Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise A, Question 1

Question:

Find the mid-point of the line joining these pairs of points:

(a) ( 4 , 2 ) , ( 6 , 8 )
(b) ( 0 , 6 ) , ( 12 , 2 )
(c) ( 2 , 2 ) , ( -4 , 6 )
(d) ( -6 , 4 ) , ( 6 , -4 )
(e) ( -5 , 3 ) , ( 7 , 5 )
(f) ( 7 , -4 ) , ( -3 , 6 )
(g) ( -5 , -5 ) , ( -11 , 8 )
(h) ( 6a , 4b ) , ( 2a , -4b )
(i) ( 2p , -q ) , ( 4p , 5q )
(j) ( -2s , -7t ) , ( 5s , t )
(k) ( -4u , 0 ) , ( 3u , -2v )
(l) ( a + b , 2a - b ) , ( 3a - b , -b )
(m) ( 4√2 , 1 ) , ( 2√2 , 7 )
(n) ( -√3 , 3√5 ) , ( 5√3 , 2√5 )
(o) ( √2 - √3 , 3√2 + 4√3 ) , ( 3√2 + √3 , -√2 + 2√3 )

Solution:

(a) \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{4 + 6}{2}, \frac{2 + 8}{2} \right) = \left( \frac{10}{2}, \frac{10}{2} \right) = \left( 5, 5 \right) \]

(b) \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 12}{2}, \frac{6 + 2}{2} \right) = \left( \frac{12}{2}, \frac{8}{2} \right) = \left( 6, 4 \right) \]

(c) \[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{2 + (-4)}{2}, \frac{2 + 6}{2} \right) = \left( \frac{-2}{2}, \frac{8}{2} \right) = \left( -1, 4 \right) \]
(d) \((x_1, y_1) = (-6, 4), (x_2, y_2) = (6, -4)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-6+6}{2}, \frac{4+(-4)}{2}\right) = \left(0, 0\right)\)

(e) \((x_1, y_1) = (-5, 3), (x_2, y_2) = (7, 5)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-5+7}{2}, \frac{3+5}{2}\right) = \left(\frac{2}{2}, \frac{8}{2}\right) = \left(1, 4\right)\)

(f) \((x_1, y_1) = (7, -4), (x_2, y_2) = (-3, 6)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{7+(-3)}{2}, \frac{-4+6}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = \left(2, 1\right)\)

(g) \((x_1, y_1) = (-5, -5), (x_2, y_2) = (11, 8)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-5+(-5)}{2}, \frac{-5+8}{2}\right) = \left(\frac{-10}{2}, \frac{3}{2}\right) = \left(-5, \frac{3}{2}\right)\)

(h) \((x_1, y_1) = (6a, 4b), (x_2, y_2) = (2a, -4b)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{6a+2a}{2}, \frac{4b+(-4b)}{2}\right) = \left(\frac{8a}{2}, \frac{0}{2}\right) = \left(4a, 0\right)\)

(i) \((x_1, y_1) = (2p, -q), (x_2, y_2) = (4p, 5q)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2p+4p}{2}, \frac{-q+5q}{2}\right) = \left(\frac{6p}{2}, \frac{4q}{2}\right) = \left(3p, 2q\right)\)

(j) \((x_1, y_1) = (-2s, -7t), (x_2, y_2) = (5s, t)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2s+5s}{2}, \frac{-7t+t}{2}\right) = \left(\frac{3s}{2}, \frac{-6t}{2}\right) = \left(\frac{3s}{2}, -3t\right)\)

(k) \((x_1, y_1) = (-4u, 0), (x_2, y_2) = (3u, -2v)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-4u+3u}{2}, \frac{0+(-2v)}{2}\right) = \left(\frac{-u}{2}, \frac{-v}{2}\right)\)

(l) \((x_1, y_1) = (a+b, 2a-b), (x_2, y_2) = (3a-b, -b)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{a+b+3a-b}{2}, \frac{2a-b+(-b)}{2}\right) = \left(\frac{4a}{2}, \frac{2a-2b}{2}\right) = \left(2a, a-b\right)\)

(m) \((x_1, y_1) = (4\sqrt{2}, 1), (x_2, y_2) = (2\sqrt{2}, 7)\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{4\sqrt{2}+2\sqrt{2}}{2}, \frac{1+7}{2}\right) = \left(\frac{6\sqrt{2}}{2}, \frac{8}{2}\right) = \left(3\sqrt{2}, 4\right)\)

(n) \((x_1, y_1) = (-\sqrt{3}, 3\sqrt{5}), (x_2, y_2) = (5\sqrt{3}, 2\sqrt{5})\)

So \(\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-\sqrt{3}+5\sqrt{3}}{2}, \frac{3\sqrt{5}+2\sqrt{5}}{2}\right) = \left(\frac{4\sqrt{3}}{2}, \frac{5\sqrt{5}}{2}\right) = \left(2\sqrt{3}, \frac{5\sqrt{5}}{2}\right)\)
(o) \((x_1, y_1) = (\sqrt{2} - \sqrt{3}, 3\sqrt{2} + 4\sqrt{3})\), \((x_2, y_2) = (3\sqrt{2} + \sqrt{3}, -\sqrt{2} + 2\sqrt{3})\)

So \(\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{\sqrt{2} - \sqrt{3} + 3\sqrt{2} + \sqrt{3}}{2}, \frac{3\sqrt{2} + 4\sqrt{3} - \sqrt{2} + 2\sqrt{3}}{2}\right)\)

\[\left(\frac{4\sqrt{2}}{2}, \frac{2\sqrt{2} + 6\sqrt{3}}{2}\right)\]

\[= (2\sqrt{2}, \sqrt{2} + 3\sqrt{3})\]
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Coordinate geometry in the (x,y) plane
Exercise A, Question 2

Question:
The line $PQ$ is a diameter of a circle, where $P$ and $Q$ are $(-4, 6)$ and $(7, 8)$ respectively. Find the coordinates of the centre of the circle.

Solution:

$\left( x_1, y_1 \right) = (-4, 6), \left( x_2, y_2 \right) = (7, 8)$

So $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-4 + 7}{2}, \frac{6 + 8}{2} \right) = \left( \frac{3}{2}, \frac{14}{2} \right) = \left( \frac{3}{2}, 7 \right)$

The centre is $\left( \frac{3}{2}, 7 \right)$.
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Coordinate geometry in the (x,y) plane
Exercise A, Question 3

Question:

The line RS is a diameter of a circle, where R and S are \( \left( \frac{4a}{5}, -\frac{3b}{4} \right) \) and \( \left( \frac{2a}{5}, \frac{5b}{4} \right) \) respectively. Find the coordinates of the centre of the circle.

Solution:

\[
\left( x_1, y_1 \right) = \left( \frac{4a}{5}, -\frac{3b}{4} \right), \quad \left( x_2, y_2 \right) = \left( \frac{2a}{5}, \frac{5b}{4} \right)
\]

So
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{\frac{4a}{5} + \frac{2a}{5}}{2}, \frac{-\frac{3b}{4} + \frac{5b}{4}}{2} \right) = \left( \frac{\frac{6a}{5}}{2}, \frac{\frac{2b}{4}}{2} \right) = \left( \frac{3a}{5}, \frac{b}{4} \right)
\]

The centre is \( \left( \frac{3a}{5}, \frac{b}{4} \right) \).

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Question:

The line $AB$ is a diameter of a circle, where $A$ and $B$ are $(-3, -4)$ and $(6, 10)$ respectively. Show that the centre of the circle lies on the line $y = 2x$.

Solution:

$\left( x_1 , y_1 \right) = (-3, -4)$, $\left( x_2 , y_2 \right) = (6, 10)$

So: $\left( \frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left( \frac{-3 + 6}{2} , \frac{-4 + 10}{2} \right) = \left( \frac{3}{2} , 3 \right)$

Substitute $x = \frac{3}{2}$ into $y = 2x$:

$y = 2 \left( \frac{3}{2} \right) = 3 \checkmark$

So the centre is on the line $y = 2x$. 

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**Coordinate geometry in the (x,y) plane**  
**Exercise A, Question 5**

**Question:**

The line \(JK\) is a diameter of a circle, where \(J\) and \(K\) are \(\left(\frac{3}{4}, \frac{4}{3}\right)\) and \(\left(-\frac{1}{2}, 2\right)\) respectively. Show that the centre of the circle lies on the line \(y = 8x + \frac{2}{3}\).

**Solution:**

\[
\begin{align*}
\left( x_1, y_1 \right) &= \left( \frac{3}{4}, \frac{4}{3} \right), \quad \left( x_2, y_2 \right) &= \left( -\frac{1}{2}, 2 \right) \\
\text{So } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left( \frac{\frac{3}{4} + (-\frac{1}{2})}{2}, \frac{\frac{4}{3} + 2}{2} \right) = \left( \frac{1}{4}, \frac{10}{3} \right) \\
\text{Substitute } x = \frac{1}{8} \text{ into } y = 8x + \frac{2}{3}. \\
y &= 8 \left( \frac{1}{8} \right) + \frac{2}{3} = 1 + \frac{2}{3} = \frac{5}{3} \checkmark
\end{align*}
\]

So the centre is on the line \(y = 8x + \frac{2}{3}\).
Question:

The line AB is a diameter of a circle, where A and B are \((0, -2)\) and \((6, -5)\) respectively. Show that the centre of the circle lies on the line \(x - 2y - 10 = 0\).

Solution:

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 6}{2}, \frac{-2 + (-5)}{2} \right) = \left( \frac{6}{2}, \frac{-7}{2} \right) = \left( 3, \frac{-7}{2} \right)
\]

Substitute \(x = 3\) and \(y = \frac{-7}{2}\) into \(x - 2y - 10 = 0\):

\[
3 - 2 \left( \frac{-7}{2} \right) - 10 = 3 + 7 - 10 = 0 \checkmark
\]

So the centre is on the line \(x - 2y - 10 = 0\).
Question:
The line $FG$ is a diameter of the circle centre $(6, 1)$. Given $F$ is $(2, -3)$, find the coordinates of $G$.

Solution:

The centre is $(6, 1)$ so

$$
\left( \frac{a+2}{2}, \frac{b+(-3)}{2} \right) = \left( 6, 1 \right)
$$

$$
\frac{a+2}{2} = 6
$$

$$
a + 2 = 12
a = 10
$$

$$
\frac{b+(-3)}{2} = 1
$$

$$
b - 3 = 2
b = 5
$$

The coordinates of $G$ are $(10, 5)$.
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Coordinate geometry in the (x,y) plane
Exercise A, Question 8

Question:
The line $CD$ is a diameter of the circle centre $( -2a , 5a )$. Given $D$ has coordinates $(3a , -7a )$, find the coordinates of $C$.

Solution:

The centre is $( -2a , 5a )$ so

\[
\left( \frac{p + 3a}{2} , \frac{q + (-7a)}{2} \right) = ( -2a , 5a )
\]

\[
p + 3a = -2a
\]
\[
p = -4a
\]
\[
q + (-7a) = 5a
\]
\[
q = 17a
\]

The coordinates of $C$ are $( -7a , 17a )$. 

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Coordinate geometry in the (x,y) plane
Exercise A, Question 9

Question:
The points $M(3, p)$ and $N(q, 4)$ lie on the circle centre $(5, 6)$. The line $MN$ is a diameter of the circle. Find the value of $p$ and $q$.

Solution:

\[
\left( \frac{3+q}{2}, \frac{p+4}{2} \right) = (5, 6)
\]

\[
\frac{3+q}{2} = 5 \\
3 + q = 10 \\
q = 7
\]

\[
\frac{p+4}{2} = 6 \\
p + 4 = 12 \\
p = 8
\]

So $p = 8, q = 7$

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Question:

The points \( V(-4, 2a) \) and \( W(3b, -4) \) lie on the circle centre \((b, 2a)\). The line \( VW \) is a diameter of the circle. Find the value of \( a \) and \( b \).

