1. (a) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q$$

where p and q are integers to be found.

(2)

(b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

(c) Find the value of the discriminant of $x^2 + 6x + 11$

(2)
(Total 6 marks)

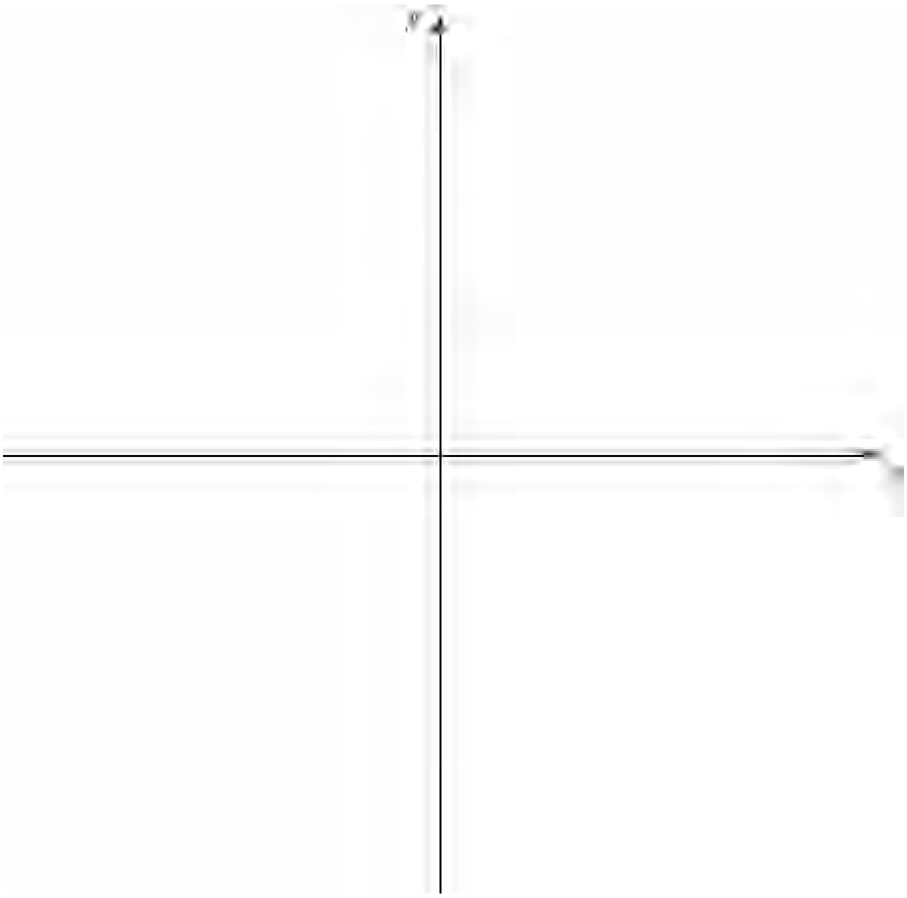
---

2. (a) On the axes below sketch the graphs of

(i) $y = x(4 - x)$

(ii) $y = x^2(7 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.



(5)

(b) Show that the x-coordinates of the points of intersection of

$$y = x(4 - x) \text{ and } y = x^2(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A, leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p, q, r and s are integers.

(7)
(Total 15 marks)

3. $f(x) = x^2 + 4kx + (3 + 11k)$, where k is a constant.

(a) Express $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k.

(3)

Given that the equation $f(x) = 0$ has no real roots,

(b) find the set of possible values of k.

(4)

Given that $k = 1$,

(c) sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis.

(3)
(Total 10 marks)

---

4. The equation $x^2 + 3px + p = 0$, where p is a non-zero constant, has equal roots.

Find the value of p.

(Total 4 marks)

---

5. (a) Factorise completely $x^3 - 6x^2 + 9x$
(3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis.
(4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the *x*-axis.
(2)
(Total 9 marks)

---

6. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x.

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0.$$

(3)

(b) Hence find the set of possible values of k.
(4)
(Total 7 marks)

---

7. Given that the equation $2qx^2 + qx - 1 = 0$, where q is a constant, has no real roots,

(a) show that $q^2 + 8q < 0$.

(2)

(b) Hence find the set of possible values of q.

(3)
(Total 5 marks)

---

8. The equation

$$x^2 + kx + 8 = k$$

has no real solutions for x.

(a) Show that k satisfies $k^2 + 4k - 32 < 0$.

(3)

(b) Hence find the set of possible values of k.

(4)
(Total 7 marks)

9. Find the set of values of x for which

$$x^2 - 7x - 18 > 0.$$

(Total 4 marks)

---

10. The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

(a) Find the value of p.

(4)

(b) For this value of p, solve the equation $x^2 + 2px + (3p + 4) = 0$.

(2)
(Total 6 marks)

11.

$$x^2 + 2x + 3 \equiv (x + a)^2 + b.$$

(a) Find the values of the constants a and b.

(2)

(b) In the space provided below, sketch the graph of $y = x^2 + 2x + 3$, indicating clearly the coordinates of any intersections with the coordinate axes.

(3)

(c) Find the value of the discriminant of $x^2 + 2x + 3$. Explain how the sign of the discriminant relates to your sketch in part (b).

