

Oscillations MS

1. D

[1]

2. (a) (Net force) $(\Delta)F = -k(\Delta)x$ (1)

Used with $F = ma$ (1)

2

(b) Use of $F = (-)kx$

Correct answer for k OR substitution of expression for k into formula below (1)

Use of $\omega^2 = k/m$ OR $T = 2\pi\sqrt{\frac{m}{k}}$ OR $c_{\max} = -\omega^2 A$,

with $a_{\max} = 9.81 \text{ Nkg}^{-1}$ (1)

Use of $\omega = 2\pi f$ OR $f = 1/T$ (1)

Correct answer for f (1)

Example of calculation:

$$k = \frac{0.15 \text{ kg} \times 9.81 \text{ Nkg}^{-1}}{0.2 \text{ m}} = 7.4 \text{ Nm}^{-1}$$

$$\omega = \sqrt{\frac{7.4 \text{ Nm}^{-1}}{0.15 \text{ kg}}} = 7.0 \text{ (rad s}^{-1}\text{)}$$

$$f = \frac{\omega}{2\pi} = \frac{\text{s}^{-1}}{2\pi} = 1.1 \text{ Hz}$$

5

[7]

3. (a) Resonance (1)

System driven at / near its natural frequency (1)

2

(b) (i) Any zero velocity point

1

(ii) Any maximum/minimum velocity point

1

- (c) Select 70 mm distance from passage/see 35 mm (1)
 Use of $a = -\omega^2x$ (1)
 Use of $v = \omega A$ (1)
 Correct answer (1) 4

Example of calculation:

$$\omega = \sqrt{\frac{0.89\text{ms}^{-1}}{3.5 \times 10^{-2}\text{m}}} = 5.04\text{rad s}^{-1}$$

$$v = \omega A = 5.04\text{s}^{-1} \times 3.5 \times 10^{-2}\text{m} = 0.18\text{ms}^{-1}$$

- (d) The answer must be clear and be organised in a logical sequence
 The springs/dampers absorb energy (from the bridge) (1)

QWC

(Because) the springs deform/oscillate with natural frequency of the bridge (1)

Hence there is an efficient/maximum transfer of energy (1)

Springs/dampers must not return energy to bridge / must dissipate the energy (1)

Max 3

[11]

4. B

[1]

5. D

[1]

6. (a) (i) Use of $F = -kx$ (1)
 $F = ma$ (1)

2

- (ii) cf with $a = -\omega^2x$ ie $\omega^2 = k/m$ (1)
 use of $T = 2\pi/\omega$ to result (1)

2

- (b) (i) resonance (1)

1

- (ii) natural freq = forcing frequency (1)

1

- (iii) use of $c = f\lambda$ (1)
 answer 9.1×10^{13} Hz (1) 2
- (iv) use of $T = 1/f$ eg $T = 1.1 \times 10^{-14}$ (1)
 rearrange formula (1)
 to give 550 N/m(1) 3

[11]

7. (a) (i) Amplitude and frequency
 0.17 m (1)
 0.8(3) Hz or s^{-1} (1) 2
- (ii) Maximum velocity
 Use of $v_{\max} = 2\pi fx_0$ (1)
 Correct answer (1)
 Example calculation:
 $v_{\max} = 2\pi \times 0.83 \text{ Hz} \times 0.17 \text{ m}$
 OR
 Use of maximum gradient of h versus t graph
 Answer to 2 sig fig minimum 2
- (iii) Velocity-time graph
 Wave from origin, period 1.2 s (1)
 Inverted sine wave with scale on velocity axis & initial peak value
 0.9 m s^{-1} (1) 2
- (b) (i) Definition of SHM
 Acceleration / resultant force proportional to displacement
 OR Acceleration / resultant force proportional to distance from a
 fixed point [not just distance from equilibrium but "distance from
 equilibrium position" is acceptable]
 OR $a = (-) \text{ constant} \times x$ [with a and x defined]
 OR $F = (-) \text{ constant} \times x$ [with F and x defined] (1)
 Acceleration /resultant force directed towards the fixed point / in
 opposite direction (to displacement)
 OR negative sign in equation explained [e.g a and x in opposite
 directions] (1) 2

- (ii) Verifying SHM
- Read off h value and use it to get displacement (1)
 [only penalise the first mark if h used for displacement throughout]
 Plot acceleration-displacement graph OR calculate ratios eg $a \div x$ (1)
 Straight line through the origin OR check ratios to see if constant (1)
 Negative gradient / observe acceleration and displacement have opposite signs OR constant is negative (1)
- OR
- Use $x = x_0 \cos(2\pi ft)$ for a range of t OR Read off h and get x (1)
 Use values of x_0 and f from part (a) OR Use $a = -(2\pi f)^2 x$ for range of x (1)
 Add equilibrium value to x to get h OR Use value of f from part (a) (1)
 If results agree with values of h (or a) from graph it is SHM (1) 4

[12]

8. (a) Name of effect
- Resonance
 - Idea that step frequency = natural frequency of bridge 2

- (b) (i) Why oscillations are forced
- E.g. amplitude is increasing OR oscillations are driven by the wind 1

