

1. Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x$ (2)

(b) $(2x - 7)(1 + x) < 0$ (3)

(c) both $3(x - 2) < 8 - 2x$ **and** $(2x - 7)(1 + x) < 0$ (1)
(Total 6 marks)

2. $f(x) = x^2 + 4kx + (3 + 11k)$, where k is a constant.

(a) Express $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k . (3)

Given that the equation $f(x) = 0$ has no real roots,

(b) find the set of possible values of k . (4)

Given that $k = 1$,

(c) sketch the graph of $y = f(x)$, showing the coordinates of any point at which the graph crosses a coordinate axis. (3)
(Total 10 marks)

3. Find the set of values of x for which

(a) $4x - 3 > 7 - x$ (2)

(b) $2x^2 - 5x - 12 < 0$ (4)

(c) **both** $4x - 3 > 7 - x$ **and** $2x^2 - 5x - 12 < 0$ (1)

(Total 7 marks)

4. The equation $kx^2 + 4x + (5 - k) = 0$, where k is a constant, has 2 different real solutions for x .

(a) Show that k satisfies

$$k^2 - 5k + 4 > 0. \quad (3)$$

(b) Hence find the set of possible values of k . (4)

(Total 7 marks)

5. The equation $x^2 + kx + (k + 3) = 0$, where k is a constant, has different real roots.

(a) Show that $k^2 - 4k - 12 > 0$. (2)

(b) Find the set of possible values of k . (4)

(Total 6 marks)

6. The equation $2x^2 - 3x - (k + 1) = 0$, where k is a constant, has no real roots.

Find the set of possible values of k .

(Total 4 marks)

7. Find the set of values of x for which

(a) $3(2x + 1) > 5 - 2x$,

(2)

(b) $2x^2 - 7x + 3 > 0$,

(4)

(c) **both** $3(2x + 1) > 5 - 2x$ **and** $2x^2 - 7x + 3 > 0$.

(2)

(Total 8 marks)

8. The width of a rectangular sports pitch is x metres, $x > 0$. The length of the pitch is 20 m more than its width. Given that the perimeter of the pitch must be less than 300 m,

(a) form a linear inequality in x .

(2)

Given that the area of the pitch must be greater than 4800 m^2 ,

(b) form a quadratic inequality in x .

(2)

(c) by solving your inequalities, find the set of possible values of x .

(4)

(Total 8 marks)

9. Find the set of values for x for which

(a) $6x - 7 < 2x + 3,$

(2)

(b) $2x^2 - 11x + 5 < 0,$

(4)

(c) both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0.$

(1)

(Total 7 marks)

10. Find the set of values of x for which

$$(2x + 1)(x - 2) > 2(x + 5).$$

(Total 7 marks)

11. Solve the inequality

$$10 + x^2 > x(x - 2).$$

(Total 3 marks)

1. (a) $3x - 6 < 8 - 2x \rightarrow 5x < 14$ (Accept $5x - 14 < 0$ (o.e.))
 $x < 2.8$ or $\frac{14}{5}$ or $2\frac{4}{5}$ (condone \leq) A1 2

Note

for attempt to rearrange to $kx < m$ (o.e.) Either $k = 5$ or $m = 14$ should be correct Allow $5x = 14$ or even $5x > 14$

- (b) Critical values are $x = \frac{7}{2}$ and -1 B1
 Choosing “inside” $-1 < x < \frac{7}{2}$ A1 3

Note

B1 for both correct critical values. (May be implied by a correct inequality)

ft their values and choose the “inside” region

A1 for fully correct inequality (Must be in part (b): do not give marks if only seen in (c)) Condone seeing $x < -1$ in working provided $-1 < x$ is in the final answer

e.g. $x > -1, x < \frac{7}{2}$ or $x > -1$ “or” $x < \frac{7}{2}$ or $x > -1$ “blank space”

$x < \frac{7}{2}$ score M1A0

BUT allow $x > -1$ and $x < \frac{7}{2}$ to score M1A1 (the “and” must be seen)

Also $(-1, \frac{7}{2})$ will score M1A1

NB $x < -1, x < \frac{7}{2}$ is of course M0A0 and a number line even with “open” ends is M0A0

Allow 3.5 instead of $\frac{7}{2}$

- (c) $-1 < x < 2.8$ B1ft 1

Accept any exact equivalents to $-1, 2.8, 3.5$

Note

B1ft for $-1 < x < 2.8$ (ignoring their previous answers) or ft their answers to part (a) and part (b) provided both answers were regions and not single values.