Solution:

\[(x_1, y_1) = (-4, 2a), \ (x_2, y_2) = (3b, -4) \] so

\[
\left(\frac{-4 + 3b}{2}, \frac{2a - 4}{2}\right) = \left(b, 2a\right)
\]

\[
\frac{-4 + 3b}{2} = b
\]

\[-4 + 3b = 2b
\]

\[-4 = -b
\]

\[b = 4
\]

\[
\frac{2a - 4}{2} = 2a
\]

\[2a - 4 = 4a
\]

\[-4 = 2a
\]

\[a = -2
\]

So \( a = -2, b = 4 \)

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Coordinate geometry in the (x,y) plane
Exercise B, Question 1

Question:

The line $FG$ is a diameter of the circle centre $C$, where $F$ and $G$ are $(-2, 5)$ and $(2, 9)$ respectively. The line $l$ passes through $C$ and is perpendicular to $FG$. Find the equation of $l$.

Solution:

(1)

(2) The gradient of $FG$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{2 - (-2)} = \frac{4}{4} = 1$$

(3) The gradient of a line perpendicular to $FG$ is $\frac{-1}{1} = -1$.

(4) $C$ is the mid-point of $FG$, so the coordinates of $C$ are

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-2 + 2}{2}, \frac{5 + 9}{2} \right) = \left( \frac{0}{2}, \frac{14}{2} \right) = \left( 0, 7 \right)$$

(5) The equation of $l$ is

$$y - y_1 = m (x - x_1)$$

$$y - 7 = -1 (x - 0)$$

$$y = -x + 7$$

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Coordinate geometry in the (x,y) plane
Exercise B, Question 2

Question:

The line \( JK \) is a diameter of the circle centre \( P \), where \( J \) and \( K \) are \( (0, -3) \) and \( (4, -5) \) respectively. The line \( l \) passes through \( P \) and is perpendicular to \( JK \). Find the equation of \( l \). Write your answer in the form \( ax + by + c = 0 \), where \( a \), \( b \) and \( c \) are integers.

Solution:

(1)

(2) The gradient of \( JK \) is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-3)}{4 - 0} = \frac{-2}{4} = \frac{-1}{2}
\]

(3) The gradient of a line perpendicular to \( JK \) is \( \frac{-1}{\frac{-1}{2}} = 2 \)

(4) \( P \) is the mid-point of \( JK \), so the coordinates of \( P \) are
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 4}{2}, \frac{-3 + (-5)}{2} \right) = \left( 2, -4 \right)
\]

(5) The equation of \( l \) is
\[
y - y_1 = m ( x - x_1 )
\]
\[
y - (-4) = 2 ( x - 2 )
\]
\[
y + 4 = 2x - 4
\]
\[
0 = 2x - y - 4 - 4
\]
\[
2x - y - 8 = 0
\]

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Coordinate geometry in the (x,y) plane
Exercise B, Question 3

Question:

The line $AB$ is a diameter of the circle centre $(4, -2)$. The line $l$ passes through $B$ and is perpendicular to $AB$. Given that $A$ is $(-2, 6)$,

(a) find the coordinates of $B$.

(b) Hence, find the equation of $l$.

Solution:

(1)

Let the coordinates of $B$ be $(a, b)$.

$(4, -2)$ is the mid-point of $AB$ so

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( 4, -2 \right)
\]

i.e.

\[
\left( \frac{-2 + a}{2}, \frac{6 + b}{2} \right) = \left( 4, -2 \right)
\]

So

\[
\frac{-2 + a}{2} = 4
\]

\[-2 + a = 8\]

\[a = 10\]

and

\[
\frac{6 + b}{2} = -2
\]

\[6 + b = -4\]

\[b = -10\]
(a) The coordinates of $B$ are $(10, -10)$.

(3) Using $(-2, 6)$ and $(4, -2)$, the gradient of $AB$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 6}{4 - (-2)} = \frac{-8}{6} = -\frac{4}{3}$$

(4) The gradient of a line perpendicular to $AB$ is $\frac{-1}{-\frac{4}{3}} = \frac{3}{4}$.

(5) The equation of $l$ is

$$y - y_1 = m(x - x_1)$$

$$y = \left(-10\right) = \frac{3}{4} \left(x - 10\right)$$

$$y + 10 = \frac{3x}{4} - \frac{30}{4}$$

$$y = \frac{3x}{4} - \frac{30}{4} - 10$$

$$y = \frac{3x}{4} - \frac{70}{4}$$

$$y = \frac{3x}{4} - \frac{35}{2}$$

(b) The equation of $l$ is $y = \frac{3}{4}x - \frac{35}{2}$.

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Coordinate geometry in the (x,y) plane
Exercise B, Question 4

Question:

The line $PQ$ is a diameter of the circle centre $(-4, -2)$. The line $l$ passes through $P$ and is perpendicular to $PQ$. Given that $Q$ is $(10, 4)$, find the equation of $l$.

Solution:

(1) 

(2) Let the coordinates of $P$ be $(a, b)$. 

$(-4, -2)$ is the mid-point of $PQ$ so

$\left(\frac{10 + a}{2}, \frac{4 + b}{2}\right) = (-4, -2)$

\[
\frac{10 + a}{2} = -4 \\
10 + a = -8 \\
a = -18
\]

\[
\frac{4 + b}{2} = -2 \\
4 + b = -4 \\
b = -8
\]

The coordinates of $P$ are $(-18, -8)$.

(3) The gradient of $PQ$ is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{10 - (-4)} = \frac{6}{14} = \frac{3}{7}
\]

(4) The gradient of a line perpendicular to $PQ$ is $\frac{-1}{\left(\frac{3}{7}\right)} = \frac{-7}{3}$.

(5) The equation of $l$ is

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\[ y - y_1 = m \left( x - x_1 \right) \]
\[ y - \left( -8 \right) = \frac{-7}{3} \left[ x - \left( -18 \right) \right] \]
\[ y + 8 = \frac{-7}{3} \left( x + 18 \right) \]
\[ y + 8 = \frac{-7}{3} x - 42 \]
\[ y = \frac{-7}{3} x - 50 \]
The line RS is a chord of the circle centre \((5, -2)\), where \(R\) and \(S\) are \((2, 3)\) and \((10, 1)\) respectively. The line \(l\) is perpendicular to RS and bisects it. Show that \(l\) passes through the centre of the circle.

**Solution:**

(1) The gradient of RS is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{10 - 2} = \frac{-2}{8} = \frac{-1}{4}
\]

(2) The gradient of a line perpendicular to RS is \(-\frac{1}{\frac{-1}{4}} = 4\).

(3) The mid-point of RS is
\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 10}{2}, \frac{3 + 1}{2}\right) = \left(\frac{12}{2}, \frac{4}{2}\right) = \left(6, 2\right)
\]

(4) The equation of \(l\) is
\[
y - y_1 = m (x - x_1)
\]
\[
y - 2 = 4 (x - 6)
\]
\[
y = 4x - 22
\]

(5) Substitute \(x = 5\) into \(y = 4x - 22\):
\[
y = 4 (5) - 22 = 20 - 22 = -2
\]
So \(l\) passes through the centre of the circle.
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Coordinate geometry in the (x,y) plane
Exercise B, Question 6

Question:

The line MN is a chord of the circle centre \( \left( 1, -\frac{1}{2} \right) \), where M and N are \( (-5, -5) \) and \( (7, 4) \) respectively. The line \( l \) is perpendicular to MN and bisects it. Find the equation of \( l \). Write your answer in the form \( ax + by + c = 0 \), where \( a, b \) and \( c \) are integers.

Solution:

(1) The gradient of MN is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-5)}{7 - (-5)} = \frac{4 + 5}{7 + 5} = \frac{9}{12} = \frac{3}{4}
\]

(2) The gradient of a line perpendicular to MN is \( \frac{-1}{\frac{3}{4}} = \frac{-4}{3} \).

(3) The coordinates of the mid-point of MN are
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-5 + 7}{2}, \frac{-5 + 4}{2} \right) = \left( \frac{2}{2}, \frac{-1}{2} \right) = \left( 1, -\frac{1}{2} \right)
\]

(4) The equation of \( l \) is
\[
y - y_1 = m \left( x - x_1 \right)
\]
\[
y - \left( \frac{-1}{2} \right) = \frac{-4}{3} \left( x - 1 \right)
\]
\[
y + \frac{1}{2} = \frac{-4}{3} \left( x - 1 \right)
\]
\[
y + \frac{1}{2} = \frac{-4}{3} x + \frac{4}{3}
\]
\[
(\times 6) \quad 6y + 3 = -8x + 8
\]
\[
8x + 6y + 3 = 8
\]
\[
8x + 6y - 5 = 0
\]

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The lines $AB$ and $CD$ are chords of a circle. The line $y = 2x + 8$ is the perpendicular bisector of $AB$. The line $y = -2x - 4$ is the perpendicular bisector of $CD$. Find the coordinates of the centre of the circle.

**Solution:**

\[
\begin{align*}
y &= 2x + 8 \\
y &= -2x - 4 \\
\frac{2y}{2} &= 4 \\
y &= 2 \\
\text{Substitute } y &= 2 \text{ into } y = 2x + 8: \\
2 &= 2x + 8 \\
-6 &= 2x \\
x &= -3 \\
\text{Check.} \\
\text{Substitute } x &= -3 \text{ and } y &= 2 \text{ into } y = -2x - 4: \\
(2) &= -2(-3) - 4 \\
2 &= 6 - 4 \\
2 &= 2 \checkmark \\
\end{align*}
\]

The coordinates of the centre of the circle are $( -3, 2 )$. 

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Coordinate geometry in the (x,y) plane
Exercise B, Question 8

Question:

The lines $EF$ and $GH$ are chords of a circle. The line $y = 3x - 24$ is the perpendicular bisector of $EF$. Given $G$ and $F$ are $(-2, 4)$ and $(4, 10)$ respectively, find the coordinates of the centre of the circle.

Solution:

(1) The gradient of $GF$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{4 - (-2)} = \frac{6}{6} = 1$$

(2) The gradient of a line perpendicular to $GF$ is $-\frac{1}{1} = -1$.

(3) The mid-point of $GF$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 4}{2}, \frac{4 + 10}{2}\right) = \left(\frac{2}{2}, \frac{14}{2}\right) = (1, 7)$$

(4) The equation of the perpendicular bisector is

$$y - y_1 = m(x - x_1)$$
$$y - 7 = -1(x - 1)$$
$$y = -x + 8$$

(5) Solving $y = -x + 8$ and $y = 3x - 24$ simultaneously:

$$-x + 8 = 3x - 24$$
$$-4x = -32$$
$$x = 8$$

Substitute $x = 8$ into $y = -x + 8$:

$$y = -(8) + 8$$
$$y = 0$$

So the centre of the circle is $(8, 0)$.
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Coordinate geometry in the (x,y) plane
Exercise B, Question 9

Question:
The points $P(3, 16)$, $Q(11, 12)$ and $R(-7, 6)$ lie on the circumference of a circle.

(a) Find the equation of the perpendicular bisector of
(i) $PQ$
(ii) $PR$.

(b) Hence, find the coordinates of the centre of the circle.

Solution:

(a) (i) The gradient $PQ$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 16}{11 - 3} = \frac{-4}{8} = -\frac{1}{2}$$

The gradient of a line perpendicular to $PQ$ is $\frac{1}{2}$.

The mid-point of $PQ$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 11}{2}, \frac{16 + 12}{2}\right) = \left(7, 14\right)$$

The equation of the perpendicular bisector of $PQ$ is

$$y - y_1 = m (x - x_1)$$

$$y - 14 = \frac{1}{2} (x - 7)$$

$$y = \frac{1}{2} x - \frac{7}{2} + 14$$

$$y = \frac{1}{2} x + \frac{21}{2}$$

(ii) The gradient of $PR$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 16}{-7 - 3} = \frac{-10}{-10} = 1$$

The gradient of a line perpendicular to $PR$ is $-1$.