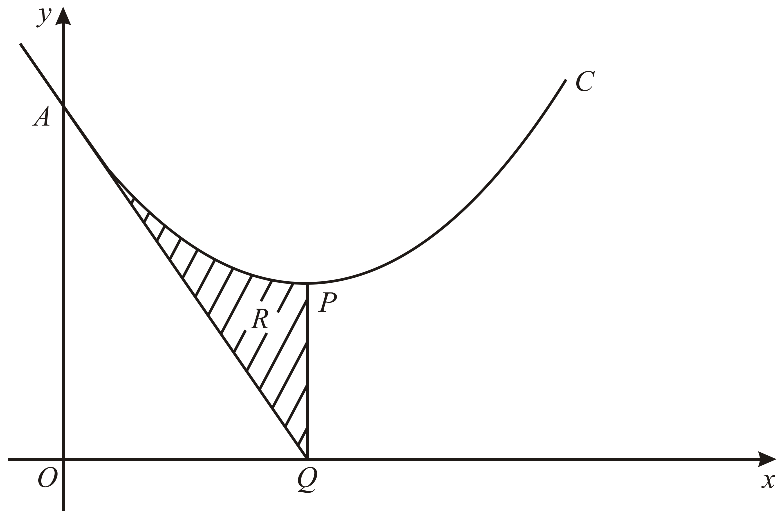
(2)

The equation $x^2 + kx + 3 = 0$, where k is a constant, has no real roots.

(d) Find the set of possible values of k, giving your answer in surd form.

(4)
(Total 11 marks)

12.



The diagram above shows part of the curve C with equation $y = x^2 - 6x + 18$. The curve meets the y-axis at the point A and has a minimum at the point P.

(a) Express $x^2 - 6x + 18$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

(b) Find the coordinates of P.

(2)

(c) Find an equation of the tangent to C at A.

(4)

The tangent to C at A meets the x-axis at the point Q.

(d) Verify that PQ is parallel to the y-axis.

(1)

The shaded region R in the diagram is enclosed by C, the tangent at A and the line PQ.

(e) Use calculus to find the area of R.

(5)
(Total 15 marks)

13. Given that the equation $kx^2 + 12x + k = 0$, where k is a positive constant, has equal roots, find the value of k.

(Total 4 marks)

---

14. Given that $f(x) = x^2 - 6x + 18$, $x \geq 0$,

(a) express $f(x)$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

(4)

The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

(5)
(Total 12 marks)

---

15. $f(x) = x^2 - kx + 9$, where k is a constant.

(a) Find the set of values of k for which the equation $f(x) = 0$ has no real solutions.

(4)

Given that $k = 4$,

(b) express $f(x)$ in the form $(x - p)^2 + q$, where p and q are constants to be found,

(3)

(c) write down the minimum value of $f(x)$ and the value of x for which this occurs.

(2)
(Total 9 marks)

---

16. (a) Solve the equation $4x^2 + 12x = 0$.

(3)

$f(x) = 4x^2 + 12x + c$, where c is a constant.

(b) Given that $f(x) = 0$ has equal roots, find the value of c and hence solve $f(x) = 0$.

(4)

(Total 7 marks)

---

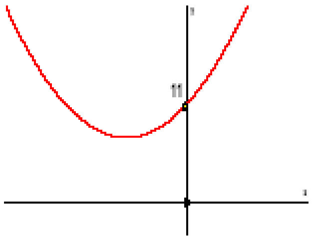
1. (a) $(x+3)^2+2$ or $p=3$ or $\frac{6}{2}$ B1

$q=2$ B1 2

<u>Note</u>

Ignore an "=0" so $(x+3)^2+2=0$ can score both marks

(b)



U shape with min in 2nd quad
(Must be above x-axis and not on y = axis) B1

U shape crossing y-axis at (0, 11) only
(Condone (11,0) marked on y-axis) B1 2

<u>Note</u>

The U shape can be interpreted fairly generously. Penalise an obvious V on 1st B1 only.
The U needn't have equal "arms" as long as there is a clear min that "holds water"

1st B1 for U shape with minimum in 2nd quad. Curve need not cross the y-axis but minimum should NOT touch x-axis and should be left of (not on) y-axis

2nd B1for U shaped curve crossing at (0, 11). Just 11 marked on y-axis is fine. The point must be marked on the sketch (do not allow from a table of values) Condone stopping at (0, 11)

(c) $b^2-4ac=6^2-4\times 11$

$= \underline{-8}$ A1 2

Note

for some correct substitution into $b^2 - 4ac$. This may be as part of the quadratic formula but must be in part (c) and must be only numbers (no x terms present).

Substitution into $b^2 < 4ac$ or $b^2 = 4ac$ or $b^2 > 4ac$ is M0

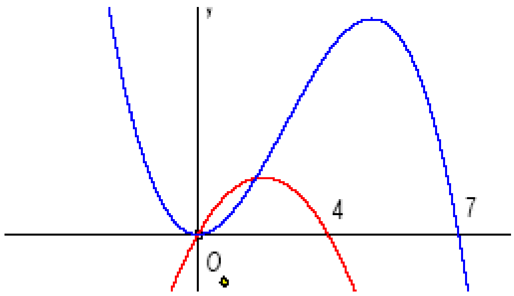
A1 for – 8 only.
If they write – 8 < 0 treat the < 0 as ISW and award A1
If they write – 8 ≥ 0 then score A0
A substitution in the quadratic formula leading to – 8 inside the square root is A0.

So substituting into $b^2 - 4ac<0$ leading to – 8 < 0 can score M1A1.

Only award marks for use of the discriminant in part (c)

[6]

2. (a)



(i) ∩ shape (anywhere on diagram) B1

Passing through or stopping at (0, 0) and (4, 0) only(Needn't be ∩ shape) B1

(ii) correct shape (-ve cubic) with a max and min drawn anywhere B1
Minimum or maximum at (0, 0) B1
Passes through or stops at (7, 0) but NOT touching. B1 5
(7, 0) should be to right of (4, 0) or B0
Condone (0, 4) or (0, 7) marked correctly on x-axis.
Don't penalise poor overlap near origin.
Points must be marked on the sketch...not in the text

(b) $x(4-x) = x^2(7-x)$ $(0=)x[7x-x^2-(4-x)]$

$(0=)x[7x-x^2-(4-x)]$ (o.e.) B1ft

$0 = x(x^2-8x+4)$ * A1 cso 3

Note

	for forming a suitable equation
B1	for a common factor of x taken out legitimately. Can treat this as an M mark. Can ft their cubic = 0 found from an attempt at solving their equations e.g. $x^3 - 8x^2 - 4x = x(...$
A1cso	no incorrect working seen. The "= 0" is required but condone missing from some lines of working. Cancelling the x scores B0A0.