Allow use of “and” between inequalities as in part (b)

If their set is empty allow a suitable description in words or the symbol \emptyset .

Common error: If (a) is correct and in (b) they simply leave their answer as $x < -1, x < 3.5$ then in (c) $x < -1$ would get B1ft as this is a correct follow through of these 3 inequalities.

Penalise use of \leq only on the A1 in part (b). [i.e. condone in part (a)]

[6]

2. (a) $(x + 2k)^2$ or $\left(x + \frac{4k}{2}\right)^2$

$(x \pm F)^2 \pm G \pm 3 \pm 11k$ (where F and G are any functions of k , not involving x)

$(x + 2k)^2 - 4k^2 + (3+11k)$ Accept unsimplified equivalents such as

A1 3

$$\left(x + \frac{4k}{2}\right)^2 - \left(\frac{4k}{2}\right)^2 + 3 + 11k,$$

and i.s.w. if necessary.

(b) Accept part (b) solutions seen in part (a).

$$“4k^2 - 11k - 3” = 0 \quad (4k + 1)(k - 3) = 0 \quad k = \dots,$$

[Or, ‘starting again’, $b^2 - 4ac = (4k)^2 - 4(3 + 11k)$

and proceed to $k = \dots$] $-\frac{1}{4} < k < 3$ (Ignore any inequalities

for the first 2 marks in (b)).

A1

Using $b^2 - 4ac < 0$ for no real roots, i.e. “ $4k^2 - 11k - 3$ ” < 0 , to establish inequalities involving their two critical values m and n

(even if the inequalities are wrong, e.g. $k < m$, $k < n$).

$-\frac{1}{4} < k < 3$ (See conditions below) Follow through

their critical values.

A1ft 4

The final A1ft is still scored if the answer $m < k < n$ follows $k < m$, $k < n$.

Using x instead of k in the final answer loses only the 2nd A mark, (condone use of x in earlier working).

Note

1st M: Forming and solving a 3-term quadratic in k (usual rules.. see general principles at end of scheme). The quadratic must come from “ $b^2 - 4ac$ ”, or from the “ q ” in part (a).

Using wrong discriminant, e.g. “ $b^2 + 4ac$ ” will score no marks in part (b).

2nd M: As defined in main scheme above.

2nd A1ft: $m < k < n$, where $m < n$, for their critical values m and n .

Other possible forms of the answer (in each case $m < n$):

(i) $n > k > m$

(ii) $k > m$ and $k < n$

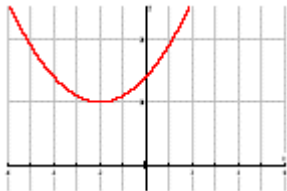
In this case the word “and” must be seen (implying intersection).


(iii) $k \in (m, n)$ (iv) $\{k : k > m\} \cap \{k : k < n\}$

Not just a number line.

Not just $k > m, k < n$ (without the word “and”).

(c)



Shape  (seen in (c))

B1

Minimum in correct quadrant, not touching the x -axis, not on the y -axis, and there must be no other minimum or maximum.

B1

(0, 14) or 14 on y -axis.

Allow (14, 0) marked on y -axis.

n.b. Minimum is at $(-2, 10)$, (but there is no mark for this).

B1

3

Note

Final B1 is dependent upon a sketch having been attempted in part (c).