The mid-point of $PR$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 - 7}{2}, \frac{16 + 6}{2}\right) = \left(-2, 11\right)$$

The equation of the perpendicular bisector of $PR$ is

$$y - y_1 = m (x - x_1)$$

$$y - 11 = -1 (x - (-2))$$

$$y = -x + 9$$

(b) Solving $y = 2x$ and $y = -x + 9$ simultaneously:

$$2x = -x + 9$$

$$3x = 9$$

$$x = 3$$

Substitute $x = 3$ in $y = 2x$:

$$y = 2 (3)$$
Check.

Substitute \( x = 3 \) and \( y = 6 \) into \( y = -x + 9 \):

\[
(6) = -(3) + 9
\]

\[
6 = -3 + 9
\]

\[
6 = 6 \checkmark
\]

The coordinates of the centre are \((3, 6)\).
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Coordinate geometry in the (x,y) plane
Exercise B, Question 10

Question:

The points \( A (-3, 19) \), \( B (9, 11) \) and \( C (-15, 1) \) lie on the circumference of a circle. Find the coordinates of the centre of the circle.

Solution:

(1) The gradient of \( AB \) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 19}{9 - (-3)} = \frac{-8}{12} = -\frac{2}{3}
\]

The gradient of a line perpendicular to \( AB \) is \( \frac{-1}{-\frac{2}{3}} = \frac{3}{2} \).

The mid-point of \( AB \) is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-3 + 9}{2}, \frac{19 + 11}{2} \right) = \left( \frac{6}{2}, \frac{30}{2} \right) = \left( 3, 15 \right)
\]

The equation of the perpendicular bisector of \( AB \) is

\[
y - y_1 = m \left( x - x_1 \right)
\]

\[
y - 15 = \frac{3}{2} \left( x - 3 \right)
\]

\[
y - 15 = \frac{3}{2}x - \frac{9}{2}
\]

\[
y = \frac{3}{2}x + \frac{21}{2}
\]

(2) The gradient of \( BC \) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 11}{-15 - 9} = \frac{-10}{-24} = \frac{5}{12}
\]

The gradient of a line perpendicular to \( BC \) is \( \frac{-1}{\frac{5}{12}} = \frac{-12}{5} \).

The mid-point of \( BC \) is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{9 + (-15)}{2}, \frac{11 + 1}{2} \right) = \left( \frac{-6}{2}, \frac{12}{2} \right) = \left( -3, 6 \right)
\]

The equation of the perpendicular bisector of \( BC \) is

\[
y - y_1 = m \left( x - x_1 \right)
\]

\[
y - 6 = \frac{-12}{5} \left[ x - \left( -3 \right) \right]
\]
\[ y - 6 = \frac{-12}{5} \left( x + 3 \right) \]
\[ y - 6 = \frac{-12}{5} x - \frac{36}{5} \]
\[ y = \frac{-12}{5} x - \frac{6}{5} \]

(3) Solving \( y = \frac{-12}{5} x - \frac{6}{5} \) and \( y = \frac{3}{2} x + \frac{21}{2} \) simultaneously:

\[ \frac{3}{2} x + \frac{21}{2} = -\frac{12}{5} x - \frac{6}{5} \]
\[ \frac{3}{2} x + \frac{12}{5} x = -\frac{6}{5} - \frac{21}{2} \]
\[ \frac{39}{10} x = -\frac{117}{10} \]
\[ 39x = -117 \]
\[ x = -3 \]

Substitute \( x = -3 \) into \( y = \frac{3}{2} x + \frac{21}{2} \):

\[ y = \frac{3}{2} \left( -3 \right) + \frac{21}{2} \]
\[ y = \frac{-9}{2} + \frac{21}{2} \]
\[ y = \frac{12}{2} \]
\[ y = 6 \]

Check.

Substitute \( x = -3 \) and \( y = 6 \) into \( y = \frac{-12}{5} x - \frac{6}{5} \):

\[ \left( 6 \right) = \frac{-12}{5} \left( -3 \right) - \frac{6}{5} \]
\[ 6 = \frac{36}{5} - \frac{6}{5} \]
\[ 6 = \frac{30}{5} \]
\[ 6 = 6 \]

The centre of the circle is \( ( -3 , 6 ) \)
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Edexcel Modular Mathematics for AS and A-Level
Coordinate geometry in the (x,y) plane
Exercise C, Question 1

Question:

Find the distance between these pairs of points:

(a) (0, 1), (6, 9)
(b) (4, -6), (9, 6)
(c) (3, 1), (-1, 4)
(d) (3, 5), (4, 7)
(e) (2, 9), (4, 3)
(f) (0, -4), (5, 5)
(g) (-2, -7), (5, 1)
(h) (-4a, 0), (3a, -2a)
(i) (-b, 4b), (-4b, -2b)
(j) (2c, c), (6c, 4c)
(k) (-4d, d), (2d, -4d)
(l) (-e, -e), (-3e, -5e)
(m) (3√2, 6√2), (2√2, 4√2)
(n) (-√3, 2√3), (3√3, 5√3)
(o) (2√3 - √2, 5 + √3), (4√3 - √2, 3√5 + √3)

Solution:

(a) \((x_1, y_1) = (0, 1), (x_2, y_2) = (6, 9)\)
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((6 - 0)^2 + (9 - 1)^2} = \sqrt{36 + 64} = \sqrt{100} = 10
\]

(b) \((x_1, y_1) = (4, -6), (x_2, y_2) = (9, 6)\)
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{((9 - 4)^2 + [6 - (-6)]^2} = \sqrt{25 + 144} = \sqrt{169} = 13
\]
(c) \((x_1, y_1) = (3, 1), (x_2, y_2) = (-1, 4)\)

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 3)^2 + (4 - 1)^2} = \sqrt{16 + 9} = \sqrt{25} = 5
\]

(d) \((x_1, y_1) = (3, 5), (x_2, y_2) = (4, 7)\)

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 3)^2 + (7 - 5)^2} = \sqrt{1 + 4} = \sqrt{5}
\]

(e) \((x_1, y_1) = (2, 9), (x_2, y_2) = (4, 3)\)

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 2)^2 + (3 - 9)^2} = \sqrt{4 + 36} = \sqrt{40} = 2\sqrt{10}
\]

(f) \((x_1, y_1) = (0, -4), (x_2, y_2) = (5, 5)\)

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - 0)^2 + [5 - (-4)]^2} = \sqrt{25 + 81} = \sqrt{106}
\]

(g) \((x_1, y_1) = (-2, -7), (x_2, y_2) = (5, 1)\)

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - (-2))^2 + [1 - (-7)]^2} = \sqrt{7^2 + 8^2} = \sqrt{113}
\]

(h) \((x_1, y_1) = (-4a, 0), (x_2, y_2) = (3a, -2a)\)

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3a - (-4a))^2 + (-2a - 0)^2} = \sqrt{53a^2} = a\sqrt{53}
\]

(i) \((x_1, y_1) = (-b, 4b), (x_2, y_2) = (4b, -2b)\)

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4b - (-b))^2 + (-2b - 4b)^2} = \sqrt{45b^2}
\]
= \sqrt{9 \times 5 \times b^2}
= \sqrt{9 \sqrt{5} b^2}
= 3b \sqrt{5}

(j) \, (x_1, y_1) = (2e, c), \, (x_2, y_2) = (6e, 4c)
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{(6e - 2c)^2 + (4c - c)^2}
= \sqrt{(4c)^2 + (3c)^2}
= \sqrt{16c^2 + 9c^2}
= \sqrt{25c^2}
= 5c

(k) \, (x_1, y_1) = (-4d, d), \, (x_2, y_2) = (2d, -4d)
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{(2d - (-4d))^2 + (-4d - d)^2}
= \sqrt{(2d + 4d)^2 + (-5d)^2}
= \sqrt{(6d)^2 + (-5d)^2}
= \sqrt{36d^2 + 25d^2}
= \sqrt{61d^2}
= d\sqrt{61}

(l) \, (x_1, y_1) = (-e, -e), \, (x_2, y_2) = (-3e, -5e)
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{(-3e - (-e))^2 + (-5e - (-e))^2}
= \sqrt{(-3e + e)^2 + (-5e + e)^2}
= \sqrt{(-2e)^2 + (-4e)^2}
= \sqrt{4e^2 + 16e^2}
= \sqrt{20e^2}
= \sqrt{4 \times 5 \times e^2}
= \sqrt{4 \times \sqrt{5} \times e^2}
= 2\sqrt{5}e

(m) \, (x_1, y_1) = (3\sqrt{2}, 6\sqrt{2}), \, (x_2, y_2) = (2\sqrt{2}, 4\sqrt{2})
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{(2\sqrt{2} - 3\sqrt{2})^2 + (4\sqrt{2} - 6\sqrt{2})^2}
= \sqrt{(-\sqrt{2})^2 + (-2\sqrt{2})^2}
= \sqrt{10}

(n) \, (x_1, y_1) = (-\sqrt{3}, 2\sqrt{3}), \, (x_2, y_2) = (3\sqrt{3}, 5\sqrt{3})
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{(3\sqrt{3} - (-\sqrt{3}))^2 + (5\sqrt{3} - 2\sqrt{3})^2}
= \sqrt{(3\sqrt{3} + \sqrt{3})^2 + (3\sqrt{3})^2}
= \sqrt{(4\sqrt{3})^2 + (3\sqrt{3})^2}
= \sqrt{48 + 27}
= \sqrt{75}
= \sqrt{25 \times 3}
= \sqrt{25} \times \sqrt{3}
= 5\sqrt{3}

(o) \, (x_1, y_1) = (2\sqrt{3} - \sqrt{2}, \sqrt{5} + \sqrt{3}), \, (x_2, y_2) = (4\sqrt{3} - \sqrt{2}, 3\sqrt{5} + \sqrt{3})
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{(4\sqrt{3} - \sqrt{2} - (2\sqrt{3} - \sqrt{2}))^2 + (3\sqrt{5} + \sqrt{3} - (\sqrt{5} + \sqrt{3}))^2}
= \sqrt{(2\sqrt{3})^2 + (2\sqrt{5})^2
\[ \sqrt{12 + 20} \]
\[ = \sqrt{32} \]
\[ = \sqrt{16 \times 2} \]
\[ = \sqrt{16} \times \sqrt{2} \]
\[ = 4 \sqrt{2} \]
Question:

The point \((4, -3)\) lies on the circle centre \((-2, 5)\). Find the radius of the circle.

Solution:

\[
(x_1, y_1) = (4, -3), (x_2, y_2) = (-2, 5)
\]

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(-2 - 4)^2 + [5 - (-3)]^2}
\]

\[
= \sqrt{36 + 64}
\]

\[
= \sqrt{100}
\]

\[
= 10
\]

Radius of circle = 10.
The point \((14, 9)\) is the centre of the circle radius 25. Show that \((-10, 2)\) lies on the circle.

Solution:

\[(x_1, y_1) = (-10, 2), (x_2, y_2) = (14, 9)\]

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(14 - (-10))^2 + (9 - 2)^2}
\]

\[
= \sqrt{24^2 + 7^2}
\]

\[
= \sqrt{576 + 49}
\]

\[
= \sqrt{625}
\]

\[
= 25
\]

So \((-10, 2)\) is on the circle.
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Coordinate geometry in the (x,y) plane
Exercise C, Question 4

Question:

The line MN is a diameter of a circle, where M and N are (6, -4) and (0, -2) respectively. Find the radius of the circle.

Solution:

\[(x_1, y_1) = (6, -4), (x_2, y_2) = (0, -2)\]
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 6)^2 + [-2 - (-4)]^2}
\]
\[
= \sqrt{(-6)^2 + (2)^2}
\]
\[
= \sqrt{36 + 4}
\]
\[
= \sqrt{40}
\]
\[
= \sqrt{4 \times 10}
\]
\[
= \sqrt{4} \times \sqrt{10}
\]
\[
= 2 \sqrt{10}
\]

The diameter has length \(2 \sqrt{10}\).

So the radius has length \(\frac{2 \sqrt{10}}{2} = \sqrt{10}\).
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Coordinate geometry in the (x,y) plane  
Exercise C, Question 5  

Question:  
The line $QR$ is a diameter of the circle centre $C$, where $Q$ and $R$ have coordinates $(11, 12)$ and $(-5, 0)$ respectively. The point $P$ is $(13, 6)$.

