(ii) Write the equation \( P = a \times 10^{-kt} \) in logarithmic form using base 10, giving your answer as simply as possible. [2]

(iii) Complete the table and draw the graph of \( \log_{10} P \) against \( t \), drawing a line of best fit by eye. [3]

(iv) Use your graph to find the values of \( a \) and \( k \) and hence the equation for \( P \) in terms of \( t \). [4]

(v) Find the size of the population in 2015 as predicted by this model.

Would the house sparrow still be on red alert? Give a reason for your answer. [3]

6. (i) Sketch the graph of \( y = 3^x \). [2]

(ii) Use logarithms to solve \( 3^{2x+1} = 10 \), giving your answer correct to 2 decimal places. [3]
7 Answer part (ii) of this question on the insert provided.

Since 1945 the populations of many countries have been growing. The table shows the estimated population of 15- to 59-year-olds in Africa during the period 1955 to 2005.

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>Population (millions)</td>
<td>131</td>
<td>161</td>
<td>209</td>
<td>277</td>
<td>372</td>
<td>492</td>
</tr>
</tbody>
</table>

Source: United Nations

Such estimates are used to model future population growth and world needs of resources. One model is $P = a10^{bt}$, where the population is $P$ millions, $t$ is the number of years after 1945 and $a$ and $b$ are constants.

(i) Show that, using this model, the graph of $\log_{10} P$ against $t$ is a straight line of gradient $b$. State the intercept of this line on the vertical axis. [3]

(ii) On the insert, complete the table, giving values correct to 2 decimal places, and plot the graph of $\log_{10} P$ against $t$. Draw, by eye, a line of best fit on your graph. [3]

(iii) Use your graph to find the equation for $P$ in terms of $t$. [4]

(iv) Use your results to estimate the population of 15- to 59-year-olds in Africa in 2050. Comment, with a reason, on the reliability of this estimate. [3]

8 (i) State the value of $\log_{a} a$. [1]

(ii) Express each of the following in terms of $\log_{a} x$.

(A) $\log_{a} x^3 + \log_{a} \sqrt{x}$ [2]

(B) $\log_{a} \frac{1}{x}$ [1]