- (ii) Calculation of maximum acceleration
- Use of $\omega = 2\pi / T$ to obtain value for ω
 - Correct answer for acceleration [14 m s^{-2}]
- Example of calculation:
 $\omega = 2\pi / T = 2\pi / (60 \text{ s} / 38) = 4.0 \text{ s}^{-1}$
 $a_{\max} = \omega^2 A = (4.0 \text{ s}^{-1})^2 \times 0.90 \text{ m} = 14 \text{ m s}^{-2}$ 2

- (iii) Show that car loses contact
- Required a_{\max} is greater than g
 - So, at a position (of 0.61 m) above the equilibrium position, vehicle loses contact with the road
- Example of calculation:
 $x = g / \omega^2 = 9.81 \text{ m s}^{-2} / (4 \text{ s}^{-1})^2 = 0.61 \text{ m}$ 2

[7]

9. (a) Experiment

Scheme for timing methods:

QOWC (1)

Use $f = 1/T$ OR $f = (\text{number of cycles/time taken})$ (1)

Apparatus (1)

Principle of method (1)

One precaution for accuracy (1)

Max 4

Examples for last three marks:

Stopclock / stopwatch (1)

Measure time taken for a number of cycles (1)

ensure vertical oscillations / not exceeding elastic limit (1)

Or

Motion (OR position) sensor and datalogger (OR computer) (1)

Read time for one (or more) cycles from displacement-time graph (1)

Read time for several cycles / ensure vertical oscillations (1)

Or

Light gate and datalogger (OR computer) (1)

Computer measures time interval between beam interruptions (1)

Use narrow light beam / position gate so beam cut at equilibrium position / ensure vertical oscillations (1)

Or

Video Camera (1)

Read time for one (or more) cycles from video (1)

Read time for several cycles / ensure vertical oscillations (1)

[Mark other reasonable techniques on the same principles]

Scheme for strobe method:

QOWC (1)

Use stroboscope [Accept strobe] (1)

Adjust frequency until mass appears at rest (1)

Find highest frequency at which this happens (1)

Repeat and average / ensure vertical oscillations (1)

Max 4

Scheme for use of $T = m/k$ / $T = e/g$

QOWC (1)

Calculate T from measured (OR known) m and k using

$T = m/k$ (1) or calculate T from measured e known g using

$T = e/g$ (1)

Use $f = 1/T$ (1)

Max 2

[e is the extension of the spring produced by weight of mass m]

[Do not give any credit for experiments to measure the resonant frequency of the system]

- (b) (i) Graph
 Axis labels and single peak (1)
 Rounded top and concave sides (1)
 f_0 marked on the frequency axis at, or just to right of, peak.
 [Amplitude at f_0 should be at least 75% of maximum amplitude] (1)
 [A sharp kink loses mark 2 only]
 [Graphs with multiple peaks lose marks 1 and 2; f_0 marked correctly on lowest frequency peak for mark 3]
 [Ignore whether or not curve goes to origin] 3
- (ii) Name of phenomenon
 Resonance (1)
 [Mark this independent of whether graph is correct
 Do not accept “resonant frequency”] 1
- (iii) Footbridge application
 People walking / wind / earthquake can cause vibration / act as a driver / apply regular impulses (1)
 If resonance occurs OR if frequency equals / is close to f_0 we may get large / dangerous / violent oscillations OR large energy transfer OR damage to bridge (1) 2

[10]

10. (a) Diagram
 At least 3 crests drawn, with correct even spacing (1)
 [Judge by eye. Allow $\pm 20\%$. Check in centre of pattern.]
 Crests approximately straight opposite harbour entrance and curved in the “shadow” region (1)
 Wavefronts get longer, but diffraction at the edges through no more than 45° (1) 3
- (b) (i) Values from graph
 Period: 3.0 (1)
 [Accept 3]
 Maximum acceleration: 1.2
 [Accept 1.17, 1.18, 1.19, 1.20]
 [Both values are needed for the mark. Ignore written units, e.g. “3.0 s”, “1.2 m s⁻².”] 1

- (ii) Calculation of amplitude
 Use of acceleration = $(-)(2\pi f)^2 x$ (1)
 Use of $f = 1/T$ (1)
 Correct answer [0.27 m] (1) 3
 [Negative amplitude loses third mark]
 e.g.
 $\omega = 2\pi / (3.0 \text{ s})$
 $= 2.09 \text{ rad s}^{-1}$
 $A = (1.2 \text{ m s}^{-2}) / (2.09 \text{ rad s}^{-1})^2$
 $= 0.27 \text{ m}$
- (iii) Displacement graph
 Cosine curve [Correct way up] (1)
 Axis labelled (i.e. displacement / x / y / z) plus all (1)
 amplitudes consistent with previous answer (i.e. within 1 square)
 plus unit 2 cycles shown with $T = 3 \text{ s}$ (1) 3
 [All zero crossings correct within 1 square]

[10]