(a) Find the coordinates of $C$.  
(b) Show that $P$ lies on the circle.

Solution:  

(a) The mid-point of $QR$ is  
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{11 + (-5)}{2}, \frac{12 + 0}{2} \right) = \left( \frac{16}{2}, \frac{12}{2} \right) = \left( 8, 6 \right)
\]

(b) The radius of the circle is  
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(11 - 3)^2 + (12 - 6)^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10
\]

The distance between $C$ and $P$ is  
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(13 - 3)^2 + (6 - 6)^2} = \sqrt{10^2 + 0^2} = \sqrt{10^2} = 10
\]

So $P$ is on the circle.
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Coordinate geometry in the (x,y) plane
Exercise C, Question 6

Question:
The points \((-3, 19)\), \((-15, 1)\) and \((9, 1)\) are vertices of a triangle. Show that a circle centre \((-3, 6)\) can be drawn through the vertices of the triangle.

Solution:

(1) \((x_1, y_1) = (-3, 6), (x_2, y_2) = (-3, 19)\)
\[\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-3+3)^2+(19-6)^2} = \sqrt{13^2} = 13\]

(2) \((x_1, y_1) = (-3, 6), (x_2, y_2) = (-15, 1)\)
\[\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-15+3)^2+(1-6)^2} = \sqrt{144+25} = \sqrt{169} = 13\]

(3) \((x_1, y_1) = (-3, 6), (x_2, y_2) = (9, 1)\)
\[\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(9+3)^2+(1-6)^2} = \sqrt{144+25} = \sqrt{169} = 13\]

The distance of each vertex of the triangle to \((-3, 6)\) is 13. So a circle centre \((-3, 6)\) and radius 13 can be drawn through the vertices of the triangle.

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Question:

The line $ST$ is a diameter of the circle $c_1$, where $S$ and $T$ are $(5, 3)$ and $(-3, 7)$ respectively. The line $UV$ is a diameter of the circle $c_2$ centre $(4, 4)$. The point $U$ is $(1, 8)$.

(a) Find the radius of (i) $c_1$ (ii) $c_2$.

(b) Find the distance between the centres of $c_1$ and $c_2$.

Solution:

(a) (i) The centre of $c_1$ is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{5 + (-3)}{2}, \frac{3 + 7}{2} \right) = \left( \frac{2}{2}, \frac{10}{2} \right) = \left( 1, 5 \right)
\]

The radius of $c_1$ is

\[
\sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2} = \sqrt{\left( 5 - 1 \right)^2 + \left( 3 - 5 \right)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}
\]

(ii) The radius of $c_2$ is

\[
\sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2} = \sqrt{\left( 4 - 1 \right)^2 + \left( 4 - 8 \right)^2} = \sqrt{9 + 16} = \sqrt{25} = 5
\]

(b) The distance between the centres is

\[
\sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2} = \sqrt{\left( 1 - 4 \right)^2 + \left( 5 - 4 \right)^2} = \sqrt{9 + 1} = \sqrt{10}
\]
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Coordinate geometry in the (x,y) plane
Exercise C, Question 8

Question:
The points $U (-2, 8), V (7, 7)$ and $W (-3, -1)$ lie on a circle.

(a) Show that $\triangle UVW$ has a right angle.

(b) Find the coordinates of the centre of the circle.

Solution:

(a) (1) The distance $UV$ is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(7-(-2))^2+(7-8)^2}$$

$$= \sqrt{(7+2)^2+(-1)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10}$$

(2) The distance $VW$ is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-3-7)^2+(-8)^2}$$

$$= \sqrt{100+64}$$

$$= \sqrt{164}$$

(3) The distance $UW$ is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-3-(-2))^2+(-1-8)^2}$$

$$= \sqrt{(-1)^2+(-9)^2}$$

$$= \sqrt{1+81}$$

$$= \sqrt{82}$$

Now $(\sqrt{82})^2 + (\sqrt{82})^2 = (\sqrt{164})^2$

(b) The angle in a semicircle is a right angle. So $VW$ is a diameter of the circle. The mid-point of $VW$ is...
The centre of the circle is \((2, 3)\).
The points $A(2, 6), B(5, 7)$ and $C(8, -2)$ lie on a circle.

(a) Show that $\triangle ABC$ has a right angle.

(b) Find the area of the triangle.

Solution:

(a) (1) The distance $AB$ is

\[
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(5-2)^2 + (7-6)^2} = \sqrt{3^2 + 1^2} = \sqrt{9 + 1} = \sqrt{10}
\]

(2) The distance $BC$ is

\[
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(8-5)^2 + (-2-7)^2} = \sqrt{3^2 + (-9)^2} = \sqrt{9 + 81} = \sqrt{90}
\]

(3) The distance $AC$ is

\[
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(8-2)^2 + (-2-6)^2} = \sqrt{6^2 + (-8)^2} = \sqrt{36 + 64} = \sqrt{100}
\]

Now $(\sqrt{10})^2 + (\sqrt{90})^2 = (\sqrt{100})^2$

i.e. $AB^2 + BC^2 = AC^2$

So, by Pythagoras’ theorem, there is a right angle at $B$.

(b) The area of the triangle is

\[
\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AB \times BC = \frac{1}{2} \times \sqrt{10} \times \sqrt{90} = \frac{1}{2} \sqrt{900} = \frac{1}{2} \times 30 = 15
\]
The points $A(-1, 9)$, $B(6, 10)$, $C(7, 3)$ and $D(0, 2)$ lie on a circle.

(a) Show that $ABCD$ is a square.

(b) Find the area of $ABCD$.

(c) Find the centre of the circle.

Solution:

(a) (1) The length of $AB$ is
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(6-(-1))^2+(10-9)^2}$$
$$= \sqrt{7^2+1^2}$$
$$= \sqrt{49+1}$$
$$= \sqrt{50}$$

(2) The length of $BC$ is
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(7-6)^2+(3-10)^2}$$
$$= \sqrt{1^2+(-7)^2}$$
$$= \sqrt{1+49}$$
$$= \sqrt{50}$$

(3) The length of $CD$ is
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(0-7)^2+(2-3)^2}$$
$$= \sqrt{(-7)^2+(-1)^2}$$
$$= \sqrt{49+1}$$
$$= \sqrt{50}$$

(4) The length of $DA$ is
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(0-7)^2+(2-3)^2}$$
$$= \sqrt{(-7)^2+(-1)^2}$$
$$= \sqrt{49+1}$$
$$= \sqrt{50}$$

The sides of the quadrilateral are equal.

(5) The gradient of $AB$ is
$$\frac{y_2-y_1}{x_2-x_1} = \frac{10-9}{6-(-1)} = \frac{1}{7}$$

The gradient of $BC$ is
$$\frac{y_2-y_1}{x_2-x_1} = \frac{3-10}{7-6} = \frac{-7}{1} = -7$$

The product of the gradients $= -1 \cdot \left( \frac{1}{7} \times -7 = -1 \right)$.

So the line $AB$ is perpendicular to $BC$.

So the quadrilateral $ABCD$ is a square.

(b) The area $= \sqrt{50} \times \sqrt{50} = 50$
(c) The mid-point of $AC$ is
\[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + 7}{2}, \frac{9 + 3}{2}\right) = \left(\frac{6}{2}, \frac{12}{2}\right) = \left(3, 6\right)
\]
So the centre of the circle is $(3, 6)$.
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Edexcel Modular Mathematics for AS and A-Level
Coordinate geometry in the (x,y) plane
Exercise D, Question 1

Question:
Write down the equation of these circles:

(a) Centre (3, 2), radius 4
(b) Centre (−4, 5), radius 6
(c) Centre (5, −6), radius 2√3
(d) Centre (2a, 7a), radius 5a
(e) Centre (−2√2, −3√2), radius 1

Solution:

(a) \((x_1, y_1) = (3, 2), r = 4\)
So \((x - 3)^2 + (y - 2)^2 = 4^2\)
or \((x - 3)^2 + (y - 2)^2 = 16\)

(b) \((x_1, y_1) = (−4, 5), r = 6\)
So \([x - (−4)]^2 + (y - 5)^2 = 6^2\)
or \((x + 4)^2 + (y - 5)^2 = 36\)

(c) \((x_1, y_1) = (5, −6), r = 2\sqrt{3}\)
So \((x - 5)^2 + [y - (−6)]^2 = (2\sqrt{3})^2\)
\((x - 5)^2 + (y + 6)^2 = 2^2(\sqrt{3})^2\)
\((x - 5)^2 + (y + 6)^2 = 4 \times 3\)
\((x - 5)^2 + (y + 6)^2 = 12\)

(d) \((x_1, y_1) = (2a, 7a), r = 5a\)
So \((x - 2a)^2 + (y - 7a)^2 = (5a)^2\)
or \((x - 2a)^2 + (y - 7a)^2 = 25a^2\)

(e) \((x_1, y_1) = (−2\sqrt{2}, −3\sqrt{2}), r = 1\)
So \([x - (−2\sqrt{2})]^2 + [y - (−3\sqrt{2})]^2 = 1^2\)
or \((x + 2\sqrt{2})^2 + (y + 3\sqrt{2})^2 = 1\)

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Solution to the question:

Write down the coordinates of the centre and the radius of these circles:

(a) \((x + 5)^2 + (y - 4)^2 = 9^2\)

(b) \((x - 7)^2 + (y - 1)^2 = 16\)

(c) \((x + 4)^2 + y^2 = 25\)

(d) \((x + 4a)^2 + (y + a)^2 = 144a^2\)

(e) \((x - 3\sqrt{5})^2 + (y + \sqrt{5})^2 = 27\)

### Solution:

(a) \((x + 5)^2 + (y - 4)^2 = 9^2\)

or \([x - (-5)]^2 + (y - 4)^2 = 9^2\)

The centre of the circle is \((-5, 4)\) and the radius is 9.

(b) \((x - 7)^2 + (y - 1)^2 = 16\)

or \((x - 7)^2 + (y - 1)^2 = 4^2\)

The centre of the circle is \((7, 1)\) and the radius is 4.

(c) \((x + 4)^2 + y^2 = 25\)

or \([x - (-4)]^2 + (y - 0)^2 = 5^2\)

The centre of the circle is \((-4, 0)\) and the radius is 5.

(d) \((x + 4a)^2 + (y + a)^2 = 144a^2\)

or \([x - (-4a)]^2 + [y - (-a)]^2 = (12a)^2\)

The centre of the circle is \((-4a, -a)\) and the radius is 12a.

(e) \((x - 3\sqrt{5})^2 + (y + \sqrt{5})^2 = 27\)

or \((x - 3\sqrt{5})^2 + [y - (-\sqrt{5})]^2 = (\sqrt{27})^2\)

Now \(\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}\)

The centre of the circle is \((3\sqrt{5}, -\sqrt{5})\) and the radius is \(3\sqrt{3}\).
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Coordinate geometry in the \((x,y)\) plane
Exercise D, Question 3

Question:

Find the centre and radius of these circles by first writing in the form \((x-a)^2 + (y-b)^2 = r^2\)

(a) \(x^2 + y^2 + 4x + 9y + 3 = 0\)

(b) \(x^2 + y^2 + 5x - 3y - 8 = 0\)

(c) \(2x^2 + 2y^2 + 8x + 15y - 1 = 0\)

(d) \(2x^2 + 2y^2 - 8x + 8y + 3 = 0\)

Solution:

(a) \(x^2 + y^2 + 4x + 9y + 3 = 0\)

\[x^2 + 4x + y^2 + 9y = -3\]

\[\left(x + 2\right)^2 - 4 + \left(y + \frac{9}{2}\right)^2 = \frac{81}{4} = -3\]

\[\left(x + 2\right)^2 + \left(y + \frac{9}{2}\right)^2 = \frac{85}{4}\]

So the centre is \((-2, -4.5)\) and the radius is 4.61 (2 d.p.)