[10]

3. (a) $5x > 10, x > 2$ [Condone $x > \frac{10}{2} = 2$ for M1A1] A1 2

Note

for attempt to collect like terms on each side leading to $ax > b$, or $ax < b$, or $ax = b$

Must have a or b correct so eg $3x > 4$ scores M0

- (b) $(2x + 3)(x - 4) = 0$, ‘Critical values’ are $-\frac{3}{2}$ and 4 A1

$$-\frac{3}{2} < x < 4 \quad \text{A1ft} \quad 4$$

Note

1st for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values

1st A1 for $-\frac{3}{2}$ and 4 seen. They may write $x < -\frac{3}{2}, x < 4$ and still get this A1

2nd for choosing the “inside region” for their critical values

2nd A1ft follow through their 2 distinct critical values

Allow $x > -\frac{3}{2}$ with “or” “,” “ \cup ” $x < 4$ to score M1A0 but “and”

or “ \cap ” score M1A1

$x \in (-\frac{3}{2}, 4)$ is M1A1 but $x \in [-\frac{3}{2}, 4]$ is M1A0. Score M0A0 for a number line or graph only

- (c) $2 < x < 4$ B1ft 1

Note

B1ft Allow if a **correct answer is seen** or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) must be regions.

Do not follow through single values.

If their follow through answer is the empty set accept \emptyset or $\{\}$ or equivalent in words

If (a) or (b) are not given then score this mark for cao

NB You may see $x < 4$ (with anything or nothing in-between) $x < -1.5$ in (b) and empty set in (c) for B1ft

Do not award marks for part (b) if only seen in part (c)

Use of \leq instead of $<$ (or \geq instead of $>$) loses one accuracy mark only, at first occurrence.

[7]

4. (a) $b^2 - 4ac > 0 \Rightarrow 16 - 4k(5 - k) > 0$ or equiv., e.g. $16 > 4k(5 - k)$ M1A1

So $k^2 - 5k + 4 > 0$ (Allow any order of terms,

e.g. $4 - 5k + k^2 > 0$)

(*)A1cso 3

Note

for attempting to use the discriminant of the initial equation (> 0 not required, but substitution of a , b and c in the correct formula is required).

If the formula $b^2 - 4ac$ is seen, at least 2 of a , b and c must be correct.

If the formula $b^2 - 4ac$ is not seen, all 3 (a , b and c) must be correct.

This mark can still be scored if substitution in $b^2 - 4ac$ is within the quadratic formula.

This mark can also be scored by comparing b^2 and $4ac$ (with substitution).

However, use of $b^2 + 4ac$ is M0.

1st A1 for fully correct expression, possibly unsimplified, with > symbol. NB must appear before the last line, even if this is simply in a statement such as $b^2 - 4ac > 0$ or ‘discriminant positive’.

Condone a bracketing slip, e.g. $16 - 4 \times k \times 5 - k$ if subsequent work is correct and convincing.

2nd A1 for a fully correct derivation with no incorrect working seen. Condone a bracketing slip if otherwise correct and convincing.

Using $\sqrt{b^2 - 4ac} > 0$:

Only available mark is the first (unless recovery is seen).

(b) Critical Values $(k - 4)(k - 1) = 0$ $k = \dots$

$k = 1$ or 4 A1

Choosing “outside” region

$k < 1$ or $k > 4$ A1 4

Note

1st for attempt to solve an appropriate 3TQ

1st A1 for both $k = 1$ and 4 (only the critical values are required, so accept, e.g. $k > 1$ and $k > 4$). * *

2nd for choosing the “outside” region. A diagram or table alone is not sufficient. Follow through their values of k . The set of values must be ‘narrowed down’ to score this M mark... listing everything

$k < 1, 1 < k < 4, k > 4$ is M0

2nd A1 for correct answer only, condone “ $k < 1, k > 4$ ” and even “ $k < 1$ and $k > 4$ ”, but “ $1 > k > 4$ ” is A0.

** Often the statement $k > 1$ and $k > 4$ is followed by the correct final answer. Allow full marks.

Seeing 1 and 4 used as critical values gives the first A1 by implication.

In part (b), condone working with x 's except for the final mark, where the set of values must be a set of values of k (i.e. 3 marks out of 4).

Use of \leq (or \geq) in the final answer loses the final mark.

[7]

5. (a) Attempt to use discriminant $b^2 - 4ac$
 $k^2 - 4(k+3) > 0$ $k^2 - 4k - 12 > 0$ (*) A1cso 2

for use of $b^2 - 4ac$, one of b or c must be correct.