(c) $\left(0 = x^2 - 8x + 4 \Rightarrow\right) x = \dfrac{8 \pm \sqrt{64-16}}{2}$ or $(x \pm 4)^2 - 4^2 + 4 (= 0)$

$(x-4)^2 = 12$

A1

$= \dfrac{8 \pm 4\sqrt{3}}{2}$ or $(x-4) = \pm 2\sqrt{3}$ B1

$x = 4 \pm 2\sqrt{3}$ A1

From sketch A is $x = 4 - 2\sqrt{3}$

So $y = \left(4-2\sqrt{3}\right)\left(4-[4-2\sqrt{3}]\right)$ (dependent on 1st

$\underline{= -12 + 8\sqrt{3}}$ A1 7

Note

1st	for some use of the correct formula or attempt to complete the square
1st A1	for a fully correct expression: condone + instead of ± **or** for $(x-4)^2 = 12$
B1	for simplifying $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{12} = 2\sqrt{3}$ .Can be scored independently of this expression
2nd A1	for correct solution of the form $p + q\sqrt{3}$: can be ± or + or –
2nd	for selecting their answer in the interval (0, 4). If they have no value in (0, 4) score M0
3rd	for attempting $y =$…using their x in correct equation. An expression needed for M1A0
3rd A1	for correct answer. If 2 answers are given A0.

[15]

3.	(a)	$(x + 2k)^2$ or $\left(x + \frac{4k}{2}\right)^2$		
		$(x \pm F)^2 \pm G \pm 3 \pm 11k$ (where F and G are <u>any</u> functions of k, not involving x)		
		$(x + 2k)^2 - 4k^2 + (3+11k)$ Accept unsimplified equivalents such as	A1	3
		$\left(x + \frac{4k}{2}\right)^2 - \left(\frac{4k}{2}\right)^2 + 3 + 11k,$		
		<u>and i.s.w. if necessary</u>.		
	(b)	Accept part (b) solutions seen in part (a).		
		"$4k^2 - 11k - 3$"$= 0$ $(4k + 1)(k - 3) = 0$ $k = \ldots,$		
		[Or, 'starting again', $b^2 - 4ac = (4k)^2 - 4(3 + 11k)$ and proceed to $k = \ldots$] $-\frac{1}{4} < k < 3$ (Ignore any inequalities for the first 2 marks in (b)).	A1	
		Using $b^2 - 4ac < 0$ for no real roots, i.e. "$4k^2 - 11k - 3$"< 0, to establish inequalities involving their <u>two</u> critical values m and n		
		(<u>even if the inequalities are wrong</u>, e.g. $k < m$, $k < n$).		
		$-\frac{1}{4} < k < 3$ (See conditions below) Follow through their critical values.	A1ft	4
		The final A1ft is still scored if the answer $m < k < n$ follows $k < m$, $k < n$.		
		<u>Using x instead of k in the final answer</u> loses only the 2nd A mark, (condone use of x in earlier working).		

Note

1st M: Forming and solving a 3-term quadratic in k (usual rules.. see general principles at end of scheme). The quadratic must come from "$b^2 - 4ac$", or from the "q" in part (a).

Using wrong discriminant, e.g. "$b^2 + 4ac$" will score no marks in part (b).

2nd M: As defined in main scheme above.

2nd A1ft: $m < k < n$, where $m < n$, for their critical values m and n.

Other possible forms of the answer (in each case $m < n$):

(i) $n > k > m$

(ii) $k > m$ and $k < n$

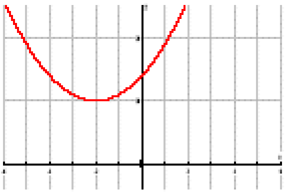
In this case the word "and" must be seen (implying intersection).

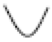
(iii) $k \in (m,n)$ (iv) $\{k : k > m\} \cap \{k : k < n\}$

Not just a number line.

Not just $k > m$, $k < n$ (without the word "and").

(c)



Shape  (seen in (c))	B1	
Minimum in correct quadrant, not touching the x-axis, not on the y-axis, and there must be no other minimum or maximum.	B1	
(0, 14) or 14 on y-axis. Allow (14, 0) marked on y-axis. n.b. Minimum is at (–2,10), (but there is no mark for this).	B1	3

Note

Final B1 is dependent upon a sketch having been attempted in part (c).

[10]

4. $b^2 - 4ac$ attempted, in terms of p.

$(3p)^2 - 4p = 0$ o.e. A1

Attempt to solve for p e.g. $P(9p - 4) = 0$ Must potentially lead to $p = k, k \neq 0$

$p = \frac{4}{9}$ (Ignore $p = 0$, if seen) A1cso

<u>Note</u>

1st for an attempt to substitute into $b^2 - 4ac$ or $b^2 = 4ac$ with b or c correct
Condone x's in one term only.
This can be inside a square root as part of the quadratic formula for example.
Use of inequalities can score the M marks only

1st A1 for any correct equation: $(3p)^2 - 4 \times 1 \times p = 0$ or better

2nd for an attempt to factorize or solve their quadratic expression in p.

Method must be sufficient to lead to their $p = \frac{4}{9}$.