(b) \(x^2 + y^2 + 5x - 3y - 8 = 0\)

\[x^2 + 5x + y^2 - 3y = 8\]

\[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4} + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4} = 8\]

\[\left(x + \frac{5}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = 16.5\]

So the centre is \((-2.5, 1.5)\) and the radius is 4.06 (2 d.p.)

(c) \(2x^2 + 2y^2 + 8x + 15y - 1 = 0\)

\[x^2 + y^2 + 4x + \frac{15}{2}y = \frac{1}{2}\]

\[x^2 + 4x + y^2 + \frac{15}{2}y = \frac{1}{2}\]

\[\left(x + 2\right)^2 - 4 + \left(y + \frac{15}{4}\right)^2 = \frac{225}{16} = \frac{1}{2}\]

\[\left(x + 2\right)^2 + \left(y + \frac{15}{4}\right)^2 = 18 \frac{9}{16}\]

So the centre is \((-2, -3.75)\) and the radius is 4.31 (2 d.p.)

(d) \(2x^2 + 2y^2 - 8x + 8y + 3 = 0\)

\[x^2 + y^2 - 4x + 4y + \frac{3}{2} = 0\]
\[ x^2 - 4x + y^2 + 4y = - \frac{3}{2} \]

\[ (x - 2)^2 - 4 + (y + 2)^2 - 4 = - \frac{3}{2} \]

\[ (x - 2)^2 + (y + 2)^2 = \frac{13}{2} \]

So the centre is \((2, -2)\) and the radius is 2.55 (2 d.p.)
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Coordinate geometry in the (x,y) plane
Exercise D, Question 4

Question:
In each case, show that the circle passes through the given point:

(a) \((x - 2)^2 + (y - 5)^2 = 13\), \((4, 8)\)
(b) \((x + 7)^2 + (y - 2)^2 = 65\), \((0, -2)\)
(c) \(x^2 + y^2 = 25^2\), \((7, -24)\)
(d) \((x - 2a)^2 + (y + 5a)^2 = 20a^2\), \((6a, -3a)\)
(e) \((x - 3\sqrt{5})^2 + (y - \sqrt{5})^2 = (2\sqrt{10})^2\), \((\sqrt{5}, -\sqrt{5})\)

Solution:

(a) Substitute \(x = 4, y = 8\) into \((x - 2)^2 + (y - 5)^2 = 13\)
\((x - 2)^2 + (y - 5)^2 = (4 - 2)^2 + (8 - 5)^2 = 2^2 + 3^2 = 4 + 9 = 13\) ✓
So the circle passes through \((4, 8)\).

(b) Substitute \(x = 0, y = -2\) into \((x + 7)^2 + (y - 2)^2 = 65\)
\((x + 7)^2 + (y - 2)^2 = (0 + 7)^2 + (-2 - 2)^2 = 7^2 + (-4)^2 = 49 + 16 = 65\)
So the circle passes through \((0, -2)\).

(c) Substitute \(x = 7\) and \(y = -24\) into \(x^2 + y^2 = 25^2\)
\(x^2 + y^2 = 7^2 + (-24)^2 = 49 + 576 = 625 = 25^2\)
So the circle passes through \((7, -24)\).

(d) Substitute \(x = 6a, y = -3a\) into \((x - 2a)^2 + (y + 5a)^2 = 20a^2\)
\((x - 2a)^2 + (y + 5a)^2 = (6a - 2a)^2 + (-3a + 5a)^2 = (4a)^2 + (2a)^2 = 16a^2 + 4a^2 = 20a^2\)
So the circle passes through \((6a, -3a)\).

(e) Substitute \(x = \sqrt{5}, y = -\sqrt{5}\) into \((x - 3\sqrt{5})^2 + (y - \sqrt{5})^2 = (2\sqrt{10})^2\)
\((x - 3\sqrt{5})^2 + (y - \sqrt{5})^2 = (\sqrt{5} - 3\sqrt{5})^2 + (-\sqrt{5} - \sqrt{5})^2 = (-2\sqrt{5})^2 + (-2\sqrt{5})^2 = 4 \times 5 + 4 \times 5 = 20 + 20 = 40 = (\sqrt{40})^2\)
Now \(\sqrt{40} = \sqrt{4 \times 10} = \sqrt{4} \times \sqrt{10} = 2 \sqrt{10}\)
So the circle passes through \((\sqrt{5}, -\sqrt{5})\).
The point \((4, -2)\) lies on the circle centre \((8, 1)\). Find the equation of the circle.

Solution:

The radius of the circle is

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
\sqrt{(8 - 4)^2 + [1 - (-2)]^2}
\]

\[
\sqrt{16 + 9}
\]

\[
= \sqrt{25}
\]

\[
= 5
\]

The centre of the circle is \((8, 1)\) and the radius is 5.

So \((x - 8)^2 + (y - 1)^2 = 5^2\)

or \((x - 8)^2 + (y - 1)^2 = 25\)
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Coordinate geometry in the (x,y) plane
Exercise D, Question 6

Question:
The line $PQ$ is the diameter of the circle, where $P$ and $Q$ are $(5, 6)$ and $(-2, 2)$ respectively. Find the equation of the circle.

Solution:

(1) The centre of the circle is
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{5 + (-2)}{2}, \frac{6 + 2}{2} \right) = \left( \frac{3}{2}, \frac{8}{2} \right) = \left( \frac{3}{2}, 4 \right)
\]

(2) The radius of the circle is
\[
\sqrt{\left( x_2 - x_1 \right)^2 + \left( y_2 - y_1 \right)^2}
\]
\[
= \sqrt{\left( 5 - \frac{3}{2} \right)^2 + \left( 6 - 4 \right)^2}
\]
\[
= \sqrt{\left( \frac{7}{2} \right)^2 + (2)^2}
\]
\[
= \sqrt{\frac{49}{4} + 4}
\]
\[
= \sqrt{\frac{49}{4} + \frac{16}{4}}
\]
\[
= \sqrt{\frac{65}{4}}
\]

So the equation of the circle is
\[
\left( x - \frac{3}{2} \right)^2 + (y - 4)^2 = \left( \sqrt{\frac{65}{4}} \right)^2
\]
or
\[
\left( x - \frac{3}{2} \right)^2 + (y - 4)^2 = \frac{65}{4}
\]
Question:

The point \((1, -3)\) lies on the circle \((x - 3)^2 + (y + 4)^2 = r^2\). Find the value of \(r\).

Solution:

Substitute \(x = 1\), \(y = -3\) into \((x - 3)^2 + (y + 4)^2 = r^2\)

\[(1 - 3)^2 + (-3 + 4)^2 = r^2\]

\[(-2)^2 + (1)^2 = r^2\]

\[4 + 1 = r^2\]

\[5 = r^2\]

So \(r = \sqrt{5}\)
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Coordinate geometry in the (x,y) plane
Exercise D, Question 8

Question:
The line $y = 2x + 13$ touches the circle $x^2 + (y - 3)^2 = 20$ at $(-4, 5)$. Show that the radius at $(-4, 5)$ is perpendicular to the line.

Solution:

1. The centre of the circle $x^2 + (y - 3)^2 = 20$ is $(0, 3)$.
2. The gradient of the line joining $(0, 3)$ and $(-4, 5)$ is
   \[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{-4 - 0} = \frac{2}{-4} = -\frac{1}{2} \]
3. The gradient of $y = 2x + 13$ is 2.
4. The product of the gradients is
   \[ -\frac{1}{2} \times 2 = -1 \]

So the radius is perpendicular to the line.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 9

Question:

The line \( x + 3y - 11 = 0 \) touches the circle \((x + 1)^2 + (y + 6)^2 = 90\) at \((2, 3)\).

(a) Find the radius of the circle.

(b) Show that the radius at \((2, 3)\) is perpendicular to the line.

Solution:

(a) The radius of the circle \((x + 1)^2 + (y + 6)^2 = 90\) is \(\sqrt{90}\).

\[
\sqrt{90} = \sqrt{9 \times 10} = \sqrt{9} \times \sqrt{10} = 3\sqrt{10}
\]

(b) (1) The centre of the circle \((x + 1)^2 + (y + 6)^2 = 90\) is \((-1, -6)\).

(2) The gradient of the line joining \((-1, -6)\) and \((2, 3)\) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-6)}{2 - (-1)} = \frac{3 + 6}{2 + 1} = \frac{9}{3} = 3
\]

(3) Rearrange \(x + 3y - 11 = 0\) into the form \(y = mx + c\)

\[
x + 3y - 11 = 0
\]

\[
3y = -x + 11
\]

\[
y = -\frac{1}{3}x + \frac{11}{3}
\]

So the gradient of \(x + 3y - 11 = 0\) is \(-\frac{1}{3}\).

(4) The product of the gradients is

\[
3 \times -\frac{1}{3} = -1
\]

So the radius is perpendicular to the line.

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Question:
The point \( P (1, -2) \) lies on the circle centre \( (4, 6) \).

(a) Find the equation of the circle.

(b) Find the equation of the tangent to the circle at \( P \).

Solution:

(a) (1) The radius of the circle is

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 1)^2 + (6 - (-2))^2} = \sqrt{3^2 + 8^2} = \sqrt{9 + 64} = \sqrt{73}
\]

(2) The equation of the circle is

\[
(x - 4)^2 + (y - 6)^2 = (\sqrt{73})^2
\]

or \((x - 4)^2 + (y - 6)^2 = 73\)

(b) (1) The gradient of the line joining \( (1, -2) \) and \( (4, 6) \) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{4 - 1} = \frac{6 + 2}{3} = \frac{8}{3}
\]

(2) The gradient of the tangent is \(-\frac{1}{\frac{8}{3}} = -\frac{3}{8}\).

(3) The equation of the tangent to the circle at \( (1, -2) \) is

\[
y - y_1 = m (x - x_1)
\]

\[
y - (-2) = -\frac{3}{8} (x - 1)
\]

\[
y + 2 = -\frac{3}{8} (x - 1)
\]

\[
8y + 16 = -3 (x - 1)
\]

\[
8y + 16 = -3x + 3
\]

\[
3x + 8y + 16 = 3
\]

\[
3x + 8y + 13 = 0
\]
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Coordinate geometry in the (x,y) plane
Exercise E, Question 1

Question:

Find where the circle \((x - 1)^2 + (y - 3)^2 = 45\) meets the x-axis.

Solution:

Substitute \(y = 0\) into \((x - 1)^2 + (y - 3)^2 = 45\)
\((x - 1)^2 + (3)^2 = 45\)
\((x - 1)^2 + 9 = 45\)
\((x - 1)^2 = 36\)
\(x - 1 = \pm \sqrt{36}\)
\(x - 1 = \pm 6\)
So \(x - 1 = 6 \Rightarrow x = 7\)
and \(x - 1 = -6 \Rightarrow x = -5\)
The circle meets the x-axis at \((7, 0)\) and \((-5, 0)\).
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Coordinate geometry in the (x,y) plane
Exercise E, Question 2

Question:
Find where the circle \((x - 2)^2 + (y + 3)^2 = 29\) meets the y-axis.

Solution:

Substitute \(x = 0\) into \((x - 2)^2 + (y + 3)^2 = 29\)
\((-2)^2 + (y + 3)^2 = 29\)
\(4 + (y + 3)^2 = 29\)
\((y + 3)^2 = 25\)
\(y + 3 = \pm \sqrt{25}\)
\(y + 3 = \pm 5\)

So \(y + 3 = 5 \Rightarrow y = 2\)
and \(y + 3 = -5 \Rightarrow y = -8\)
The circle meets the y-axis at \((0, 2)\) and \((0, -8)\).
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Coordinate geometry in the (x,y) plane
Exercise E, Question 3

Question:

The circle \((x - 3)^2 + (y + 3)^2 = 34\) meets the x-axis at \((a, 0)\) and the y-axis at \((0, b)\). Find the possible values of \(a\) and \(b\).