Or full attempt using completing the square that leads to a 3TQ in k

$$\text{e.g. } \left(\left[x + \frac{k}{2} \right]^2 = \right) \frac{k^2}{4} - (k+3)$$

A1cso Correct argument to printed result. Need to state (or imply) that $b^2 - 4ac > 0$ and no incorrect working seen. Must have > 0 . If > 0 just appears with $k^2 - 4(k+3) > 0$ that is OK. If > 0 appears on last line only with no explanation give A0. $b^2 - 4ac$ followed by $k^2 - 4k - 12 > 0$ only is insufficient so M0A0

e.g. $k^2 - 4 \times 1 \times k + 3$ (missing brackets) can get M1A0 but $k^2 + 4(k+3)$ is M0A0 (wrong formula) Using $\sqrt{b^2 - 4ac} > 0$ is M0.

- (b) $k^2 - 4k - 12 = 0 \Rightarrow (k - 6)(k + 2)$, with $ab = 12$
 or $(k =) \frac{4 \pm \sqrt{4^2 - 4 \times (-12)}}{2}$ or $(k - 2)^2 = 2^2 - 12$
 $k = -2$ and 6 (both) A1
 $k < -2, k > 6$ or $(-\infty, -2); (6, \infty)$ M: choosing “outside” A1ft 4
- 1stM1 for attempting to find critical regions. Factors, formula or completing the square
- 1st A1 for $k = 6$ and -2 only
- 2nd for choosing the outside regions
- 2nd A1ft as printed on f.t. their (non identical) critical values
- $6 < k < -2$ is M1A0 but ignore if it follows a correct version
 $-2 < k < 6$ is M0A0 whatever their diagram looks like
- Condone use of x instead of k for critical values and final answers in (b).

Treat this question as 3 two mark parts. If part (a) is seen in (b) or vice versa marks can be awarded.

[6]

6. Use of $b^2 - 4ac$, perhaps implicit (e.g. in quadratic formula)
 $(-3)^2 - 4 \times 2 \times -(k + 1) < 0 \quad (9 + 8(k + 1) < 0)$ A1
 $8k < -17$ (Manipulate to get $pk < q$, or $pk > q$, or $pk = q$)
 $k < -\frac{17}{8}$ (Or equiv: $k < -2\frac{1}{8}$ or $k < -2.125$) A1cso 4

- 1st M: Could also be, for example, comparing or equating b^2 and $4ac$.
 Must be considering the given quadratic equation.
 There must not be x terms in the expression, but there must be a k term.
- 1st A: Correct expression (need not be simplified) and correct inequality sign.
 Allow also $-3^2 - 4 \times 2 \times -(k + 1) < 0$.
- 2nd M: Condone sign or bracketing mistakes in manipulation.
 Not dependent on 1st M, but should not be given for irrelevant work.
 M0 could be scored:
 e.g. where $b^2 + 4ac$ is used instead of $b^2 - 4ac$.

Special cases:

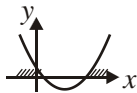
1. Where there are x terms in the discriminant expression, but then division by x^2 gives an inequality/equation in k .
(This could score M0 A0 A1).
 2. Use of \square instead of $<$ loses one A mark only, at first occurrence, so an otherwise correct solution leading to $k \square -\frac{17}{8}$ scores A0 A1.
- N.B. Use of $b = 3$ instead of $b = -3$ implies no A marks.

[4]

7. (a) $6x + 3 > 5 - 2x \square 8x > 2$
 $x > \frac{1}{4}$ or 0.25 or $\frac{2}{8}$ A1 2

Multiply out and collect terms (allow one slip and allow use of = here)

(b) $(2x - 1)(x - 3) (> 0)$
 Critical values $x = \frac{1}{2}, 3$ (both) A1



Choosing “outside” region

$x > 3$ or $x < \frac{1}{2}$ A1 ft 4

1st Attempting to factorise 3TQ $\square x = \dots$

2nd Choosing the outside region

2nd A1 f.t. f.t. their critical values N.B. ($x > 3, x > \frac{1}{2}$ is

M0A0)

For $p < x < q$ where $p > q$ penalise the final A1 in (b)

(c) $x > 3$ or $\frac{1}{4} < x < \frac{1}{2}$ B1f.t. B1f.t. 2

f.t. their answers to (a) and (b)

1st B1 a correct f.t. leading to an infinite region

2nd B1 a correct f.t. leading to a finite region

Penalise \leq or \geq once only at first offence.

