Fig. 12 shows the graph of a cubic curve. It intersects the axes at \((-5, 0), (-2, 0), (1.5, 0)\) and \((0, -30)\).

(i) Use the intersections with both axes to express the equation of the curve in a factorised form. [2]

(ii) Hence show that the equation of the curve may be written as \(y = 2x^3 + 11x^2 - x - 30\). [2]

(iii) Draw the line \(y = 5x + 10\) accurately on the graph. The curve and this line intersect at \((-2, 0)\); find graphically the \(x\)-coordinates of the other points of intersection. [3]

(iv) Show algebraically that the \(x\)-coordinates of the other points of intersection satisfy the equation

\[2x^2 + 7x - 20 = 0.\]

Hence find the exact values of the \(x\)-coordinates of the other points of intersection. [5]
Fig. 12 shows the graph of \( y = \frac{1}{x-2} \).

(i) Draw accurately the graph of \( y = 2x + 3 \) on the copy of Fig. 12 and use it to estimate the coordinates of the points of intersection of \( y = \frac{1}{x-2} \) and \( y = 2x + 3 \). [3]

(ii) Show algebraically that the \( x \)-coordinates of the points of intersection of \( y = \frac{1}{x-2} \) and \( y = 2x + 3 \) satisfy the equation \( 2x^2 - x - 7 = 0 \). Hence find the exact values of the \( x \)-coordinates of the points of intersection. [5]

(iii) Find the quadratic equation satisfied by the \( x \)-coordinates of the points of intersection of \( y = \frac{1}{x-2} \) and \( y = -x + k \). Hence find the exact values of \( k \) for which \( y = -x + k \) is a tangent to \( y = \frac{1}{x-2} \). [4]
Fig. 12 shows the graph of $y = \frac{1}{x-3}$.

(i) Draw accurately, on the copy of Fig. 12, the graph of $y = x^2 - 4x + 1$ for $-1 \leq x \leq 5$. Use your graph to estimate the coordinates of the intersections of $y = \frac{1}{x-3}$ and $y = x^2 - 4x + 1$. [5]

(ii) Show algebraically that, where the curves intersect, $x^3 - 7x^2 + 13x - 4 = 0$. [3]

(iii) Use the fact that $x = 4$ is a root of $x^3 - 7x^2 + 13x - 4 = 0$ to find a quadratic factor of $x^3 - 7x^2 + 13x - 4$. Hence find the exact values of the other two roots of this equation. [5]
4 (i) Find algebraically the coordinates of the points of intersection of the curve \( y = 4x^2 + 24x + 31 \) and the line \( x + y = 10 \). [5]

(ii) Express \( 4x^2 + 24x + 31 \) in the form \( a(x+b)^2 + c \). [4]

(iii) For the curve \( y = 4x^2 + 24x + 31 \),

\( (A) \) write down the equation of the line of symmetry. [1]

\( (B) \) write down the minimum \( y \)-value on the curve. [1]

5 (i) Solve, by factorising, the equation \( 2x^2 - x - 3 = 0 \). [3]

(ii) Sketch the graph of \( y = 2x^2 - x - 3 \). [3]

(iii) Show that the equation \( x^2 - 5x + 10 = 0 \) has no real roots. [2]

(iv) Find the \( x \)-coordinates of the points of intersection of the graphs of \( y = 2x^2 - x - 3 \) and \( y = x^2 - 5x + 10 \). Give your answer in the form \( a \pm \sqrt{b} \). [4]

6 Answer the whole of this question on the insert provided.

The insert shows the graph of \( y = \frac{1}{x} \), \( x \neq 0 \).

(i) Use the graph to find approximate roots of the equation \( \frac{1}{x} = 2x + 3 \), showing your method clearly. [3]

(ii) Rearrange the equation \( \frac{1}{x} = 2x + 3 \) to form a quadratic equation. Solve the resulting equation,

leaving your answers in the form \( \frac{p \pm \sqrt{q}}{r} \). [5]

(iii) Draw the graph of \( y = \frac{1}{x} + 2 \), \( x \neq 0 \), on the grid used for part (i). [2]

(iv) Write down the values of \( x \) which satisfy the equation \( \frac{1}{x} + 2 = 2x + 3 \). [2]