Solution:

(1) Substitute \(x = a, y = 0\) into \((x - 3)^2 + (y + 3)^2 = 34\)

\[(a - 3)^2 + (3)^2 = 34\]
\[(a - 3)^2 + 9 = 34\]
\[(a - 3)^2 = 25\]
\[a - 3 = \pm \sqrt{25}\]
\[a - 3 = \pm 5\]

So \(a - 3 = 5 \Rightarrow a = 8\)
and \(a - 3 = -5 \Rightarrow a = -2\)
The circle meets the x-axis at \((8, 0)\) and \((-2, 0)\).

(2) Substitute \(x = 0, y = b\) into \((x - 3)^2 + (y + 3)^2 = 34\)

\[(-3)^2 + (b + 3)^2 = 34\]
\[9 + (b + 3)^2 = 34\]
\[(b + 3)^2 = 25\]
\[b + 3 = \pm \sqrt{25}\]
\[b + 3 = \pm 5\]

So \(b + 3 = 5 \Rightarrow b = 2\)
and \(b + 3 = -5 \Rightarrow b = -8\)
The circle meets the y-axis at \((0, 2)\) and \((0, -8)\).
The line \( y = x + 4 \) meets the circle \((x - 3)^2 + (y - 5)^2 = 34\) at \(A\) and \(B\). Find the coordinates of \(A\) and \(B\).

**Solution:**

Substitute \(y = x + 4\) into \((x - 3)^2 + (y - 5)^2 = 34\) \[
\begin{align*}
(x - 3)^2 + [(x + 4) - 5]^2 &= 34 \\
(x - 3)^2 + (x + 4 - 5)^2 &= 34 \\
(x - 3)^2 + (x - 1)^2 &= 34 \\
x^2 - 6x + 9 + x^2 - 2x + 1 &= 34 \\
2x^2 - 8x + 10 &= 34 \\
2x^2 - 8x - 24 &= 0 \\
x^2 - 4x - 12 &= 0 \\
(x - 6)(x + 2) &= 0 \\
\text{So } x &= 6 \text{ and } x = -2
\end{align*}
\]

Substitute \(x = 6\) into \(y = x + 4\)
\[
\begin{align*}
y &= 6 + 4 \\
y &= 10
\end{align*}
\]

Substitute \(x = -2\) into \(y = x + 4\)
\[
\begin{align*}
y &= -2 + 4 \\
y &= 2
\end{align*}
\]

The coordinates of \(A\) and \(B\) are \((6, 10)\) and \((-2, 2)\).
Question:

Find where the line \( x + y + 5 = 0 \) meets the circle \( (x + 3)^2 + (y + 5)^2 = 65 \).

Solution:

Rearranging \( x + y + 5 = 0 \)
\[
y + 5 = -x
\]
\[
y = -x - 5
\]
Substitute \( y = -x - 5 \) into \( (x + 3)^2 + (y + 5)^2 = 65 \)
\[
(x + 3)^2 + [(−x−5)+5]^2 = 65
\]
\[
(x + 3)^2 + (−x−5+5)^2 = 65
\]
\[
(x + 3)^2 + (−x)^2 = 65
\]
\[
x^2 + 6x + 9 + x^2 = 65
\]
\[
2x^2 + 6x + 9 = 65
\]
\[
2x^2 + 6x - 56 = 0
\]
\[
x^2 + 3x - 28 = 0
\]
\[
(x + 7)(x - 4) = 0
\]
So \( x = -7 \) and \( x = 4 \)

Substitute \( x = -7 \) into \( y = -x - 5 \)
\[
y = -(−7)−5
\]
\[
y = 7 - 5
\]
\[
y = 2
\]
Substitute \( x = 4 \) into \( y = -x - 5 \)
\[
y = - (4)−5
\]
\[
y = -4 - 5
\]
\[
y = -9
\]
So the line meets the circle at \((-7, 2)\) and \((4, -9)\).
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Coordinate geometry in the (x,y) plane
Exercise E, Question 6

Question:

Show that the line \( y = x - 10 \) does not meet the circle \( (x - 2)^2 + y^2 = 25 \).

Solution:

Substitute \( y = x - 10 \) into \( (x - 2)^2 + y^2 = 25 \)

\[
(x - 2)^2 + (x - 10)^2 = 25
\]

\[
x^2 - 4x + 4 + x^2 - 20x + 100 = 25
\]

\[
2x^2 - 24x + 104 = 25
\]

\[
2x^2 - 24x + 79 = 0
\]

Now \( b^2 - 4ac = (-24)^2 - 4(2)(79) = 576 - 632 = -56 \)

As \( b^2 - 4ac < 0 \) then \( 2x^2 - 24x + 79 = 0 \) has no real roots.

So the line does not meet the circle.
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**Edexcel Modular Mathematics for AS and A-Level**

**Coordinate geometry in the (x,y) plane**

**Exercise E, Question 7**

**Question:**

Show that the line $x + y = 11$ is a tangent to the circle $x^2 + (y - 3)^2 = 32$.

**Solution:**

Rearranging $x + y = 11$

$y = 11 - x$

Substitute $y = 11 - x$ into $x^2 + (y - 3)^2 = 32$

$x^2 + (11 - x - 3)^2 = 32$

$x^2 + (8 - x)^2 = 32$

$x^2 + 64 - 16x + x^2 = 32$

$2x^2 - 16x + 64 = 32$

$2x^2 - 16x + 32 = 0$

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

The line meets the circle at $x = 4$ (only).

So the line is a tangent.
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Coordinate geometry in the (x,y) plane
Exercise E, Question 8

Question:
Show that the line $3x - 4y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$.

Solution:
Rearrange $3x - 4y + 25 = 0$

$3x + 25 = 4y$

$4y = 3x + 25$

$y = \frac{3}{4}x + \frac{25}{4}$

Substitute $y = \frac{3}{4}x + \frac{25}{4}$ into $x^2 + y^2 = 25$

$x^2 + \left( \frac{3}{4}x + \frac{25}{4} \right)^2 = 25$

$x^2 + \frac{9}{16}x^2 + \frac{150}{16}x + \frac{625}{16} = 25$

$\frac{25}{16}x^2 + \frac{150}{16}x + \frac{225}{16} = 0$

$25x^2 + 150x + 225 = 0$

$x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

The line meets the circle at $x = -3$ (only).

So the line is a tangent.
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Coordinate geometry in the (x,y) plane
Exercise E, Question 9

Question:

The line \( y = 2x - 2 \) meets the circle \( (x - 2)^2 + (y - 2)^2 = 20 \) at \( A \) and \( B \).

(a) Find the coordinates of \( A \) and \( B \).

(b) Show that \( AB \) is a diameter of the circle.

Solution:

(a) Substitute \( y = 2x - 2 \) into \( (x - 2)^2 + (y - 2)^2 = 20 \)

\begin{align*}
(x - 2)^2 + [2x - 2 - 2]^2 &= 20 \\
(x - 2)^2 + (2x - 4)^2 &= 20 \\
x^2 - 4x + 4 + 4x^2 - 16x + 16 &= 20 \\
5x^2 - 20x + 20 &= 20 \\
5x^2 - 20x &= 0 \\
5x(x - 4) &= 0
\end{align*}

So \( x = 0 \) and \( x = 4 \)

Substitute \( x = 0 \) into \( y = 2x - 2 \)

\begin{align*}
y &= 2(0) - 2 \\
y &= 0 - 2 \\
y &= -2
\end{align*}

Substitute \( x = 4 \) into \( y = 2x - 2 \)

\begin{align*}
y &= 2(4) - 2 \\
y &= 8 - 2 \\
y &= 6
\end{align*}

So the coordinates of \( A \) and \( B \) are \((0, -2)\) and \((4, 6)\).

(b) (1) The length of \( AB \) is

\begin{align*}
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\
&= \sqrt{(4 - 0)^2 + [6 - ( -2 )]^2} \\
&= \sqrt{16 + 64} \\
&= \sqrt{80} \\
&= 4 \times \sqrt{5} \\
&= 2 \times \sqrt{20}
\end{align*}

The radius of the circle \( (x - 2)^2 + (y - 2)^2 = 20 \) is \( \sqrt{20} \).

So the length of the chord \( AB \) is twice the length of the radius.

\( AB \) is a diameter of the circle.

(2) Substitute \( x = 2, y = 2 \) into \( y = 2x - 2 \)

\begin{align*}
2 &= 2(2) - 2 = 2 = 2 \\
\checkmark
\end{align*}

So the line \( y = 2x - 2 \) joining \( A \) and \( B \) passes through the centre \((2, 2)\) of the circle.

So \( AB \) is a diameter of the circle.

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The line $x + y = a$ meets the circle $(x - p)^2 + (y - 6)^2 = 20$ at $(3, 10)$, where $a$ and $p$ are constants.

(a) Work out the value of $a$.

(b) Work out the two possible values of $p$.

Solution:

(a) Substitute $x = 3$, $y = 10$ into $x + y = a$

$3 + 10 = a$

So $a = 13$

(b) Substitute $x = 3$, $y = 10$ into $(x - p)^2 + (y - 6)^2 = 20$

$(3 - p)^2 + (10 - 6)^2 = 20$

$(3 - p)^2 + 16 = 20$

$(3 - p)^2 = 4$

$3 - p = \pm 2$

So $3 - p = 2 \Rightarrow p = 1$

and $3 - p = -2 \Rightarrow p = 5$

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Question:

The line \( y = 2x - 8 \) meets the coordinate axes at \( A \) and \( B \). The line \( AB \) is a diameter of the circle. Find the equation of the circle.

Solution:

Substitute \( x = 0 \) into \( y = 2x - 8 \)
\[
y = 2 \cdot 0 - 8 = -8
\]

Substitute \( y = 0 \) into \( y = 2x - 8 \)
\[
0 = 2x - 8
\]
\[
x = 4
\]
The line meets the coordinate axes at \( (0, -8) \) and \( (4, 0) \).
The coordinates of the centre of the circle is
\[
\left(\frac{0+4}{2}, \frac{-8+0}{2}\right) = \left(\frac{4}{2}, \frac{-8}{2}\right) = (2, -4)
\]
The length of the diameter is
\[
\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}
\]
\[
= \sqrt{(4-0)^2 + [0-(-8)]^2}
\]
\[
= \sqrt{16 + 64}
\]
\[
= \sqrt{80}
\]
\[
= \sqrt{16 \times 5}
\]
\[
= 4\sqrt{5}
\]
So the length of the radius is \( \frac{4\sqrt{5}}{2} = 2\sqrt{5} \).

The centre of the circle is \( (2, -4) \) and the radius is \( 2\sqrt{5} \).
So the equation is
\[
(x-x_1)^2 + (y-y_1)^2 = r^2
\]
\[
(x-2)^2 + [y-(4)]^2 = (2\sqrt{5})^2
\]
\[
(x-2)^2 + (y+4)^2 = 20
\]

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Coordinate geometry in the (x,y) plane
Exercise F, Question 2

Question:

The circle centre (8, 10) meets the x-axis at (4, 0) and (a, 0).

(a) Find the radius of the circle.

(b) Find the value of a.

Solution:

(a) The radius is

\[ r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(8 - 4)^2 + (10 - 0)^2} \]

\[ = \sqrt{4^2 + 10^2} \]

\[ = \sqrt{16 + 100} \]

\[ = \sqrt{116} \]

\[ = 2\sqrt{29} \]

(b) The centre is on the perpendicular bisector of (4, 0) and (a, 0). So

\[ \frac{4 + a}{2} = 8 \]

4 + a = 16

a = 12

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Coordinate geometry in the (x,y) plane
Exercise F, Question 3

Question:

The circle \((x - 5)^2 + y^2 = 36\) meets the x-axis at \(P\) and \(Q\). Find the coordinates of \(P\) and \(Q\).

Solution:

Substitute \(y = 0\) into \((x - 5)^2 + y^2 = 36\)
\((x - 5)^2 = 36\)
\(x - 5 = \sqrt{36}\)
\(x - 5 = \pm 6\)
So \(x - 5 = 6 \Rightarrow x = 11\)
and \(x - 5 = -6 \Rightarrow x = -1\)
The coordinates of \(P\) and \(Q\) are \((-1, 0)\) and \((11, 0)\).

