Completing the Square [Ch. 4]

1. Express \(2x^2 + 12x + 13\) in the form \(a(x + b)^2 + c\). [4]

2. (i) Express \(x^2 - 8x + 24\) in the form \((x - a)^2 + b\). [3]
   (ii) Hence write down the coordinates of the vertex of the graph of \(y = x^2 - 8x + 24\). [2]

3. Express \(3x^2 + 12x + 7\) in the form \(3(x + a)^2 + b\). [4]

4. (i) Find the constants \(a\), \(b\) and \(c\) such that, for all values of \(x\),
    \[4x^2 + 40x + 97 = a(x + b)^2 + c.\] [4]
   (ii) Hence write down the equation of the line of symmetry of the curve \(y = 4x^2 + 40x + 97\). [1]

5. (i) Express \(x^2 + 8x + 18\) in the form \((x + a)^2 + b\). [2]
   (ii) Sketch the graph of \(y = x^2 + 8x + 18\), stating the coordinates of its vertex. [3]

6. (i) Find the constants \(a\) and \(b\) such that, for all values of \(x\),
    \[x^2 + 6x + 20 = (x + a)^2 + b.\] [3]
   (ii) Hence state the least value of \(x^2 + 6x + 20\), and state also the value of \(x\) for which this least value occurs. [2]
   (iii) Write down the greatest value of \(\frac{1}{x^2 + 6x + 20}\). [1]

7. (i) Express \(2x^2 + 4x - 1\) in the form
    \[a[(x + p)^2 + q],\]
    stating the values of the constants \(a\), \(p\) and \(q\). [4]
   (ii) Sketch the graph of \(y = 2x^2 + 4x - 1\), stating the coordinates of the vertex. [4]
   (iii) The graph of \(y = 2x^2 + 4x - 1\) is obtained from the graph of \(y = x^2\) by a sequence of transformations. Describe such a sequence, specifying each transformation fully, and stating the order in which they are applied. [4]
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8 (i) Express \(3x^2 + 4x + 1\) in the form \(a(x + b)^2 + c\).

(ii) Hence or otherwise find the coordinates of the vertex of the graph of \(y = 3x^2 + 4x + 1\).

9 (i) Express \(4x^2 - 16x + 8\) in the form \(a(x + b)^2 + c\).

(ii) Hence find the coordinates of the vertex of the graph of \(y = 4x^2 - 16x + 8\).

(iii) Sketch the graph of \(y = 4x^2 - 16x + 8\), giving the \(x\)-coordinates of the points where the graph meets the \(x\)-axis.

10 (a) (i) Express the quadratic polynomial \(x^2 - (2\sqrt{2})x + 4\) in the form \((x + a)^2 + b\), stating the exact values of the constants \(a\) and \(b\).

(ii) Hence write down the equation of the line of symmetry of the curve \(y = x^2 - (2\sqrt{2})x + 4\).

(b) The quadratic equation

\[x^2 + (k+1)x + 16 = 0\]

has two distinct real roots. Find the set of possible values of the constant \(k\).