e.g. (a) (b) (c) Mark

$x > \frac{1}{4}$ $\frac{1}{2} < x < 3$ $\frac{1}{2} < x < 3$ B0 B1

$x > \frac{1}{4}$ $x > 3, x > \frac{1}{2}$ $x > 3$ B1 B0

[8]

Algebra – Inequalities

8. (a) $2x + 2(x + 20) < 300$ A1 2
 (Using $x - 20$ is A0)

(b) $x(x + 20) > 4800$ A1 2
 (Using $x - 20$ is A0)

(c) 65 (i.e. Allow wrong inequality sign or $x = 65$). B1ft
 Solving 3 term quadratic, $(x + 80)(x - 60) = 0$ $x = \dots$
 $x > 60$ ($x < -80$ may be included here, but there must be no other
wrong solution to the quadratic inequality such as $x > -80$) A1
 $60 < x < 65$ A1 4

[8]

9. (a) $6x - 2x < 3 + 7$ $x < 2\frac{1}{2}$ A1 2

(b) $(2x \square 1)(x \square 5)$ Critical values $\frac{1}{2}$ and 5 A1
 $\frac{1}{2} < x < 5$ A1 ft 4

(c) $\frac{1}{2} < x < 2\frac{1}{2}$ B1 ft 1

[7]

10. $(2x + 1)(x - 2) > 2(x + 5)$
 $2x^2 - 4x + x - 2 > 2x + 10$ A1
 $2x^2 - 5x - 12 > 0$ A1 ft
 $(2x + 3)(x - 4) > 0$ (or solving A1) A1 ft
 $x < -\frac{3}{2}, x > 4$ A1 7

[7]

(c) 65 (i.e. Allow wrong inequality sign or $x = 65$). B1ft
 Solving 3 term quadratic, $(x + 80)(x - 60) = 0$ $x = \dots$
 $x > 60$ ($x < -80$ may be included here, but there must be no other
wrong solution to the quadratic inequality such as $x > -80$) A1
 $60 < x < 65$ A1 4

[8]

Algebra – Inequalities

11. $10 + x^2 \square x^2 \square 2x$
10 \square \square \square \square \square \square

B1

A1 3

[3]

1. Most handled the linear inequality in part (a) very well with only occasional errors in rearranging terms. The responses to part (b) though were less encouraging. It was surprising how many multiplied out the brackets and then tried factorising again (often incorrectly) or used the formula to find the critical values rather than simply writing them down as was intended. Those who found the critical values did not always go on to solve the inequality and those who did often gave their answer as $x < -1, x < 3.5$. Those who sketched a graph of the function were usually more successful in establishing the correct interval.

Part (c) was answered well by many of those who had correct solutions to parts (a) and (b) and some successfully followed through their incorrect answers to gain the mark here. Some did not seem to realise that the intersection of the two intervals was required and simply restated their previous answers making no attempt to combine them. Drawing a number line was helpful for some candidates.

2. This was a demanding question on which few candidates scored full marks. In part (a), many found the algebra challenging and their attempts to complete the square often led to mistakes such as $x^2 + 4kx = (x + 2k)^2 - 4k$. Rather than using the result of part (a) to answer part (b), the vast majority used the discriminant of the given equation. Numerical and algebraic errors were extremely common at this stage, and even those who obtained the correct condition $4k^2 - 11k - 3 < 0$ were often unable to solve this inequality to find the required set of values for k . The sketch in part (c) could have been done independently of the rest of the question, so it was disappointing to see so many poor attempts. Methods were too often overcomplicated, with many candidates wasting time by unnecessarily solving the equation with $k = 1$. Where a sketch was eventually seen, common mistakes were to have the curve touching the x -axis or to have the minimum on the y -axis.

3. Part (a) provided a simple start for the majority of the candidates and apart from a few arithmetic errors most scored full marks.

In part (b) the quadratic expression was factorised and the critical values were usually found correctly however many candidates were unable to identify the solution as a closed region. Many just left their answer as $x < -\frac{3}{2}$ and $4 < x$, others chose the outside regions and some just stopped after finding the critical values. Candidates who successfully answered part (b) often answered part (c) correctly as well although some repeated their previous working to achieve this result.