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Coordinate geometry in the (x,y) plane
Exercise F, Question 4

Question:

The circle $(x + 4)^2 + (y - 7)^2 = 121$ meets the y-axis at $(0, m)$ and $(0, n)$. Find the value of $m$ and $n$.

Solution:

Substitute $x = 0$ into $(x + 4)^2 + (y - 7)^2 = 121$
$4^2 + (y - 7)^2 = 121$
$16 + (y - 7)^2 = 121$
$(y - 7)^2 = 105$
$y - 7 = \pm \sqrt{105}$
So $y = 7 \pm \sqrt{105}$
The values of $m$ and $n$ are $7 + \sqrt{105}$ and $7 - \sqrt{105}$.

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Question:

The line $y = 0$ is a tangent to the circle $(x - 8)^2 + (y - a)^2 = 16$. Find the value of $a$.

Solution:

The radius of the circle is $\sqrt{16} = 4$.

So $a = 4$.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 6

Question:

The point \( A \left( -3, -7 \right) \) lies on the circle centre \( (5, 1) \).
Find the equation of the tangent to the circle at \( A \).

Solution:

The gradient of the line joining \( ( -3, -7 ) \) and \( (5, 1) \) is
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-7)}{5 - (-3)} = \frac{8}{8} = 1
\]

So the gradient of the tangent is \( -\frac{1}{(1)} = -1 \).

The equation of the tangent is
\[
y - y_1 = m \left( x - x_1 \right)
\]
\[
y - ( -7 ) = -1 \left[ x - ( -3 ) \right]
\]
\[
y + 7 = -1 \left( x + 3 \right)
\]
\[
y + 7 = -x - 3
\]
\[
y = -x - 10 \text{ or } x + y + 10 = 0
\]

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The circle \((x + 3)^2 + (y + 8)^2 = 100\) meets the positive coordinate axes at \(A(a, 0)\) and \(B(0, b)\).

(a) Find the value of \(a\) and \(b\).

(b) Find the equation of the line \(AB\).

Solution:

(a) Substitute \(y = 0\) into \((x + 3)^2 + (y + 8)^2 = 100\)
\[
(x + 3)^2 + 8^2 = 100
\]
\[
(x + 3)^2 + 64 = 100
\]
\[
(x + 3)^2 = 36
\]
\[
x + 3 = \pm \sqrt{36}
\]
\[
x + 3 = \pm 6
\]
So \(x + 3 = 6 \Rightarrow x = 3\)
and \(x + 3 = -6 \Rightarrow x = -9\)
As \(a > 0\), \(a = 3\).

Substitute \(x = 0\) into \((x + 3)^2 + (y + 8)^2 = 100\)
\[
3^2 + (y + 8)^2 = 100
\]
\[
9 + (y + 8)^2 = 100
\]
\[
(y + 8)^2 = 91
\]
\[
y + 8 = \pm \sqrt{91}
\]
So \(y + 8 = \sqrt{91} \Rightarrow y = \sqrt{91} - 8\)
and \(y + 8 = -\sqrt{91} \Rightarrow y = -\sqrt{91} - 8\)
As \(b > 0\), \(b = \sqrt{91} - 8\).

(b) The equation of the line joining \((3, 0)\) and \((0, \sqrt{91} - 8)\) is
\[
\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}
\]
\[
\frac{y - 0}{(\sqrt{91} - 8) - 0} = \frac{x - 3}{0 - 3}
\]
\[
y = \frac{x - 3}{-3}
\]
\[
y = \left(\sqrt{91} - 8\right) \times \left(\frac{x - 3}{-3}\right)
\]
\[
y = \left(\frac{\sqrt{91} - 8}{-3}\right) \left(x - 3\right)
\]
\[
y = \left(\frac{8 - \sqrt{91}}{3}\right) \left(x - 3\right)
\]
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**Coordinate geometry in the (x,y) plane**  
Exercise F, Question 8

**Question:**

The circle \((x + 2)^2 + (y - 5)^2 = 169\) meets the positive coordinate axes at \(C(c, 0)\) and \(D(0, d)\).

(a) Find the value of \(c\) and \(d\).

(b) Find the area of \(\triangle OCD\), where \(O\) is the origin.

**Solution:**

(a) Substitute \(y = 0\) into \((x + 2)^2 + (y - 5)^2 = 169\)

\[(x + 2)^2 + (-5)^2 = 169\]

\[(x + 2)^2 + 25 = 169\]

\[(x + 2)^2 = 144\]

\[x + 2 = \pm \sqrt{144}\]

\[x + 2 = \pm 12\]

So \(x + 2 = 12\) \(\Rightarrow\) \(x = 10\)

and \(x + 2 = -12\) \(\Rightarrow\) \(x = -14\)

As \(c > 0\), \(c = 10\).

Substitute \(x = 0\) into \((x + 2)^2 + (y - 5)^2 = 169\)

\[2^2 + (y - 5)^2 = 169\]

\[4 + (y - 5)^2 = 169\]

\[(y - 5)^2 = 165\]

\[y - 5 = \pm \sqrt{165}\]

So \(y - 5 = \sqrt{165}\) \(\Rightarrow\) \(y = \sqrt{165} + 5\)

and \(y - 5 = -\sqrt{165}\) \(\Rightarrow\) \(y = -\sqrt{165} + 5\)

As \(d > 0\), \(d = \sqrt{165} + 5\).

(b)

![Diagram showing points C(10,0) and D(0,√(165+5))](image)

The area of \(\triangle OCD\) is

\[
\frac{1}{2} \times 10 \times \left( \sqrt{165} + 5 \right) = 5 \left( \sqrt{165} + 5 \right)
\]
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Coordinate geometry in the (x,y) plane
Exercise F, Question 9

Question:
The circle, centre \((p, q)\) radius 25, meets the x-axis at \((-7, 0)\) and \((7, 0)\), where \(q > 0\).

(a) Find the value of \(p\) and \(q\).

(b) Find the coordinates of the points where the circle meets the y-axis.

Solution:

(a) By symmetry \(p = 0\).

Using Pythagoras’ theorem
\[q^2 + 7^2 = 25^2\]
\[q^2 + 49 = 625\]
\[q^2 = 576\]
\[q = \pm \sqrt{576}\]
\[q = \pm 24\]
As \(q > 0\), \(q = 24\).

(b) The circle meets the y-axis at \(q \pm r\); i.e.
at \(24 + 25 = 49\)
and \(24 - 25 = -1\)
So the coordinates are \((0, 49)\) and \((0, -1)\).
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Coordinate geometry in the (x,y) plane
Exercise F, Question 10

Question:

Show that \((0,0)\) lies inside the circle \((x-5)^2 + (y+2)^2 = 30\).

Solution:

The distance between \((0,0)\) and \((5, -2)\) is

\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(5 - 0)^2 + (-2 - 0)^2}
\]

\[
= \sqrt{25 + 4}
\]

\[
= \sqrt{29}
\]

The radius of the circle is \(\sqrt{30}\).

As \(\sqrt{29} < \sqrt{30}\) \((0,0)\) lies inside the circle.
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Coordinate geometry in the (x,y) plane  
Exercise F, Question 11  

Question:  
The points A (−4, 0), B (4, 8) and C (6, 0) lie on a circle. The lines AB and BC are chords of the circle. Find the coordinates of the centre of the circle.  

Solution:  
(1) The gradient of AB is  
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{4 - (-4)} = \frac{8}{4 + 4} = \frac{8}{8} = 1 
\]

(2) The gradient of a line perpendicular to AB is \(-1\).  

(3) The mid-point of AB is  
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-4 + 4}{2}, \frac{0 + 8}{2} \right) = \left( 0, 4 \right) 
\]

(4) The equation of the perpendicular bisector of AB is  
y - y_1 = m (x - x_1)
y - 4 = -1 (x - 0)
y - 4 = -x 
y = -x + 4

(5) The gradient of BC is  
\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 8}{6 - 4} = \frac{-8}{2} = -4 
\]

(6) The gradient of a line perpendicular to BC is \(-\frac{1}{4}\).  

(7) The mid-point of BC is  
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{4 + 6}{2}, \frac{8 + 0}{2} \right) = \left( \frac{10}{2}, \frac{8}{2} \right) = \left( 5, 4 \right) 
\]

(8) The equation of the perpendicular bisector of BC is  
y - y_1 = m (x - x_1)
y - 4 = \frac{1}{4} \left( x - 5 \right) 
y - 4 = \frac{1}{4}x - \frac{5}{4} 
y = \frac{1}{4}x + \frac{11}{4}

(9) Solving \( y = -x + 4 \) and \( y = \frac{1}{4}x + \frac{11}{4} \) simultaneously  
\[
\frac{1}{4}x + \frac{11}{4} = -x + 4 
\]
\[
\frac{5}{4}x + \frac{11}{4} = 4
\]
\[
\frac{5}{4}x = \frac{5}{4}
\]
x = 1
Substitute x = 1 into y = −x + 4
y = −1 + 4
y = 3
So coordinates of the centre of the circle are (1, 3).
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Coordinate geometry in the (x,y) plane
Exercise F, Question 12

Question:
The points R ( -4 , 3 ) , S ( 7 , 4 ) and T ( 8 , -7 ) lie on a circle.

(a) Show that \( \triangle RST \) has a right angle.

(b) Find the equation of the circle.

Solution:

(a) (1) The distance between R and S is

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(7 - (-4))^2 + (4 - 3)^2} \]

\[ = \sqrt{11^2 + 1^2} \]

\[ = \sqrt{122} \]

(2) The distance between S and T is

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(8 - 7)^2 + (-7 - 4)^2} \]

\[ = \sqrt{1^2 + (-11)^2} \]

\[ = \sqrt{122} \]

(3) The distance between R and T is

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(8 - (-4))^2 + (-7 - 3)^2} \]

\[ = \sqrt{12^2 + (-10)^2} \]

\[ = \sqrt{244} \]

By Pythagoras' theorem

\[ (\sqrt{122})^2 + (\sqrt{122})^2 = (\sqrt{244})^2 \]

So \( \triangle RST \) has a right angle (at S).

(b) (1) The radius of the circle is

\[ \frac{1}{2} \times \text{diameter} = \frac{1}{2} \sqrt{244} = \frac{1}{2} \sqrt{4 \times 61} = \frac{1}{2} \sqrt{4 \times \sqrt{61}} = \frac{1}{2} \times 2 \sqrt{61} = \sqrt{61} \]

(2) The centre of the circle is the mid-point of RT:

\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

\[ = \left( \frac{-4 + 8}{2}, \frac{3 + (-7)}{2} \right) = \left( \frac{4}{2}, \frac{-4}{2} \right) = \left( 2, -2 \right) \]

So the equation of the circle is

\[ (x - 2)^2 + (y + 2)^2 = (\sqrt{61})^2 \]

or

\[ (x - 2)^2 + (y + 2)^2 = 61 \]
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Coordinate geometry in the (x,y) plane
Exercise F, Question 13

Question:

The points \( A ( -7 , 7 ) \), \( B ( 1 , 9 ) \), \( C ( 3 , 1 ) \) and \( D ( -7 , 1 ) \) lie on a circle. The lines \( AB \) and \( CD \) are chords of the circle.

(a) Find the equation of the perpendicular bisector of (i) \( AB \) (ii) \( CD \).

(b) Find the coordinates of the centre of the circle.

Solution:

(a) (i) The gradient of the line joining \( A \) and \( B \) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 7}{1 - (-7)} = \frac{2}{8} = \frac{1}{4}
\]

(2) The gradient of a line perpendicular to \( AB \) is

\[-\frac{1}{m} = -4
\]

(3) The mid-point of \( AB \) is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{-7 + 1}{2}, \frac{7 + 9}{2} \right) = \left( \frac{-6}{2}, \frac{16}{2} \right) = \left( -3, 8 \right)
\]

(4) The equation of the perpendicular bisector of \( AB \) is

\[y - y_1 = m(x - x_1)\]

\[y - 8 = -4[x - (-3)]\]

\[y - 8 = -4x - 12\]

\[y = -4x - 4\]

(ii) The gradient of the line joining \( C \) and \( D \) is

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 1}{-7 - 3} = \frac{0}{-10} = 0
\]

So the line is horizontal.