11. (a) Meaning of resonance:
 Parts of building have about the same natural frequency as
 driving vibrator (1)
 so vibrate with increased/large amplitude **or** maximum/large
 energy transfer occurs (1) 2
- (b) (i) Why springs reduce vibration:
 Spring deforms (instead of building) (1)
 absorbing energy (1) 2
- (ii) Less damage during earthquake:
 vibration of building is damped / amplitude of vibration is
 reduced (so damage to building would be less) (1) 1

[5]

12. (a) Time 10 oscillations then divide by 10 / keep eye in the
 same position each time (1)
 [do not accept light gates etc] 1

- (b) Nearest 0.01 m / 1 cm (1)
 Either suitable because a 1 m length is sensibly measured to nearest 1 cm (1)
 Or could measure to nearest mm with a metre rule (1) 2 max
- (c) (i) Column headed $T^2 / l^{0.5} / \log\log$ or $\ln\ln$ (1)
 Units $s^2 / m^{0.5} / \text{no units}$ (1)
 Correct values [T^2 check last fig 5.02 / $l^{0.5}$ 0.60 row = 0.78 (1)
 Log 0.6 row : -0.22 0.210]
 Scales: points occupy more than half page (1)
 Points (1)
 Best fit straight line [not thro origin] (1)
 [T v l graph marks 4 and 5 only 2 max] 6
- (ii) Line does not go through origin (1)
 Therefore T^2 not proportional to l / T not proportional to $l^{1/2}$ (1)
 [LogLog : Need to find gradient (1)
 Round off = 0.5] 2 max
- (d) Line does not go thro origin / When T = 0 there appears to be a value of length (1)
 Intercept is about .. cm this shows an error in l (1)
 The actual length of pendulum is longer than measured (1)
 The intercept is long enough to be a possible (systematic) error (1) 3 max
 [No marks for log log graphs]
- (e) Gradient from large triangle (1)
 = 3.9 – 4.1 for $T^2 / l = 1.9 – 2.05$ for root l (1)
 Equate with either $4\pi^2/g / 2\pi / \text{root } g$ (1)
 value 9.6 – 10.5 $m s^{-2}$ (1)
 [log log intercept (1)
 = $\log (2\pi / \text{root } g)$ (1)(1)
 value 9.6 – 10.5 $m s^{-2}$] (1) 4 max

[18]

13. (a) Definition of SHM

Acceleration (OR force) is proportional to displacement/allow distance from point (from a fixed point) / a (OR F) = (-)constant x with symbols defined (1)

Acceleration (OR force) is in opposite direction to displacement/ Acceleration is towards equilibrium point [Allow “towards a fixed point” if they have said the displacement is measured from this fixed point] / Signs in equation unambiguously correct, e.g.

$$a \text{ (OR } F) = -\omega^2 x$$

[Above scheme is the only way to earn 2 marks, but allow 1 mark for motion whose period is independent of amplitude OR motion whose displacement/time graph is sinusoidal]

2

(b) (i) Calculation of period

Use of $T = 2\pi\sqrt{m/k}$ (1)

Correct answer [1.1 s] (1)

2

$$\begin{aligned} \text{e.g. } T &= 2\pi\sqrt{(0.120 \text{ kg} / 3.9 \text{ N m}^{-1})} \\ &= 1.10 \text{ s} \end{aligned}$$

(ii) Calculation of maximum speed

Use of $v_{\max} = 2\pi f x_0$ and $f = 1/T$ (1)

Correct answer [0.86 m s⁻¹] (1)

2

$$\begin{aligned} \text{e.g. } f &= 1/(1.10 \text{ s}) \\ &= 0.91 \text{ Hz} \end{aligned}$$

$$\begin{aligned} v_{\max} &= 2\pi (0.91 \text{ Hz})(0.15 \text{ m}) \\ &= 0.86 \text{ m s}^{-1} \end{aligned}$$

(iii) Calculation of maximum acceleration

Use of $a_{\max} = -(2\pi f)^2 x_0$ (1)

Correct answer [4.9 ms⁻²] (1)

2

$$\begin{aligned} \text{e.g. } a_{\max} &= (2\pi \times 0.91 \text{ Hz})^2(0.15 \text{ m}) \\ &= 4.9 \text{ m s}^{-2} \end{aligned}$$

(iv) Calculation of mass of block

Use of $T \propto \sqrt{m}$ / Use of $T = 2\pi\sqrt{m/k}$ (1) □

Correct answer [0.19 kg] (1)

2

$$\begin{aligned} \text{e.g. } m &= (0.12 \text{ kg})(1.4 \text{ s} / 1.1 \text{ s})^2 \\ &= 0.19 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{OR } m &= (3.9 \text{ N m}^{-1})(1.4 \text{ s} / 2\pi)^2 \\ &= 0.19 \text{ kg} \end{aligned}$$

[Apply ecf throughout]

[10]

14. (a) Experimental verification

QOWC (1)

Measure T using clock or motion sensor or video camera or digital (1)
camera [Don't accept light gates]

for a range of masses (or various masses) (1)

Plot T vs $m^{1/2}$ / Plot T^2 vs m / Plot $\log T$ vs $\log m$ / calculate $T/m^{1/2}$
or T^2/m (1)