Accept factors or use of quadratic formula or $\left(p \pm \frac{2}{9}\right)^2 = k^2$ (o.e. eg) $\left(3p \pm \frac{2}{3}\right)^2 = k^2$ or equivalent work on <u>their</u> eqn.

which would lead to $9p = 4$ is OK for this 2nd

ALT <u>Comparing coefficients</u>

for $(x + \alpha)^2 = x^2 + \alpha^2 + 2\alpha x$ and A1 for a correct equation eg $3p = 2\sqrt{P}$
for forming solving leading to $\sqrt{P} = \frac{2}{3}$ or better

<u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u>

If the formula is quoted accept some correct substitution leading to a partially correct expression.

If the formula is not quoted only award for a fully correct expression using their values.

[4]

5. (a) $x(x^2 - 6x + 9)$ B1

$= x(x - 3)(x - 3)$ A1 3

<u>Note</u>

B1 for correctly taking out a factor of x

for an attempt to factorize their 3TQ e.g. $(x + p)(x + q)$ where $|pq| = 9$.

So $(x - 3)(x + 3)$ will score but A0

A1 for a fully correct factorized expression – accept $x(x - 3)^2$

If they "solve" use ISW

S.C.

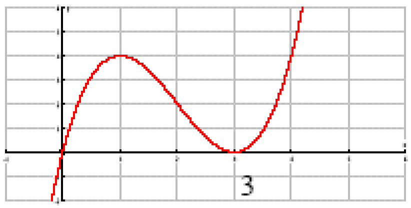
If the only correct linear factor is (x – 3), perhaps from factor theorem, award B0M1A0

Do not award marks for factorising in part (b)

<u>For the graphs</u>

"Sharp points" will lose the 1st B1 in (b) but otherwise be generous on shape Condone (0, 3) in (b) and (0, 2), (0,5) in (c) if the points are marked in the correct places.

(b)



Shape 

<u>Through</u> origin (<u>not</u> touching) B1

Touching x-axis only once B1

Touching at (3, 0), or 3 on x-axis B1ft 4

[Must be on graph not in a table]

<u>Note</u>

2nd B1 for a curve that starts or terminates at (0, 0) score B0

4th B1ft for a curve that touches (not crossing or terminating) at $(a, 0)$ where their $y = x(x - a)^2$

(c)

Moved horizontally (either way)

(2, 0) and (5, 0), or 2 and 5 on x-axis A1 2

Note

for their graph moved horizontally (only) or a fully correct graph
Condone a partial stretch if ignoring their values looks like a simple translation

A1 for their graph translated 2 to the right and crossing or touching the axis at 2 and 5 only

Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)

[9]

6. (a) $b^2 - 4ac > 0 \Rightarrow 16 - 4k(5 - k) > 0$ or equiv., e.g. $16 > 4k(5 - k)$ M1A1

So $k^2 - 5k + 4 > 0$ (Allow any order of terms,
e.g. $4 - 5k + k^2 > 0$) (*) A1cso 3

Note

for attempting to use the discriminant of the initial equation (> 0 not required, but substitution of a, b and c in the correct formula is required).

If the formula $b^2 - 4ac$ is seen, at least 2 of a, b and c must be correct.

If the formula $b^2 - 4ac$ is not seen, all 3 (a, b and c) must be correct.

This mark can still be scored if substitution in $b^2 - 4ac$ is within the quadratic formula.
This mark can also be scored by comparing $b2$ and $4ac$ (with substitution).

However, use of $b^2 + 4ac$ is M0.

1st A1 for fully correct expression, possibly unsimplified, with > symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2 - 4ac > 0$ or 'discriminant positive'.
Condone a bracketing slip, e.g. $16 - 4 \times k \times 5 - k$ if subsequent work is correct and convincing.

2nd A1 for a fully correct derivation with no incorrect working seen.
Condone a bracketing slip if otherwise correct and convincing.

Using $\sqrt{b^2} - 4ac > 0$:

Only available mark is the first (unless recovery is seen).

(b) Critical Values $(k - 4)(k - 1) = 0$ $k = ...$

$k = 1$ or 4 A1

Choosing "outside" region

$k < 1$ or $k > 4$ A1 4

Note

1st for attempt to solve an appropriate 3TQ

1st A1 for both $k = 1$ and 4 (only the critical values are required, so accept, e.g. $k > 1$ and $k > 4$). * *

2nd for choosing the "outside" region. A diagram or table alone is not sufficient. Follow through their values of k. The set of values must be 'narrowed down' to score this M mark… listing everything $k < 1$, $1 < k < 4$, $k > 4$ is M0

2nd A1 for correct answer only, condone "$k < 1$, $k > 4$" and even "$k < 1$ and $k > 4$",.
but "$1 > k > 4$" is A0.

* * Often the statement $k > 1$ and $k > 4$ is followed by the correct final answer. Allow full marks.

Seeing 1 and 4 used as critical values gives the first A1 by implication.

In part (b), condone working with x's except for the final mark, where the set of values must be a set of values of k (i.e. 3 marks out of 4).

Use of $\leq$ (or $\geq$) in the final answer loses the final mark.

[7]

7. (a) [No real roots implies $b^2 - 4ac < 0$.] $b^2 - 4ac = q^2 - 4 \times 2q \times (-1)$

So $q^2 - 4 \times 2q \times (-1) < 0$ i.e. $q^2 + 8q < 0$ (*) A1cso 2

for attempting $b^2 - 4ac$ with one of b or a correct.
< 0 not needed for
This may be inside a square root.

A1cso for simplifying to printed result with no incorrect working or statements seen.
Need an intermediate step

e.g.$q^2 - -8q < 0$ or $q^2 - 4 \times 2q \times -1 < 0$ or $q^2 - 4(2q)(-1) < 0$
or $q^2 - 8q(-1) < 0$ or $q^2 - 8q \times -1 < 0$

i.e. must have × or brackets on the $4ac$ term

< 0 must be seen at least one line before the final answer.

