

## Transformations Questions

- 6 (a) Describe the geometrical transformation that maps the curve with equation  $y = \sin x$  onto the curve with equation:
- (i)  $y = 2 \sin x$ ; (2 marks)
  - (ii)  $y = -\sin x$ ; (2 marks)
  - (iii)  $y = \sin(x - 30^\circ)$ . (2 marks)
- (b) Solve the equation  $\sin(\theta - 30^\circ) = 0.7$ , giving your answers to the nearest  $0.1^\circ$  in the interval  $0^\circ \leq \theta \leq 360^\circ$ . (3 marks)
- (c) Prove that  $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$ . (4 marks)
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- 8 (a) Describe the single geometrical transformation by which the curve with equation  $y = \tan \frac{1}{2}x$  can be obtained from the curve  $y = \tan x$ . (2 marks)
- (b) Solve the equation  $\tan \frac{1}{2}x = 3$  in the interval  $0 < x < 4\pi$ , giving your answers in radians to three significant figures. (4 marks)
- (c) Solve the equation

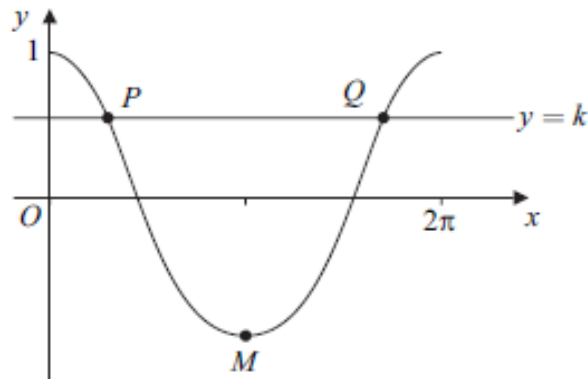
$$\cos \theta (\sin \theta - 3 \cos \theta) = 0$$

in the interval  $0 < \theta < 2\pi$ , giving your answers in radians to three significant figures. (5 marks)

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- 7 (a) Sketch the graph of  $y = \tan x$  for  $0^\circ \leq x \leq 360^\circ$ . (3 marks)
- (b) Write down the **two** solutions of the equation  $\tan x = \tan 61^\circ$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (2 marks)
- (c) (i) Given that  $\sin \theta + \cos \theta = 0$ , show that  $\tan \theta = -1$ . (1 mark)
- (ii) Hence solve the equation  $\sin(x - 20^\circ) + \cos(x - 20^\circ) = 0$  in the interval  $0^\circ \leq x \leq 360^\circ$ . (4 marks)
- (d) Describe the single geometrical transformation that maps the graph of  $y = \tan x$  onto the graph of  $y = \tan(x - 20^\circ)$ . (2 marks)
- (e) The curve  $y = \tan x$  is stretched in the  $x$ -direction with scale factor  $\frac{1}{4}$  to give the curve with equation  $y = f(x)$ . Write down an expression for  $f(x)$ . (1 mark)

- 8 (a) Solve the equation  $\cos x = 0.3$  in the interval  $0 \leq x \leq 2\pi$ , giving your answers in radians to three significant figures. (3 marks)
- (b) The diagram shows the graph of  $y = \cos x$  for  $0 \leq x \leq 2\pi$  and the line  $y = k$ .



The line  $y = k$  intersects the curve  $y = \cos x$ ,  $0 \leq x \leq 2\pi$ , at the points  $P$  and  $Q$ .  
The point  $M$  is the minimum point of the curve.

- (i) Write down the coordinates of the point  $M$ . (2 marks)
- (ii) The  $x$ -coordinate of  $P$  is  $\alpha$ .