Str line through origin / Str line through origin / Str line gradient (1)
0.5 / constant

One precaution (1)

e.g. Use fiducial (or reference) mark

Repeat and average

No permanent deformation of spring

Small amplitude or displacement

Measure at least 10T

Max 5

(b) Explanations

Natural frequency:

Freq of free vibrations / freq of unforced vibrations / freq when it (1)
oscillates by itself (or of its own accord) / freq of oscillation if
mass is displaced

[Don't accept frequency at which it resonates, frequency at which
it oscillates naturally, frequency if no external forces]

Resonance:

When vibration is forced (or driven) at natural frequency (1)

Amplitude (or displacement or oscillation) is large
(or violent or increases) (1)

Amplitude is a maximum / large energy transfer (1)

4

[Accept 4 Hz for natural frequency]

[9]

15. (a) Definition of longitudinal wave

Oscillations OR particles (of medium) move (1)

Parallel to direction of wave propagation/travel / energy transfer (1)

2

2nd mark consequent on 1st]

(b) Collapsing tower

Resonance (1)

Frequency of quake = natural frequency of tower (1)

[Allow resonant frequency for natural frequency]

Max energy transfer (1)

Very large increase in amplitude of oscillation or maximum amplitude (1) max 3

[5]

16. (i) Diagram
Component ($mg\cos\theta$) correctly drawn – good alignment and (1) approximately same length 1
- (ii) Diagram
Component ($mg\sin\theta$) correctly drawn, reasonably perpendicular (1) to T to the left 1
- (iii) Acceleration
Use of $mg\sin\theta = ma$ [must see 9.8(1) (m s^{-2}) not 10 for this mark] (1)
 $a = 0.68 \text{ m s}^{-2}$ [for this mark allow 0.69 m s^{-2} ie 10 m s^{-2} for g] (1) 2
- (iv) Direction
Directed to O along arc/in same direction as $mg\sin\theta$ /tangential to (1) arc 1

[5]

17. Gradient of graph
Gradient = 2.5 (1)
Unit s^{-2} or negative sign (1)
Frequency
 $(2\pi f)^2 = 2.5$ [or above value] (1)
 $f = 0.25 \text{ Hz}$ [ecf ONLY for gradient error] (1) 4
- Period
 $T = 4 \text{ s}$ ecf their f (1) 1
- Acceleration against time graph
Any sinusoidal curve over at least two cycles (1)
Negative sine curve (1)
 y axis scale showing $a = 20 \text{ (mm s}^{-2}\text{)}$ OR x axis scale showing $T = 4(\text{s})$ / their T 3

[8]

18. Conditions for simple harmonic motion
Acceleration OR restoring force \propto displacement (1)
in opposite direction / towards equilibrium position (1) 2
- Why child's motion only approximately simple harmonic
Any one from:
- damped / friction opposes motion / air resistance
 - swing's path is arc of circle, not straight line
 - angle too large (1) 1

Calculations

- (i) Period of the motion:

$$T = 20 \text{ s} / 6 = 3.3 \text{ s (1)}$$

$$f = 1/T = 0.30 \text{ Hz (allow e.c.f. for } T) \text{ (1)}$$

- (ii) Value of ω

$$\text{Use of } \omega = 2\pi f$$

$$= 2\pi \times 0.30$$

$$= 1.9 \text{ rad s}^{-1} \text{ (allow e.c.f. for } f, \text{ no repeat unit error) (1)}$$

- (iii) Child's acceleration:

$$a_{\max} = -\omega^2 A \text{ (1)}$$

$$= -1.9^2 \times 1.2 \text{ (allow e.c.f. for } \omega) \text{ (1)}$$

$$= (-) 4.3 \text{ ms}^{-2}$$

Swing an example of resonance

Push (driver) at same frequency as swing (driven) (1)

causes increase of amplitude / energy transfer (1)

2

[10]

19. Simple harmonic motion

Acceleration proportional to displacement (from equilibrium position/ point) (1)

and in opposite direction/directed towards equilibrium position / point) (1)

2

OR accept fully defined equation

Oscillations

$$x_0 = 0.036 \text{ m (1)}$$

$$\text{Period} = 7.60 \text{ s} / 20 = 0.380 \text{ s (1)}$$

$$f = 2.63 \text{ Hz (1)}$$

3

Displacement when $t = 1.00 \text{ s}$

$$x = (-)0.026 \text{ m (1)}$$

1

How and why motion differs from prediction

Motion is damped/amplitude decreases with time (1)

(Because of) air resistance (1)

2

[8]

- 20. Oscillating, system**
- Diagram: suitable oscillator (1)
 method of applying periodic force of variable frequency (1) 2
- Natural frequency:*
- (With no periodic force) displace oscillator and let it oscillate (freely) (1)
- Frequency of this motion is natural frequency (1)
- Forced oscillation:*
- Their system is being forced to oscillate/vibrate at driver's frequency (1)
- Resonance:*
- Vary the frequency (1)
- Oscillator has large amplitude at / near natural frequency (1) 5

[7]