(2) The mid-point of \( CD \) is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{3 + (-7)}{2}, \frac{1 + 1}{2} \right) = \left( \frac{-4}{2}, \frac{2}{2} \right) = \left( -2, 1 \right)
\]

(3) The equation of the perpendicular bisector of \( CD \) is \( x = -2 \)

i.e. the vertical line through \((-2, 1)\).

(b) Solving \( y = -4x - 4 \) and \( x = -2 \) simultaneously,

substitute \( x = -2 \) into \( y = -4x - 4\)

\[y = -4(-2) - 4 = 8 - 4 = 4\]

So the centre of the circle is \((-2, 4)\).
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Coordinate geometry in the (x,y) plane
Exercise F, Question 14

Question:

The centres of the circles \((x - 8)^2 + (y - 8)^2 = 117\) and \((x + 1)^2 + (y - 3)^2 = 106\) are \(P\) and \(Q\) respectively.

(a) Show that \(P\) lies on \((x + 1)^2 + (y - 3)^2 = 106\).

(b) Find the length of \(PQ\).

Solution:

(a) The centre of \((x - 8)^2 + (y - 8)^2 = 117\) is \((8, 8)\).

Substitute \((8, 8)\) into \((x + 1)^2 + (y - 3)^2 = 106\)

\((8 + 1)^2 + (8 - 3)^2 = 9^2 + 5^2 = 81 + 25 = 106\)

So \((8, 8)\) lies on the circle \((x + 1)^2 + (y - 3)^2 = 106\).

(b) As \(Q\) is the centre of the circle \((x + 1)^2 + (y - 3)^2 = 106\) and \(P\) lies on this circle, the length \(PQ\) must equal the radius.

So \(PQ = \sqrt{106}\)

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Coordinate geometry in the (x,y) plane
Exercise F, Question 15

Question:

The line \( y = -3x + 12 \) meets the coordinate axes at \( A \) and \( B \).

(a) Find the coordinates of \( A \) and \( B \).

(b) Find the coordinates of the mid-point of \( AB \).

(c) Find the equation of the circle that passes through \( A, B \) and \( O \), where \( O \) is the origin.

Solution:

(a) \( y = -3x + 12 \)
(1) Substitute \( x = 0 \) into \( y = -3x + 12 \)
\( y = -3(0) + 12 = 12 \)
So \( A \) is \( (0, 12) \) .

(2) Substitute \( y = 0 \) into \( y = -3x + 12 \)
\( 0 = -3x + 12 \)
\( 3x = 12 \)
\( x = 4 \)
So \( B \) is \( (4, 0) \) .

(b) The mid-point of \( AB \) is
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left( \frac{0 + 4}{2}, \frac{12 + 0}{2} \right) = \left( 2, 6 \right)
\]

(c)

\[\angle AOB = 90^\circ, \text{ so } AB \text{ is a diameter of the circle.}\]
The centre of the circle is the mid-point of \( AB \), i.e. \( (2, 6) \) .
The length of the diameter \( AB \) is
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 0)^2 + (0 - 12)^2} = \sqrt{16 + 144} = \sqrt{160}
\]
So the radius of the circle is \( \frac{\sqrt{160}}{2} \).
The equation of the circle is

\[\sqrt{(x - 2)^2 + (y - 6)^2} = \frac{\sqrt{160}}{2}\]
\[(x - 2)^2 + (y - 6)^2 = \left(\frac{\sqrt{160}}{2}\right)^2\]

\[(x - 2)^2 + (y - 6)^2 = \frac{160}{4}\]

\[(x - 2)^2 + (y - 6)^2 = 40\]
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Coordinate geometry in the (x,y) plane
Exercise F, Question 16

Question:
The points $A(-5,5)$, $B(1,5)$, $C(3,3)$ and $D(3,-3)$ lie on a circle. Find the equation of the circle.

Solution:

1. \[A(-5,5)\] \[B(1,5)\] \[C(3,3)\] \[D(3,-3)\]

2. The mid-point of $AB$ is \[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + 1}{2}, \frac{5 + 5}{2}\right) = \left(-\frac{4}{2}, \frac{10}{2}\right) = \left(-2, 5\right)
\]
So the equation of the perpendicular bisector of $AB$ is $x = -2$.

3. The mid-point of $CD$ is \[
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 3}{2}, \frac{3 + (-3)}{2}\right) = \left(\frac{6}{2}, \frac{3 - 3}{2}\right) = \left(3, \frac{0}{2}\right) = \left(3, 0\right)
\]
So the equation of the perpendicular bisector of $CD$ is $y = 0$.

4. The perpendicular bisectors intersect at \((-2, 0)\).

5. The radius is the distance between \((-2, 0)\) and \((-5, 5)\)
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5 - (-2))^2 + (5 - 0)^2} = \sqrt{(-3)^2 + (5)^2} = \sqrt{9 + 25} = \sqrt{34}
\]

6. So the equation of the circle centre \((-2, 0)\) and radius $\sqrt{34}$ is
\[
(x + 2)^2 + (y - 0)^2 = (\sqrt{34})^2
\]

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The line $AB$ is a chord of a circle centre $(2, -1)$, where $A$ and $B$ are $(3, 7)$ and $(-5, 3)$ respectively. $AC$ is a diameter of the circle. Find the area of $\triangle ABC$.

Solution:

(1) [Diagram of triangle ABC]

(2) Let the coordinates of $C$ be $(p, q)$.

$(2, -1)$ is the mid-point of $(3, 7)$ and $(p, q)$

So $\frac{3 + p}{2} = 2$ and $\frac{7 + q}{2} = -1$

$3 + p = 4$
$p = 1$

$7 + q = -2$
$q = -9$

So the coordinates of $C$ are $(1, -9)$.

(3) The length of $AB$ is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(-5 - 3)^2 + (3 - 7)^2}$$

$$\sqrt{(-8)^2 + (-4)^2}$$

$$= \sqrt{64 + 16}$$

$$= \sqrt{80}$$

The length of $BC$ is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(-5 - (-1))^2 + (3 - (-9))^2}$$

$$\sqrt{(-4)^2 + (12)^2}$$

$$= \sqrt{16 + 144}$$

$$= \sqrt{160}$$

(4) The area of $\triangle ABC$ is

$$\frac{1}{2} \sqrt{80} \times \sqrt{180} = \frac{1}{2} \sqrt{14400} = \frac{1}{2} \sqrt{144} \times \sqrt{100} = \frac{1}{2} \times \sqrt{144} \times \sqrt{100} = \frac{1}{2} \times 12 \times 10 = 60$$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise F, Question 18

Question:

The points A (−1, 0), B \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right) and C \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) are the vertices of a triangle.

(a) Show that the circle \(x^2 + y^2 = 1\) passes through the vertices of the triangle.

(b) Show that \(\triangle ABC\) is equilateral.

Solution:

(a) (1) Substitute (−1, 0) into \(x^2 + y^2 = 1\)

\((-1)^2 + (0)^2 = 1 + 0 = 1 \quad \checkmark\)

So (−1, 0) is on the circle.

(2) Substitute \(\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\) into \(x^2 + y^2 = 1\)

\(\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1 \quad \checkmark\)

So \(\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\) is on the circle.

(3) Substitute \(\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\) into \(x^2 + y^2 = 1\)

\(\left(\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1 \quad \checkmark\)

So \(\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\) is on the circle.

(b) (1) The distance between (−1, 0) and \(\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\) is

\[\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\]

\[= \sqrt{\left(\frac{1}{2} - (-1)\right)^2 + \left(\frac{\sqrt{3}}{2} - 0\right)^2}\]

\[= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\]

\[= \sqrt{\frac{9}{4} + \frac{3}{4}}\]

\[= \sqrt{3}\]

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(2) The distance between \((-1, 0)\) and \(\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\) is
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{\left(\frac{1}{2} - \left(-1\right)\right)^2 + \left(-\frac{\sqrt{3}}{2} - 0\right)^2}
= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2}
= \sqrt{\frac{9}{4} + \frac{3}{4}}
= \sqrt{\frac{12}{4}}
= \sqrt{3}
\]

(3) The distance between \(\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)\) and \(\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)\) is
\[
\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
= \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right)^2}
= \sqrt{0^2 + (\sqrt{3})^2}
= \sqrt{0 + 3}
= \sqrt{3}
\]
So \(AB, BC\) and \(AC\) all equal \(\sqrt{3}\).
\(\triangle ABC\) is equilateral.
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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise F, Question 19

Question:

The points \( P(2, 2) \), \( Q(2 + \sqrt{3}, 5) \) and \( R(2 - \sqrt{3}, 5) \) lie on the circle \( (x - 2)^2 + (y - 4)^2 = r^2 \).

(a) Find the value of \( r \).

(b) Show that \( \triangle PQR \) is equilateral.

Solution:

(a) Substitute \( (2, 2) \) into \( (x - 2)^2 + (y - 4)^2 = r^2 \)

\[ (2 - 2)^2 + (2 - 4)^2 = r^2 \]

\[ 0^2 + (-2)^2 = r^2 \]

\[ r^2 = 4 \]

\[ r = 2 \]

(b) (1) The distance between \( (2, 2) \) and \( (2 + \sqrt{3}, 5) \) is

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(2 + \sqrt{3} - 2)^2 + (5 - 2)^2} \]

\[ = \sqrt{(\sqrt{3})^2 + 3^2} \]

\[ = \sqrt{3 + 9} \]

\[ = \sqrt{12} \]

(2) The distance between \( (2, 2) \) and \( (2 - \sqrt{3}, 5) \) is

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(2 - \sqrt{3} - 2)^2 + (5 - 2)^2} \]

\[ = \sqrt{(-\sqrt{3})^2 + 3^2} \]

\[ = \sqrt{3 + 9} \]

\[ = \sqrt{12} \]

(3) The distance between \( (2 + \sqrt{3}, 5) \) and \( (2 - \sqrt{3}, 5) \) is

\[ \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{[(2 - \sqrt{3}) - (2 + \sqrt{3})]^2 + (5 - 5)^2} \]

\[ = \sqrt{(-2 \sqrt{3})^2 + 0^2} \]

\[ = \sqrt{4 \times 3} \]

\[ = \sqrt{12} \]

So \( PQ \), \( QR \) and \( PR \) all equal \( \sqrt{12} \).

\( \triangle PQR \) is equilateral.

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Question:

The points $A(-3, -2)$, $B(-6, 0)$ and $C(p, q)$ lie on a circle centre $\left(-\frac{5}{2}, 2\right)$. The line $BC$ is a diameter of the circle.

(a) Find the value of $p$ and $q$.

(b) Find the gradient of (i) $AB$ (ii) $AC$.

(c) Show that $AB$ is perpendicular to $AC$.

Solution:

(a) The mid-point of $(-6, 0)$ and $(p, q)$ is $\left(-\frac{5}{2}, 2\right)$.

So $\left(-\frac{6+p}{2}, \frac{0+q}{2}\right) = \left(-\frac{5}{2}, 2\right)$

$-\frac{6+p}{2} = -\frac{5}{2}$

$-6+p = -5$

$p = -5+6$

$p = 1$

$\frac{0+q}{2} = 2$

$q = 4$

(b) (i) The gradient of the line joining $(-3, -2)$ and $(-6, 0)$ is

$\frac{y_2-y_1}{x_2-x_1} = \frac{0 - (-2)}{-6 - (-3)} = \frac{2}{-3} = \frac{-2}{3}$

(ii) The gradient of the line joining $(-3, -2)$ and $(1, 4)$ is

$\frac{y_2-y_1}{x_2-x_1} = \frac{4 - (-2)}{1 - (-3)} = \frac{6}{4} = \frac{3}{2}$

(c) Two lines are perpendicular if $m_1 \times m_2 = -1$.

Now $-\frac{2}{3} \times \frac{3}{2} = -1$ √

So $AB$ is perpendicular to $AC$. 

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