(b) $q(q + 8) = 0$ or $(q \pm 4)^2 \pm 16 = 0$

$(q) = 0$ or -8 (2 cvs) A1

$-8 < q < 0$ <u>or</u> $q \in (-8, 0)$ <u>or</u> $q < 0$ and $q > -8$ A1ft 3

for factorizing or completing the square or attempting to solve $q^2 \pm 8q = 0$. A method that would lead to 2 values for q. The "= 0" may be implied by values appearing later.

1st A1 for $q = 0$ and $q = -8$

2nd A1 for $-8 < q < 0$. Can follow through their cvs but must choose "inside" region.
$q < 0$, $q > -8$ is A0, $q < 0$ or $q > -8$ is A0,
$(-8, 0)$ on its own is A0
BUT " $q < 0$ and $q > -8$" is A1

Do not accept a number line for final mark

[5]

8. (a) $x^2 + kx + (8 - k)\ (= 0)$ $8 - k$ need not be bracketed

$b^2 - 4ac = k^2 - 4(8 - k)$

$b^2 - 4ac < 0 \Rightarrow k^2 + 4k - 32 < 0$ (*) A1cso 3

1st M: Using the k from the right hand side to form 3-term quadratic in x ('= 0' can be implied), or…

attempting to complete the square $\left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 - k (= 0)$

or equiv., using the k from the right hand side.

For either approach, condone sign errors.

1st M may be implied when candidate moves straight to the discriminant.

2nd M: Dependent on the 1st M.

Forming expressions in k (with no x's) by using b^2 and $4ac$.

(Usually seen as the discriminant $b^2 - 4ac$, but separate expressions are fine, and also allow the use of $b^2 + 4ac$.

(For 'completing the square' approach, the expression must be clearly separated from the equation in x).

If b^2 and $4ac$ are used in the quadratic formula, they must be clearly separated from the formula to score this mark.

For any approach, condone sign errors.

If the wrong statement $\sqrt{b^2 - 4ac} < 0$ is seen, maximum score is A0.

(b) $(k + 8)(k - 4) = 0$ $k = ...$

$k = -8$ $k = 4$ A1

Choosing 'inside' region (between the two k values)

$-8 < k < 4$ or $4 > k > -8$ A1 4

Condone the use of × (instead of k) in part (b).

1st M: Attempt to solve a 3-term quadratic equation in k.

It might be different from the given quadratic in part (a).

Ignore the use of < in solving the equation. The 1st A1 can be scored if –8 and 4 are achieved, even if stated as $k < -8$, $k < 4$.

Allow the first A1 to be scored in part (a).

N.B. '$k > -8$, $k < 4$' scores 2nd A0

'$k > -8$ or $k < 4$' scores 2nd A0

'$k > -8$ and $k < 4$' scores 2nd A1

'$k = -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$' scores 2nd M0 A0

Use of ≤ (in the answer) loses the final mark.

[7]

9. Critical Values

$(x \pm a)(x \pm b)$ with $ab = 18$ or $x = \dfrac{7 \pm \sqrt{49 - -72}}{2}$ or $(x - \frac{7}{2})^2 \pm \left(\frac{7}{2}\right)^2 - 18$

$(x - 9)(x + 2)$ or $x = \dfrac{7 \pm 11}{2}$ or $x = \dfrac{7}{2} \pm \dfrac{11}{2}$ A1

Solving Inequality $x > 9$ or $x < -2$ Choosing "outside" A1

1st For attempting to find critical values.
Factors alone are OK for $x =$ appearing somewhere for the formula and as written for completing the square

1st A1 Factors alone are OK. Formula or completing the square need $x =$ as written.

2nd For choosing outside region. Can f.t. their critical values.
They must have two different critical values.

$-2 > x > 9$ is M1A0 but ignore if it follows a correct version

$-2 < x < 9$ is M0A0 whatever the diagram looks like.

2nd A1 Use of $\geq$ in final answer gets A0

[4]

10. (a) $b^2 - 4ac = 4p^2 - 4(3p + 4) = 4p^2 - 12p - 16 \; (= 0)$ A1

or $(x + p)^2 - p^2 + (3p + 4) = 0 \Rightarrow p^2 - 3p - 4 (= 0)$

$(p - 4)(p + 1) = 0$

$p = (-1 \text{ or}) \; 4$ A1c.s.o. 4

1st *For use of $b^2 - 4ac$ or a full attempt to complete the square leading to a 3TQ in p.*

May use $b^2 = 4ac$. One of b or c must be correct.

1st A1 *For a correct 3TQ in p. Condone missing "=0" but all 3 terms must be on one side.*

2nd *For attempt to solve their 3TQ leading to p = ...*

2nd A1 *For p = 4 (ignore p = –1).*

$b^2 = 4ac$ leading to $p^2 = 4(3p + 4)$ and then "spotting" p = 4 scores 4/4.

(b) $x = \dfrac{-b}{2a}$ or $(x + p)(x + p) = 0 \Rightarrow x = \ldots$

$x \; (= -p) = -4$ A1f.t. 2

For a full method leading to a repeated root x ...

A1f.t. *For x = –4 (– their p)*

<u>Trial and Improvement</u>

For substituting values of p into the equation and attempting to factorize.

