1. (i) On the same axes, sketch the curves $y = 3^x$ and $y = 3^{2x}$, identifying clearly which is which.  

(ii) Given that $3^{2x} = 729$, find in either order the values of $3^x$ and $x$. 

2. Fig. 8 shows the graph of $\log_{10} y$ against $\log_{10} x$. It is a straight line passing through the points $(2, 8)$ and $(0, 2)$.

![Graph](image)

**Fig. 8**

Find the equation relating $\log_{10} y$ and $\log_{10} x$ and hence find the equation relating $y$ and $x$. 

3. Use logarithms to solve the equation $3^{x+1} = 5^{2x}$. Give your answer correct to 3 decimal places.
4 The thickness of a glacier has been measured every five years from 1960 to 2010. The table shows the reduction in thickness from its measurement in 1960.

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An exponential model may be used for these data, assuming that the relationship between $h$ and $t$ is of the form $h = a \times 10^{bt}$, where $a$ and $b$ are constants to be determined.

(i) Show that this relationship may be expressed in the form $\log_{10} h = mt + c$, stating the values of $m$ and $c$ in terms of $a$ and $b$. [2]

(ii) Complete the table of values in the answer book, giving your answers correct to 2 decimal places, and plot the graph of $\log_{10} h$ against $t$, drawing by eye a line of best fit. [4]

(iii) Use your graph to find $h$ in terms of $t$ for this model. [4]

(iv) Calculate by how much the glacier will reduce in thickness between 2010 and 2020, according to the model. [2]

(v) Give one reason why this model will not be suitable in the long term. [1]

5 A hot drink when first made has a temperature which is 65°C higher than room temperature. The temperature difference, $d$°C, between the drink and its surroundings decreases by 1.7% each minute.

(i) Show that 3 minutes after the drink is made, $d = 61.7$ to 3 significant figures. [2]

(ii) Write down an expression for the value of $d$ at time $n$ minutes after the drink is made, where $n$ is an integer. [1]

(iii) Show that when $d < 3$, $n$ must satisfy the inequality

$$n > \frac{\log_{10} 3 - \log_{10} 65}{\log_{10} 0.983}.$$ 

Hence find the least integer value of $n$ for which $d < 3$. [4]

(iv) The temperature difference at any time $t$ minutes after the drink is made can also be expressed as $d = 65 \times 10^{-kt}$, for some constant $k$. Use the value of $d$ for 1 minute after the drink is made to calculate the value of $k$. Hence find the temperature difference 25.3 minutes after the drink is made. [4]
6. Fig. 6 shows the relationship between $\log_{10} x$ and $\log_{10} y$.

\[ y = 5x \]

Find $y$ in terms of $x$. [5]

7. The graph of $y = ab^x$ passes through the points (1, 6) and (2, 3.6). Find the values of $a$ and $b$. [3]

8. Using logarithms, rearrange $p = st^n$ to make $n$ the subject. [3]

9. You are given that

$$\log_a x = \frac{1}{2} \log_a 16 + \log_a 75 - 2 \log_a 5.$$ 

Find the value of $x$. [3]