Hidden Quadratic Equations [Ch. 4]

1. By substituting $y = x^2$, or otherwise, find the real roots of the equation $x^4 - 3x^2 - 4 = 0$.

2. By substituting $t = x^{\frac{1}{3}}$, find the values of $x$ for which $2x + 3 = 7x^{\frac{1}{3}}$.

3. (i) Given that $t^4 = y$, show that the equation $t^4 + 2t^{-4} = 3$ may be written as $y^2 - 3y + 2 = 0$.
   (ii) Hence solve the equation $t^4 + 2t^{-4} = 3$.

4. Find, correct to 3 significant figures, all the roots of the equation $x^4 - 4x^2 + 1 = 0$.

5. Solve the equation
   \[ x\sqrt{8} - 11 = \frac{3x}{\sqrt{2}}, \]
   giving your answer in the form $k\sqrt{2}$, where $k$ is an integer.

6. (i) Given that $\sqrt{x} = y$, show that the equation
   \[ \sqrt{x} + \frac{10}{\sqrt{x}} = 7 \]
   may be written as
   \[ y^2 - 7y + 10 = 0. \]
   (ii) Hence solve the equation
   \[ \sqrt{x} + \frac{10}{\sqrt{x}} = 7. \]

7. (i) By letting $t = x^{\frac{1}{3}}$, show that the equation
   \[ x^3 + (125x)^{\frac{1}{3}} = 14 \]
   may be written as
   \[ t^2 + 5t - 14 = 0. \]
   (ii) Hence solve the equation
   \[ x^3 + (125x)^{\frac{1}{3}} = 14. \]