(Really need to get to p = 4 or –1)

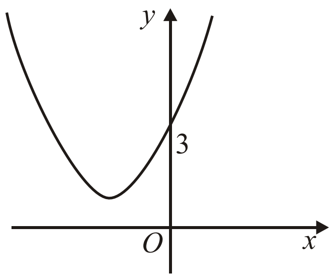
A2c.s.o. *Achieve p = 4. Don't give without valid method being seen.*

[6]

11. (a) $x^2 + 2x + 3 = (x + 1)^2 + 2$

$a = 1, b = 2$ B1, B1 2

(b)



U shape anywhere

minimum ft their a and positive b A1ft

(0, 3) marked B1 3

(c) $\Delta = b^2 - 4ac = 2^2 - 4 \times 3 = -8$ B1

The negative sign implies there are no real roots and, hence, the curve in (b) does not intersect (meet, cut, ..) the x–axis. Accept equivalent statements and the statement that the whole curve is above the x–axis. B1 2

(d) $\Delta = k^2 - 12$

$\Delta < 0 \quad \Rightarrow \quad k^2 - 12 < 0 \quad (\text{or } k^2 < 12)$ A1

$-2\sqrt{3} < k < 2\sqrt{3}$ Allow $\sqrt{12}$ A1 4

If just $k < 2\sqrt{3}$ allow A0

[11]

Alternative to (d)

$\frac{dy}{dx} = 0 \quad \Rightarrow \quad 2x + k = 0 \quad \Rightarrow \quad x = -\frac{k}{2}$

Minimum greater than 0 implies $\frac{k^2}{4} - \frac{k^2}{2} + 3 > 0$

$k^2 < 12$ A1

Then as before.

12.	(a)	$(x-3)^2$, $+9$ isw . $a = 3$ and $b = 9$ may just be written down with no method shown.	B1,	A1	3
	(b)	P is (3, 9)		B1	
	(c)	A = (0, 18)		B1	
		$\frac{dy}{dx} = 2x - 6$, at A $m = -6$		A1	
		Equation of tangent is $y - 18 = -6x$ (in any form)		A1ft	4
	(d)	Showing that line meets x axis directly below P, i.e. at $x = 3$.		A1cso	1
	(e)	$A = \int x^2 - 6x + 18x = [\frac{1}{3}x^3 - 3x^2 + 18x]$			A1
		Substituting $x = 3$ to find area A under curve A [=36]			
		Area of $R = A$ – area of triangle $= A - \frac{1}{2} \times 18\ 3, = 9$		A1	5
		Alternative: $\int x^2 - 6x + 18 - (18 - 6x)dx$			
		$= \frac{1}{3}x^3$ A1 ft			
		Use $x = 3$ to give answer 9		A1	

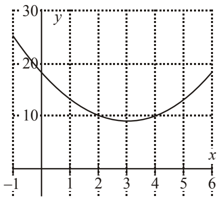
[13]

13.	Attempt to use discriminant $b^2 - 4ac$ Should have no x's			
	(Need not be equated to zero) (Could be within the quadratic formula)			
	$144 - 4 \times k \times k = 0$ or $\sqrt{144 - 4 \times k \times k} = 0$		A1	
	Attempt to solve for k (Could be an inequality)			
	$k = 6$		A1	4

[4]

14. (a) $x^2 - 6x + 18 = (x - 3)^2, +9$ B1, A1 3

(b)



"U"-shaped parabola

Vertex in correct quadrant A1ft

P: (0, 18) (or 18 on y-axis) B1

Q: (3, 9) B1ft 4

(c) $x^2 - 6x + 18 = 41$ or $(x - 3)^2 + 9 = 41$

Attempt to solve 3 term quadratic $x = \ldots$

$x = \dfrac{6 \pm \sqrt{36 - (4 \times -23)}}{2}$ (or equiv.) A1

$\sqrt{128} = \sqrt{64} \times \sqrt{2}$ (or surd manipulation $\sqrt{2a} = \sqrt{2}\sqrt{a}$)

$3 + 4\sqrt{2}$ A1 5

[12]

15. (a) $b^2 - 4ac = (-k)^2 - 36 = k^2 - 36$ A1

Or, (completing the square), $\left(x - \frac{1}{2}k\right)^2 = \frac{1}{4}k^2 - 9$

Or, if b^2 and $4ac$ are compared directly, [for finding both

[A1] for k^2 and 36. A1

No real solutions: $k^2 - 36 < 0$, $-6 < k < 6$ (ft their "36") A1ft 4

(b) $x^2 - 4x + 9 = (x - 2)^2$ ………. ($p = 2$) B1

Ignore statement $p = -2$ if otherwise correct.

$x^2 - 4x + 9 = (x - 2)^2 - 4 + 9 = (x - 2)^2 + 5$ ($q = 5$) A1 3

M: Attempting $(x \pm a)^2 \pm b \pm 9$, $a \neq 0$, $b \neq 0$.

(c) Min value 5 (or just q), occurs where $x = 2$ (or just p) B1ft, B1ft 2

Alternative: $f'(x) = 2x - 4$ (Min occurs where) $x = 2$ [B1]

Where $x = 2$, $f(x) = 5$ [B1ft]

[9]

16.	(a)	$4x(x + 3) = 0$, $x = \dots$ (or use of quadratic formula)		
		$x = 0 \quad x = -3$	A1 A1	3
	(b)	Using $b^2 - 4ac = 0$ or other method, proceed to $c = \dots$		
		$c = 9$	A1	
		$(2x + 3)(2x + 3) = 0$ $x = \dots$ (or other method to solve a 3-term quadratic)		
		$x = -\frac{3}{2}$	A1	4

[7]

1. Part (a) was answered well with many scoring both marks. Some gave $q = 20$ from adding 11 + 9 instead of subtracting but most understood the principle of completing the square.

Quite a number of candidates struggled with the sketch in part (b). Most had the correct shape but the minimum was invariably in the wrong position: on the y-axis at (0, 11) or on the x-axis at (–3, 0) were common errors but the intercept at (0, 11) was more often correct.

