Core 2 Integration Questions

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for
\[
\int_{0}^{4} \frac{1}{x^2 + 1} \, dx
\]
giving your answer to four significant figures. \(4 \text{ marks}\)

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. \(1 \text{ mark}\)

6 The diagram shows a sketch of the curve with equation \(y = 27 - 3^x\).

The curve \(y = 27 - 3^x\) intersects the \(y\)-axis at the point \(A\) and the \(x\)-axis at the point \(B\).

(a) (i) Find the \(y\)-coordinate of point \(A\). \(2 \text{ marks}\)

(ii) Verify that the \(x\)-coordinate of point \(B\) is 3. \(1 \text{ mark}\)

(b) The region, \(R\), bounded by the curve \(y = 27 - 3^x\) and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of \(R\). \(4 \text{ marks}\)

(c) (i) Use logarithms to solve the equation \(3^x = 13\), giving your answer to four decimal places. \(3 \text{ marks}\)
(ii) The line \( y = k \) intersects the curve \( y = 27 - 3^x \) at the point where \( 3^x = 13 \). Find the value of \( k \). \( (1 \text{ mark}) \)

(d) (i) Describe the single geometrical transformation by which the curve with equation \( y = -3^x \) can be obtained from the curve \( y = 27 - 3^x \). \( (2 \text{ marks}) \)

(ii) Sketch the curve \( y = -3^x \). \( (2 \text{ marks}) \)

2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

\[
\int_0^3 \sqrt{2x} \, dx
\]

giving your answer to three decimal places. \( (4 \text{ marks}) \)

6 The diagram shows a sketch of the curve with equation \( y = 3(2^x + 1) \).

The curve \( y = 3(2^x + 1) \) intersects the y-axis at the point \( A \).

(a) Find the y-coordinate of the point \( A \). \( (2 \text{ marks}) \)

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

\[
\int_0^6 3(2^x + 1) \, dx
\]

\( (4 \text{ marks}) \)

(c) The line \( y = 21 \) intersects the curve \( y = 3(2^x + 1) \) at the point \( P \).

(i) Show that the x-coordinate of \( P \) satisfies the equation

\[
2^x = 6
\]

\( (1 \text{ mark}) \)

(ii) Use logarithms to find the x-coordinate of \( P \), giving your answer to three significant figures. \( (3 \text{ marks}) \)
## Core 2 integration Answers

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2(a)</td>
<td>$h=1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Integral $= \frac{h}{2} { \ldots }$</td>
<td>B1</td>
<td></td>
<td>PI</td>
</tr>
<tr>
<td></td>
<td>${ \ldots } = f(0) + f(4) + 2[f(1) + f(2) + f(3)]$</td>
<td>M1</td>
<td>OE summing of areas of the four trapezia. $[0.75 + 0.35 + 0.15 + 0.079\ldots]$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= \left[1 + \frac{1}{17} + 2\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)\right]$</td>
<td>A1</td>
<td>Exact or to 3dp values Condone one numerical slip</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Integral $= 1.329$</td>
<td>A1</td>
<td>4</td>
<td>CSO. Must be $1.329$</td>
</tr>
<tr>
<td></td>
<td>Increase the number of ordinates</td>
<td>E1</td>
<td>1</td>
<td>OE</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

---
6(a)(i)  \( y \)-coordinate of \( A \) is \( 27 - 3^2; -26 \)  
When \( x = 3 \), \( y = 27 - 3^2 = 0 \Rightarrow B(3,0) \)

(b) \( h = 1 \)

\[ \text{Area} \approx 2 \{ \ldots \} \]
\[ \{ \ldots \} = f(0) + f(3) + 2[f(1) + f(2)] \]
\[ \{ \ldots \} = 26 + 0 + 2(24 + 18) \]
\( \text{(Area} \approx 55) \)

(c)(i) \( \log_3 3^2 = \log_3 13 \)
\( x \log_3 3 = \log_3 13 \)
\( x = \frac{\log 13}{\log 3} = 2.334717 \ldots \)
\( -2.3347 \) to 4dp

\( \{ x \} = 14 \)

(d)(i) Translation;

\[
\begin{bmatrix}
0 \\
-27
\end{bmatrix}
\]

(ii) Correct shape (translation of given curve vertically downwards)

Total 15

2  \( h = 1 \)
\( f(x) = \sqrt{2^x} \)

\[ \text{Area} \approx h/2 \{ \ldots \} \]
\[ \{ \ldots \} = f(0) + f(3) - 2[f(1) + f(2)] \]
\[ \{ \ldots \} = 1 + 7 + 2(\sqrt{2} + 2) \]
\( \text{(Area} \approx 5.3284 \ldots = 5.328 \) (to 3 dp) \)

Total 4
<table>
<thead>
<tr>
<th>(a)</th>
<th>( y_a = 3 \left(2^x + 1\right) - 6 )</th>
<th>M1</th>
<th>Substituting ( x = 0 ) ( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>( h = 2 ) ( \text{Integral} = h/2 ) [( \ldots \ldots )] ( { \ldots } = f(0) + 2[f(2) + f(4)] + f(6) ) ( { } = 6 + 2[3 \times 5 + 3 \times 17] + 3 \times 65 ) ( = 6 + 2[15 + 51] + 195 ) ( \text{Integral} = 333 )</td>
<td>A1</td>
<td>2</td>
</tr>
<tr>
<td>(c)(i)</td>
<td>( 21 = 3(2^x + 1) \Rightarrow 2^x = 6 )</td>
<td>M1</td>
<td>OE summing of areas of the three traps.</td>
</tr>
<tr>
<td>(ii)</td>
<td>( \log_{10} 2^x = \log_{10} 6 )</td>
<td>A1</td>
<td>Condense 1 numerical slip (# on (a) for ( f(0) ) if not recovered) [Sum of 3 traps. = 21 + 66 + 246]</td>
</tr>
<tr>
<td></td>
<td>( x \log_{10} 2 = \log_{10} 6 )</td>
<td>A1</td>
<td>CAO</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{\log 6}{\log 2} = 2.5849... = 2.38 ) to 3sf</td>
<td>A1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>AG (be convinced)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Take In or ( \log_{10} ) of both sides of ( a^x = b ) or other relevant base if clear. The equation ( a^x = b ) used must be correct. Use of ( \log 2^x = x \log 2 ) OE</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Both method marks must have been awarded.</td>
</tr>
</tbody>
</table>

**Total** 10