Write down the  $x$ -coordinate of  $Q$  in terms of  $\pi$  and  $\alpha$ . (1 mark)

- (c) Describe the geometrical transformation that maps the graph of  $y = \cos x$  onto the graph of  $y = \cos 2x$ . (2 marks)
- (d) Solve the equation  $\cos 2x = \cos \frac{4\pi}{5}$  in the interval  $0 \leq x \leq 2\pi$ , giving the values of  $x$  in terms of  $\pi$ . (4 marks)

## Transformations Answers

Q	SOLUTION	MARKS	TOTAL	COMMENTS
6(a)(i)	Stretch (I) in y-direction (II) Scale factor 2 (III)	M1A1	2	>1 transformation is M0. M1 for (I) <u>and</u> either (II) or (III) or (III)
(ii)	Reflection; in x-axis	M1 A1	2	'Reflection' / 'reflect(ed)' (or in y-axis or $y = 0$ or $x = 0$ )
(iii)	Translation; $\begin{bmatrix} 30 \\ 0 \end{bmatrix}$	B1 B1	2	'Translation' / 'translate(d)'  Accept full equivalent in words provided linked to 'translation/move/shift' and positive x-direction (Note: B0 B1 is possible)
(b)	$\{\theta - 30^\circ =\} \sin^{-1}(0.7) = 44.4\dots^\circ$ ..... = $180^\circ - 44.4^\circ$ $\theta = 74.4^\circ, 165.6^\circ$	M1 m1 A1	3	Inverse sine of 0.7 PI eg by sight of 44, 74 or better Valid method for 2 <sup>nd</sup> angle Condone >1dp accuracy
(c)	... = $\cos^2 x + 2\cos x \sin x + \sin^2 x +$ $\cos^2 x - 2\cos x \sin x + \sin^2 x$  .... = $2\cos^2 x + 2\sin^2 x$ = $2(\cos^2 x + \sin^2 x) = 2(1)$ = 2	M1 A1 M1 A1	4	Award for either bracket expanded correctly  OE $\cos^2 x + \sin^2 x = 1$ stated or used. AG (be convinced)
<b>Total</b>			<b>13</b>	

Question	Solution	Marks	Total	Comments
8(a)	Stretch (I) in x-direction (II) scale factor 2 (III)	M1 A1	2	Need(I) and one of (II),(III) M0 if more than one transformation
(b)	$\tan^{-1}3 = 1.2(49\dots) (= \alpha)$	M1		$\tan^{-1}3$ [PI by 71.(56..)°]
	$\{\frac{1}{2}x = \pi + \alpha;$	m1		Correct quadrant; condone degrees or mix
	$\frac{1}{2}x = 1.249\dots; 4.3906\dots$			
	$x = 2.498\dots = 2.50$ to 3 sf	A1		Condone 2.5 otherwise deduct <u>max</u> of 1 mark throughout Q8 from A marks if 'correct' rads. but to 2sf or final answers in degrees. (143°, 503°)
	$x = 8.781\dots = 8.78$ to 3 sf	A1	4	As usual, accept greater accuracy answers. Ignore extra values outside the given interval (0 to 12.6). If > 2 values inside interval lose an A mark for each one.  NB M1m0A1A0 is possible
	SC after M0 for error $\tan x = 6$ ; Either $x = 1.40(5), 4.54(7), 7.68(8), 10.8(3)$ or $x = 80.5^\circ, 260.5^\circ, 440.5^\circ, 620.5^\circ$ SC B1 (accept each rounded or truncated to 3 sf)			
(c)	$\cos \theta = 0, \sin \theta - 3 \cos \theta = 0$	M1		Need both
	$\tan \theta = \frac{\sin \theta}{\cos \theta}$ or $\tan \theta = 3$	M1		$\tan \theta = \frac{\sin \theta}{\cos \theta}$ seen/used
	$\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2} = 1.57(07\dots)$	B1		Accept $\frac{\pi}{2}$
	or $\theta = \frac{3\pi}{2} = 4.71(23\dots)$	B1		Accept $\frac{3\pi}{2}$
	$\tan \theta = 3 \Rightarrow$ $\theta = 1.249\dots; 4.3906\dots = 1.25, 4.39$ to 3sf	A1✓	5	If not correct, ft on (b) NB M0M1(B0B0)A1ft is possible  90°; 270°; 71.5(6)°; 251.5(6)°
	<b>Total</b>		<b>11</b>	