Some candidates did not know what the discriminant was. Some confused it with the derivative, others knew it was something to do with the quadratic formula and simply applied the formula to the original equation. The correct formula was used by many candidates but a few faltered over the arithmetic with "36 – 44 = –12" being quite common.

Few candidates seemed to spot the connections between the parts in this question: (a) was intended to help them with the sketch in part (b) and a negative discriminant in (c) confirmed that their sketch did not cross the x-axis. Candidates should be encouraged to identify these connections.

2. The majority sketched a quadratic and a cubic curve in part (a) but not always with the correct features. The quadratic was often U shaped and although the intercepts at (0, 0) and (4, 0) were mostly correct, sometimes the curve passes through (–4, 0) and (4, 0). The cubic was sometimes a positive cubic and whilst it often passes through (0, 0) and (7, 0) the turning point was not always at the origin and the intercept was sometimes at (–7, 0).

Part (b) caused few problems with most candidates scoring full marks here.

Most could start part (c) and the quadratic formula was usually used to solve their equation. Although many simplified $\sqrt{48}$ to $4\sqrt{3}$ several candidates failed to divide by 2 correctly and gave their answers as $x = 4 \pm 4\sqrt{3}$. Most realised they needed to find the y-coordinate as well and usually they substituted their value of x into the quadratic equation to find y, though some chose the much less friendly cubic equation instead.

The selection of the correct solution defeated all but the best candidates. Most successful solutions involved checking the y coordinates for both $x = 4 + 2\sqrt{3}$ and $x = 4 - 2\sqrt{3}$ and, if the calculations were correct, selecting the one that gave a positive y coordinate. Only a rare minority realised that the required point would have an x coordinate in the interval (0, 4) and therefore only the $x = 4 - 2\sqrt{3}$ case need be considered.

3. This was a demanding question on which few candidates scored full marks. In part (a), many found the algebra challenging and their attempts to complete the square often led to mistakes such as $x^2 + 4kx = (x + 2k)^2 - 4k$.
Rather than using the result of part (a) to answer part (b), the vast majority used the discriminant of the given equation. Numerical and algebraic errors were extremely common at this stage, and even those who obtained the correct condition $4k^2 - 11k - 3 < 0$ were often unable to solve this inequality to find the required set of values for k.
The sketch in part (c) could have been done independently of the rest of the question, so it was disappointing to see so many poor attempts. Methods were too often overcomplicated, with many candidates wasting time by unnecessarily solving the equation with $k = 1$. Where a sketch was eventually seen, common mistakes were to have the curve touching the x-axis or to have the minimum on the y-axis.

4. Some candidates opted out of this question but most realised that they needed to use the discriminant and made some progress. The most common error was to simplify $(3p)^2$ to $3p^2$ but there were many correct equations seen and usually these led to a correct answer. Many candidates chose to solve their equation by cancelling a p. In this case that was fine, since they were told that p was non-zero, but this is not good practice in general and factorising $9p^2 - 4p$ to $p(9p - 4)$ is recommended.

5. It seemed clear that many candidates did not appreciate the links between the 3 parts of this question and there was much unnecessary work carried out: differentiating to find turning points and tables of values to help draw, not sketch, the curves.

Part (a) was usually answered well but not all the successful candidates started by taking out the factor of x, rather they tried to use the factor theorem to establish a first factor. Whilst techniques from (or higher units) **may** be used in they are not required and the "best" approach will not use them.

A correct factorisation in (a) should have made the sketch in (b) straightforward. Most drew a cubic curve (but some had a negative cubic not a positive one) and usually their curve either touched or passed through the origin. The most common non-cubic curve was a parabola passing through (0, 0) and (3,0). Part (c) looked complicated but those who spotted that they were sketching f(x – 2) had few problems in securing both marks. Many candidates though embarked upon half a page or more of algebraic manipulation to no avail - this part was only worth 2 marks and a little thought may have helped them realise that such an approach was unlikely to be the correct one.

6. Candidates who understood the demands of this question usually did well, while others struggled to pick up marks. In part (a), those who correctly used the discriminant of the original equation often progressed well, but it was sometimes unclear whether they knew the condition for different real roots. An initial statement such as "$b^2 – 4ac > 0$ for different real roots" would have convinced examiners. Irrelevant work with the discriminant of $k^2 – 5k + 4$ was sometimes seen.
In part (b) by the vast majority of candidates scored two marks for finding the correct critical values, although it was disappointing to see so many resorting to the quadratic formula. It was surprising, however, that many did not manage to identify the required set of values of k. The inappropriate statement "$1 > k > 4$" was sometimes given as the final answer, rather than "$k < 1$ or $k > 4$".

7. As usual many candidates floundered on this type of question or simply omitted it. Many though knew that the discriminant was required but some lost the A mark for failing to introduce an inequality before their final statement. Those who started with $b^2 < 4ac$ tended to make fewer sign errors. In part (b) those who factorized were more successful in arriving at the two critical values than those who used the quadratic formula or tried to complete the square. A surprising number failed to give a final inequality and a few listed integers for their answer. The final mark was sometimes lost as the candidate switched the variable from q to x.

8. Although those candidates who started part (a) correctly were usually able to derive the required inequality, many were unsure of what was required here. A substantial number of candidates failed to form a three term quadratic equal to zero before attempting to write down the discriminant. Weaker candidates simply wrote down the discriminant of the given quadratic expression in k, or perhaps solved the quadratic in k to find the critical values required for part (b). Some candidates substituted $k = x^2 + kx + 8$ into $k^2 + 4k – 32 < 0$ and proceeded to waste time in producing some very complicated algebra.