- 21. (a) (i) a is acceleration 1**
- f is frequency 1
- x is displacement from equilibrium/centre/mean position 1
- [beware amplitude]
- Either minus sign means that a is always directed to centre/equilibrium position/mean position
 OR a and x in opposite directions / a opposite to displacement 1
- (ii) Attempt to use $v_m = 2\pi fa$ 1
- $v_m = 2\pi(50 \text{ Hz})(8.0 \times 10^{-6} \text{ m}) [a = 4 \times 10^{-6} \text{ m e.o.p}]$ 1
- $= 2.5 \times 10^{-3} \text{ m s}^{-1} / 2.5 \text{ mm s}^{-1} / 2500 \text{ } \mu\text{m s}^{-1}$ 1
- (b) (i) ρ in kg m^{-3} and g in $\text{m s}^{-2} / \text{N kg}^{-1}$ 1
- $A\rho g/m$ with units leading to s^{-2} 1
- (ii) At least three peaks of y/m measured [ignore numbers] 1
- EITHER
- | | | | |
|----------|---------|---------|-------|
| (5.4/5/6 | 3.7/6/8 | 2.4 | 1.6/7 |
| 1.2 | | | |
| 4.4 | 2.8/9 | 2.0/1.9 | 1.4) |
- Attempt to calculate successive ratios [e.g. 0.67 or 1.5] 1
- Logical statement re ratios and exponential change 1

OR

Read 3 values of x and y [ignore numbers]	1
Two half lives between 0.75 s and 0.95 s	1
Logical statement re half-lives and exponential change [beware Δy not constant / use of $y = Y_0 e^{-kt}$]	1

[12]

22. Simple harmonic motion – conditions

Acceleration (or force) proportional to displacement

[OR $a \propto -x$] (1)

Acceleration (or force) directed towards equilibrium position (1) 2

Graph

Horizontal line at $E_p = 19 \text{ J}$ (1) 1

Calculations

(i) Use of $E_K = E_r - E_p$ (1)
 $= 19 \text{ J} - 5 \text{ J}$
 $= 14 \text{ J}$ (1)

(ii) Use of $E_p = \frac{1}{2} kx^2$ (1)
with readings from graph (1)
e.g. $k = 2 \times 5 \text{ J} / (0.02 \text{ m})^2 = 2.5 \times \text{N m}^{-1}$ (1)

[8]

23. Simple harmonic motion

Acceleration/force is proportional to *displacement* (1)

but in the opposite direction / towards equilibrium point /mean point (1) 2

Graph

Sine curve (1)

-ve [consequent] (1)

A and B / (i) and (ii) / a & x [beware $a \vee x$] (1) 3

[5]

24. Data for speaker and equation

Equation for shm: $x = A \cos \omega t$

A amplitude = 1.0 mm or 1.0×10^{-3} m

$\omega = 2\pi f = 6.28 \times 10^2$ (rad s⁻¹) – no unit penalty for ω (1) 2

Calculations

(i) $A = A \omega^2$
 $= 1.0 \times 10^{-3} \text{ m} \times (6.28 \times 10^2 \text{ rad s}^{-1})^2 = 394 \text{ m s}^{-2}$ (1)

(ii) $v = A \omega$ (1)
 $= 1.0 \times 10^{-3} \text{ m} \times 6.28 \times 10^2 \text{ rad s}^{-1} = 0.63 \text{ m s}^{-1}$ (1) 3

Acceleration - time graph

Two cycles of sinusoidally shaped graph (1)

Period = 10 m s (1)

Amplitude = 394 m s⁻² [e.c.f from (i)] (1) 3

Explanation

Resonance (stated or implied by explanation) (1)

Increased amplitude at resonant frequency (1) 2

[10]

25. Simple harmonic motion

Force/Acceleration proportional to displacement/ $a = \omega^2 x$ (1)

[define a and x but not ω]

In opposite direction to displacement/ towards a fixed point/towards equilibrium position/minus sign (1) 2

Oscillation of mass

Clarity of written communication (1)

Down: $T > W$ [W varying 0/3] (1)

Up: $T < W$ (1)

$T > W$ gives **resultant** force/acceleration UP (1)
[or equivalent argument for displaced up] 4

Velocity-time graph

Either Cosine graph [zigzag -1 here] (1)

Starting at positive maximum [if very poorly synchronised 0/2] (1) 2

Maximum velocity of mass

Period = 0.50 **or** amplitude = 0.07 (1)

Use of $f = 1/T$ / Use of $x = x_0 \sin$ or $\cos(\omega t)$ (1)

Use of $v_{\max} = 2\pi f x_0$ (1)

0.88 m s⁻¹ (1)

OR

4

v_{\max} = gradient at 0.50 s (or equivalent) (1)

Correct method for gradient (1)

Answer in range 0.8 to 1.0 m s⁻¹ (1)

[Max 3 for gradient method]

[12]

26. Simple harmonic

Acceleration (OR force) \propto displacement (1)

and in opposite direction (1)

[OR $F = -kx$ (1) OR $x = A \cos \omega t$ (1) symbols defined (1)]

2

Graph

(i) E_k inverse of potential energy curve

(ii) T horizontal line at 2×10^{-6} J

2

Stiffness

Use of $E = \frac{1}{2} kx^2$ (1)