Q	Solution	Marks	Total	Comments
7(a)		M1		Correct shape of branch from $O$ {to $90^\circ$ } or correct shapes of branches from $90^\circ$ - $360^\circ$
		A1		Complete graph for $0^\circ \leq x \leq 360^\circ$ (Asymptotes not explicitly required but graphs should show 'tendency')
		A1	3	Correct scaling on $x$ -axis $0^\circ \leq x \leq 360^\circ$
(b)	$61^\circ$ ; $241^\circ$	B1		For $61^\circ$
		B1	2	For $241^\circ$ and no 'extras' in the interval $0^\circ \leq x \leq 360^\circ$
(c)(i)	$\sin \theta = -\cos \theta \Rightarrow \frac{\sin \theta}{\cos \theta} = -1$ $\Rightarrow \tan \theta = -1.$	B1	1	AG; be convinced that the identity $\frac{\sin \theta}{\cos \theta} = \tan \theta$ is known and validly used
(ii)	$\Rightarrow \tan(x - 20^\circ) = -1$ $x - 20^\circ = \tan^{-1}(-1)$ $x - 20^\circ = 135^\circ, 315^\circ \dots$ $x = 155^\circ ;$ $335^\circ$	M1 m1		
		A1		
		A1ft	4	Ft on $(180 + "155")$ and no 'extras' in the given interval.
(d)	Translation	B1		'Translation'/'translate(d)'
	$\begin{bmatrix} 20 \\ 0 \end{bmatrix}$	B1	2	Accept equivalent in words provided linked to 'translation/move/shift' (Note: B0B1 is possible)
(e)	$f(x) = \tan 4x$	B1	1	For $\tan 4x$
	<b>Total</b>		<b>13</b>	

<b>8(a)</b>	$\{x = \} \cos^{-1}(0.3) = 1.266\dots \{ = \beta \}$	M1		$\cos^{-1}(0.3)$ PI by eg $72^\circ$ or $73^\circ$
	$\{x = \} 2\pi - \beta$	m1		Condone degrees or mix.
	$x = 1.27, 5.02$	A1	3	Accept 1.26 to 1.27 with 5.01 to 5.02 inclusive
<b>(b)(i)</b>	$M(\pi, -1)$	B1;B1	2	B1 for each coordinate
<b>(ii)</b>	$\{x_0 = \} 2\pi - \alpha$	B1	1	OE (unsimplified)
<b>(c)</b>	Stretch <b>(I)</b> in $x$ -direction <b>(II)</b> scale factor $\frac{1}{2}$ <b>(III)</b>	M1		Need <b>(I)</b> & one of <b>(II)</b> , <b>(III)</b>
		A1	2	
<b>(d)</b>	$\cos 2x = \cos \frac{4\pi}{5} \Rightarrow 2x = \frac{4\pi}{5}$ $\Rightarrow x = \frac{2\pi}{5} (= \alpha)$ $x = \pi - \alpha$ ; OE  $x = \pi + \alpha$ ; $x = 2\pi - \alpha$ ; OE  $x = \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}$	B1		OE. (From correct work)  Condone decimals/degrees
		M1		OE eg $2x = 2\pi - \frac{4\pi}{5}$ Correct quadrant; condone degrees/decimals/mix
		m1		Need both (OE for $2x=$ ) with no extras (quadrants) within the given interval. Condone degrees/decimals/mix
		A1	4	Need all 4 solutions for $x$ but condone unsimplified provided in terms of $\pi$  Ignore extra values outside the given interval.
<b>Total</b>			<b>12</b>	