The use of a sketch in part (b) and a number line in part (c) was effective and is a highly recommended strategy for questions of this type.

4. Candidates who understood the demands of this question usually did well, while others struggled to pick up marks. In part (a), those who correctly used the discriminant of the original equation often progressed well, but it was sometimes unclear whether they knew the condition for different real roots. An initial statement such as “ $b^2 - 4ac > 0$ for different real roots” would have convinced examiners. Irrelevant work with the discriminant of $k^2 - 5k + 4$ was sometimes seen. In part (b) by the vast majority of candidates scored two marks for finding the correct critical values, although it was disappointing to see so many resorting to the quadratic formula. It was surprising, however, that many did not manage to identify the required set of values of k . The inappropriate statement “ $1 > k > 4$ ” was sometimes given as the final answer, rather than “ $k < 1$ or $k > 4$ ”.
5. The quality of answers to this question was better than to similar questions in previous years. Most used the discriminant to answer part (a) and, apart from occasional slips with signs, were able to establish the inequality correctly. A few realised that the discriminant had to be used but tried to apply it to $k^2 - 4k - 12$. In part (b) the majority were able to find the critical values of -2 and 6 but many then failed to find the correct inequalities with $x > -2$ and $x > 6$ being a common incorrect answer. Some candidates still thought that the correct regions could be written as $6 < k < -2$ but there were many fully correct solutions seen often accompanied by correct sketches.
6. Although candidates who produced a totally correct solution to this question were in the minority, most knew that the use of the discriminant was needed. The correct inequality, $(-2)^2 + 4(2)(k + 1) < 0$ or equivalent, was generally seen only from the better candidates. A very common error was to take c to be $(k + 1)$ instead of $-(k + 1)$. Sometimes $b^2 = 4ac$ was used rather than $b^2 < 4ac$, giving access to only 2 of the 4 available marks. Algebraic manipulation was quite poor in this question, with many sign and bracketing mistakes being seen. Weaker candidates sometimes tried solving the equation with various values of k , or rearranged to give $2x^2 - 3x - 1 = k$ and proceeded to solve $2x^2 - 3x - 1 = 0$, making no progress.
7. Part (a) was usually correct but there were some sign errors and a common mistake was to follow $8x > 2$ with $x > 4$. In part (b) the critical values were usually found without difficulty although even here some used the formula. Those who drew a sketch often went on to establish the correct regions easily but sometimes they expressed them in an inappropriate form such as $0.5 > x > 3$. Others made the usual errors of choosing the inside region, $0.5 < x < 3$ or simply giving the answer as $x > 0.5, x > 3$. Part (c) proved to be a good discriminator and whilst a good number of students were able to combine their answers from parts (a) and (b) successfully, a large number gave incomplete or incorrect solutions or tried to start from scratch and got lost in a mix of algebra and inequalities. The use of a simple number line would have helped some but others thought $0.25 > 0.5$ and so missed the finite region.

- 8.** Most candidates interpreted the context well and were able to write down the correct inequalities in parts (a) and (b), although some gave equations. Solving the linear inequality usually caused no problems but the quadratic inequality was sometimes badly handled, with candidates being unsure of how to deal with the negative critical value. While most candidates knew that the intersection of the two solutions would give the solution to part (c), some were unable to complete the question. Referring back to the context was clearly helpful at this final stage. There were some solutions using “trial and improvement” and others that gave only the set of possible integer values of x . Some candidates tried substituting one inequality into the other in an attempt to solve simultaneously.
- 9.** Apart from algebraic slips, solutions to the linear equality in part (a) were usually good. The quadratic inequality in part (b), however, caused many problems. While the vast majority were able to solve the equation to find the critical values $\frac{1}{2}$ and 5, solutions such as $x < \frac{1}{2}$, $x < 5$ were common. There was evidence in some cases of the use of an appropriate sketch to determine the required set of values, but incorrect use of inequality signs often led to loss of marks. Answers to part (c) were also disappointing, and here a sketch would certainly have helped some candidates to decide upon the correct intersection of their solutions to parts (a) and (b). Many candidates seemed unaware that such an intersection was required.
- 10.** No Report available for this question.
- 11.** No Report available for this question.