$2 \times 10^{-6} = \frac{1}{2} k 0.10^2$

$k = 4 \times 10^{-4}$ J m⁻² (1)

[accept N m⁻¹]

2

Variation in potential energy

Any 3 points from the following:

- $I_{150} / I_{100} = 100^2 / 150^2 = 0.44$
- So $E_{\max} = 0.44 \times 2 \times 10^{-6} = 0.88 \times 10^{-6}$ (J) [no u.e]
- Curve through (0,0), max 0.8 – 0.9
- Curve the same
- $I \propto (\text{amplitude})^2$

Max 3

[9]

27. Frequency:

Natural frequency/fundamental frequency (1)

1

[Not resonant]

Explanation:

Resonance occurs when driving frequency = natural frequency (1)

causing maximum energy transfer(1)

increased/maximum amplitude (1)

Max 2

Graph:

Undamped - marked A

Acceptable shape - narrow peak (1)

Resonant frequency marked under graph max. (1)

Damped - marked B

B - entire graph below A (1)

[Accept touching graphs] (1)

4

Peak covers greater frequency range than A

Prevention of resonance:

Damps oscillations (1)

Fewer forced oscillations (1)

Explanation of damping [e.g. in terms of energy transfers] (1)

Max 2

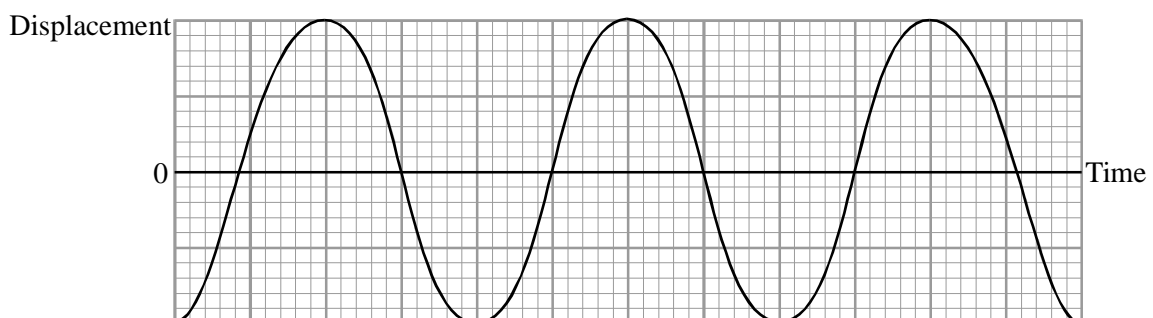
[9]

28. Displacement-time graph:

Cosine curve (1)

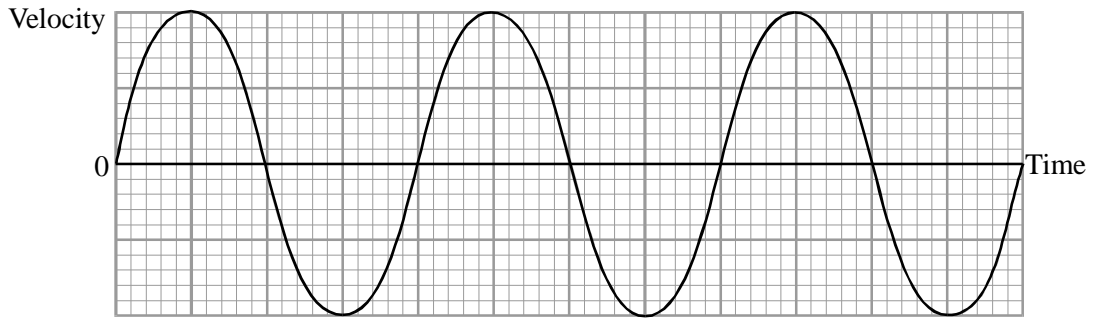
Constant period and amplitude (1)

2



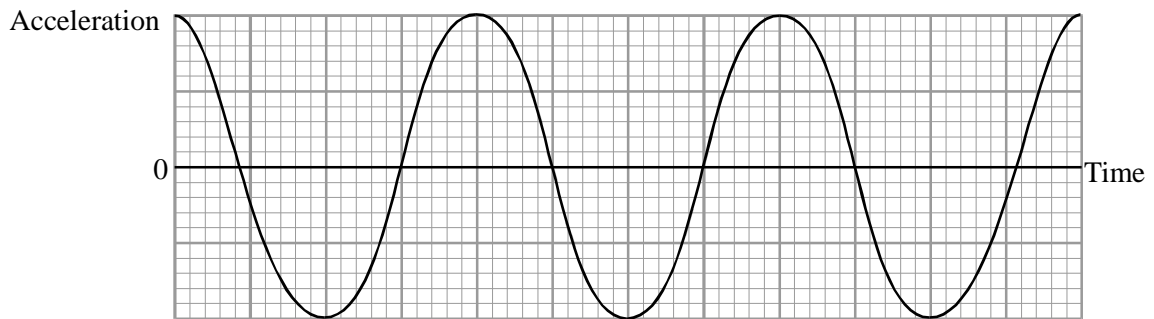
Velocity-time and acceleration-time graphs:

Velocity: sine curve; 90° out of phase with displacement-time **(1)**



Acceleration: cosine curve; 180° out of phase with displacement-time **(1)**

2



Two requirements:

Force towards centre **(1)**

a (or F) $\propto x$ **(1)**

opposite direction [acceleration and displacement acceptable] **(1)**

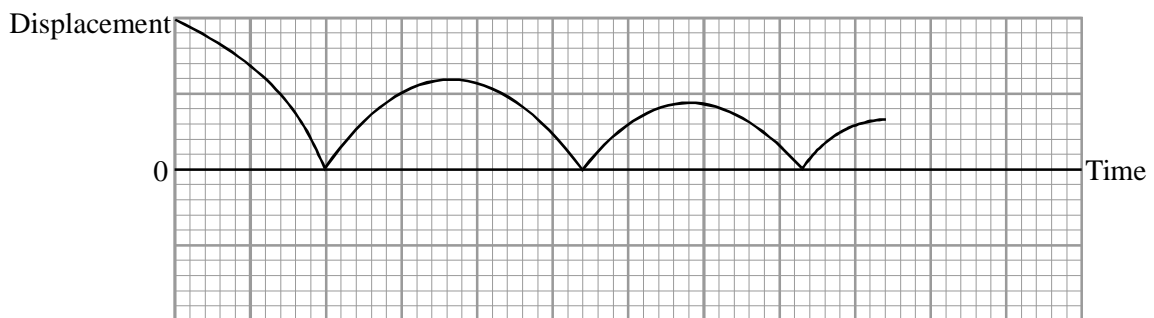
Max 2

Displacement-time and acceleration-time:

Starts positive, curved, always > 0 **(1)**

Period same as velocity **(1)**

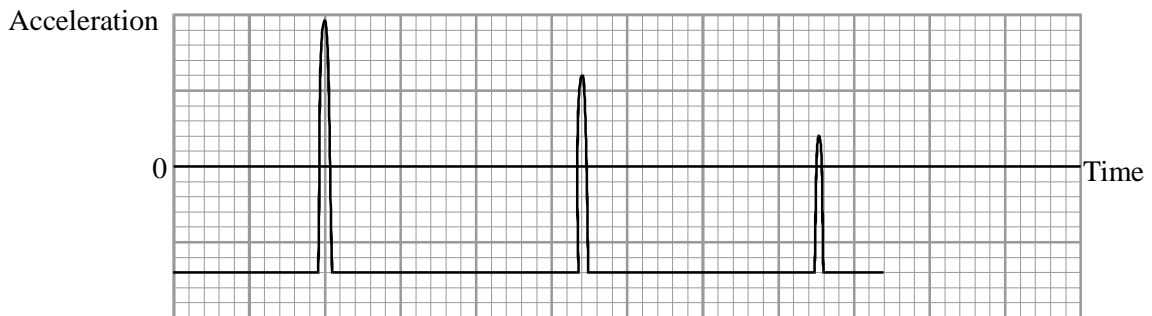
2



Acceleration constant not equal to 0 (1)

Sharp peak when ball in contact with floor (1)

2



Explanation:

Not simple harmonic motion (1)

Reason e.g. acceleration constant when ball in free fall / period not constant / acceleration not \propto displacement. (1)

2

[12]

29. Why example is resonance:

These are forced vibrations, i.e. 2 systems [driver and driven] (1)

The vibrations have max amplitude at one particular frequency or decrease both sides (1)

When forcing frequency = natural frequency of steering wheel car/ "system" (1) 3

Calculation of maximum acceleration:

Acceleration = $(2\pi \times 2.4)^2 \text{ s}^{-2} \times 6 \text{ mm}$ [accept any attempted conversion to m, e.g. 6×10^{-2}] (1)

= 1400 (1360) mm s^{-2} [no e.c.f.] (1)

(1.4 m s^{-2})

2

[5]

30. A body oscillates with simple harmonic motion when the resultant force F acting on it and its displacement x are related by the expression

$$F = -kx \text{ or } F \propto -x.$$

The acceleration of such a body is always directed **towards the centre of the oscillation** or **in the opposite direction to displacement**/ $x =$

0 /equilibrium/similar

The acceleration of the body is a maximum when its displacement is **maximum** and its velocity is **maximum** when its displacement is zero. 4

Force constant:

$$= 14$$

$$\text{N m}^{-1} \text{ or } \text{kg s}^{-2}$$

2

[6]

31. Displacement-time graph:

Sine or cosine (1)

Max/min at +1.8 and -1.8 (1)

$T = 0.05$ s and at least one cycle (1) 3

Calculation of maximum speed:

Correct use of $v = ax_0$ or $2\pi fx_0$

$$= 2\pi \times 10 \text{ s}^{-1} \times 1.8 \times 10^{-2} \text{ m} \quad (1)$$

$$\text{Maximum speed} = 2.3 \text{ m s}^{-1} \quad (226 \text{ cm s}^{-1}) \quad (1) \quad 2$$

Any two places correctly marked M (1) 1

[Total 6 marks]

32. A student was studying the motion of a simple pendulum the time period of which was given by $T = 2\pi (l/g)^{1/2}$.

He measured T for values of l given by

$$l/m = 0.10, 0.40, 0.70, 0.70, 1.00$$

and plotted a graph of T against \sqrt{l} in order to deduce a value for g , the free-fall acceleration.