In part (b), most candidates were able to find the critical values but not all offered a solution to the inequality. Many of those who found the correct set of values of k did so with the help of a sketch. The most common incorrect critical values were –4 and 8 (instead of –8 and 4). Some candidates lost the final mark because their inequalities $k >– 8$, $k < 4$ were not combined as $– 8 < k < 4$. Generally, however, part (b) was well done.

9. It was encouraging to see most candidates factorizing the quadratic expression in order to find the critical values for the inequality. Sometimes the critical values had incorrect signs, despite the factorization being correct, but the most common error was still a failure to select the outside region. Some candidates struggled with the correct symbolic notation for the answer and $-2 > x > 9$ was occasionally seen. A few candidates chose to use the formula or completing the square to find the critical values, these approaches are less efficient in this case and often gave rise to arithmetic errors.

10. Candidates who equated the discriminant $b^2 - 4ac$ to zero were often successful although b^2 was often given as $2p^2$ Sometimes a second error in multiplying out $-4(3p + 4)$ as $-12p + 16$ led to the incorrect equation $2p^2 - 12p + 16 = 0$ This of course led to $p = 4$ and although accuracy marks were lost in part (a) full marks could be gained in part (b). Some tried to complete the square usually with little success but others did find $p = 4$ by trial and improvement. A few candidates spotted that $3p + 4$ had to be a square number and indeed had to be p^2 and this often lead to a correct solution. In (b) the majority simply solved to find equal roots, very few candidates seemed to appreciate that the root will simply be $x = \frac{b}{2a}$.

11. **Pure Mathematics P1**

Parts (a) and (b) were generally well done although the error of having the minimum of the curve on the y-axis was sometimes seen. There were a few candidates who were not familiar with the term discriminant but the majority could calculate it correctly and make the connection, which is that the curve does not intersect the x-axis, with their diagram. Part (d) proved demanding and only a minority of candidates could give the complete range
$-2\sqrt{3} < k < 2\sqrt{3}$.

Core Mathematics

Few candidates scored full marks on this question.

In part (a), most were familiar with the method of 'completing the square' and were able to produce correct values for a and b. Most candidates knew that a sketch of a parabola was required in part (b), but the vertex was usually in the wrong position, often in the first quadrant. Some candidates omitted part (c) and others clearly had little idea of the definition or significance of 'discriminant'. Some were able to find its value but were not able to relate this to the sketch.

Although in part (d) most candidates realised the need to use a condition involving $b^2 - 4ac$, the condition was sometimes thought to be $b_2 - 4ac = 0$. About half of all candidates did correctly reach $k^2 - 12 < 0$, but then very few proceeded to a correct set of values for k, the most popular suggestion being $k^2 < \pm\sqrt{12}$.

12. (a) This was generally answered very well. Many candidates scored full marks, with others gaining B1, A0, for answers such as $(x – 3)^2$-9, or -27, or +27

(b) The majority of candidates did not use their (a) to get the answer in (b) but used differentiation. This meant that most candidates had this part correct, even if they had (a) incorrect or didn't attempt it.

(c) Again this part was well answered, especially by those who had done the differentiation in (b) as they went on to get the gradient of -6, and used (0,18) to get the equation of the line. Unfortunately, some assumed the coordinates of Q at (3,0), and found the gradient using the points A and Q, so didn't gain many marks at all in this part.

(d) Generally if candidates had answered part (c) correctly, they were able to do part (d) as well, although quite a few lost credibility because they stated that the gradient was 0. Many compared the x coordinates and deduced the line was parallel to the y axis and gained the credit.

(e) This was very well answered. Many candidates had full marks in this part even if they hadn't scored full marks earlier. A few candidates made the mistake of using the y value of 9 instead of the x value of 3 in the integral. Other errors included using a trapezium instead of a triangle, and some candidates made small slips such as the 18 being copied down as an 8 or integrating the 6x to get 6x/2, or $6x^2$. It is possible that these candidates were short of time.

13. Those candidates who equated the discriminant to zero to form an equation for *k* were often successful in reaching a correct answer, although a common mistake was to proceed from $4k^2 = 144$ to $4k = 12$. Other approaches, which included completing the square, factorisation attempts and trial and error, were rarely successful.

14. Most candidates were familiar with the method of "completing the square" and were able to produce a correct solution to part (a). Sketches in part (b), however, were often disappointing. Although most candidates knew that a parabola was required, the minimum point was often in the wrong position, sometimes in the fourth quadrant and sometimes at (3, 0). Some candidates omitted part (c), but most knew what was required and some very good, concise solutions were seen. Those who used the answer to part (a) and formed the equation $(x-3)^2 + 9 = 41$ were able to proceed more easily to an answer from $x = 3 \pm \sqrt{32}$. Some candidates found difficulty in manipulating surds and could not cope with the final step of expressing $\sqrt{32}$ as $4\sqrt{2}$.

15. Candidates found part (a) of this question difficult. Although they often started correctly by considering (either explicitly or implicitly) $b^2 - 4ac$, their algebra in dealing with $b = -k$ was sometimes faulty and then many failed to indicate clearly the condition for "no real solutions". Even those who did manage to reach $k^2 < 36$ were often unable to proceed to the correct set of values for k. There was greater success in part (b), where an encouraging number of candidates were able to complete the square appropriately. Some then used their expression to identify correctly the minimum value and the value of x for which it occurred, while others used differentiation. Occasionally candidates were confused between the value of the function and the value of x.

16. Many candidates had difficulty with this question. In part (a), some preferred to use the quadratic formula rather than to factorise $4x^2 + 12x$, and it was very common for the $x = 0$ root to be omitted. Having found a solution to part (a), many candidates thought that they needed to use this in part (b), and made no effective progress. Apart from this, various valid approaches were seen, including the use of the discriminant condition for equal roots, completion of the square, and use of the fact that, for equal roots, the derivative of the function is zero at the root.