Explain why these values for l are poorly chosen.

An inadequacy PLUS a reason why (many possibilities) e.g. some values too short to produce accurate T values; when the values are square rooted; spacing is unsatisfactory. (1)

(1 mark)

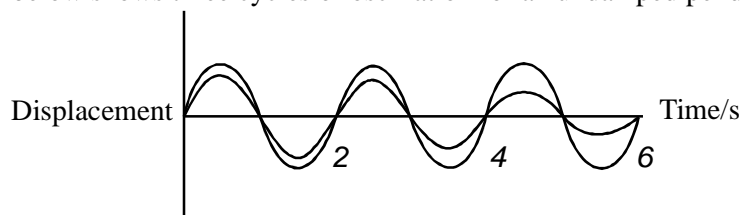
How would the student obtain a value of g from the gradient of the graph?

$$\text{Gradient} = \frac{2\pi}{\sqrt{g}} \quad (1)$$

$$\text{Hence } g = \frac{4\pi^2}{(\text{gradient})^2} \quad (1)$$

(2 marks)

The graph below shows three cycles of oscillation for an undamped pendulum of length 1.00 m.



Add magnitudes to the time axis and on the same axes show three cycles for the same pendulum when its motion is lightly air damped.

$$T = 2 \text{ s} \quad (1)$$

$$T_{\text{damped}} = T_{\text{original}} \quad (1)$$

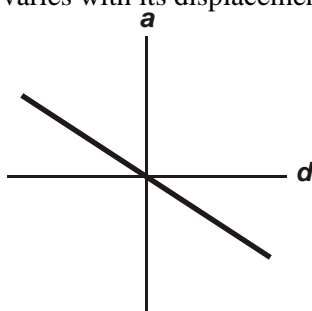
Amplitude reduced (1)

Continuous reduction in amplitude (1)

(4 marks)

[Total 7 marks]

33. A body oscillates with simple harmonic motion. On the axes below sketch a graph to show how the acceleration of the body varies with its displacement.



Straight line through origin (1)

Negative gradient (1)

(2 marks)

How could the graph be used to determine T , the period of oscillation of the body?

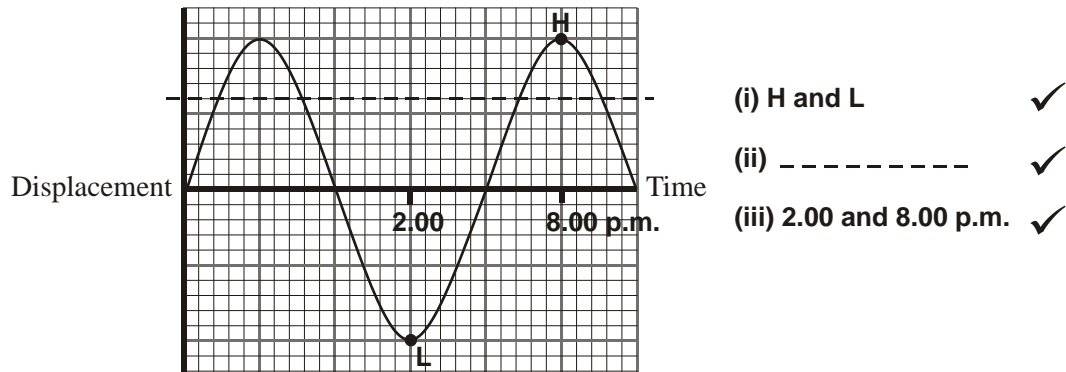
Reference to gradient of line (1)

$$\text{Gradient} = (-) \omega^2 \text{ or } = (-)(2\pi f)^2$$

$$\text{OR } T = \frac{2\pi}{\sqrt{(-)\text{gradient}}} \quad (1)$$

(2 marks)

A displacement-time graph from simple harmonic motion is drawn below.



The movement of tides can be regarded as simple harmonic, with a period of approximately 12 hours.

On a uniformly sloping beach, the distance along the sand between the high water mark and the low water mark is 50 m. A family builds a sand castle 10 m below the high water mark while the tide is on its way out. Low tide is at 2.00 p.m.

On the graph

- (i) label points L and H, showing the displacements at low tide and next high tide,
- (ii) draw a line parallel to the time axis showing the location of the sand castle,
- (iii) add the times of low and high tide.

(3 marks)

Calculate the time at which the rising tide reaches the sand castle.

Use of $x = x \sin \omega t$

$$15 = 25 \sin \omega t$$

$$\omega = \frac{\pi}{6} \text{ or } \omega t = 37^\circ / t = 1.23 \text{ hours}$$

Time = 6.14 p.m. ANY THREE LINES (3)

Full error carried forward from wrong diagram

Alternative using graph:

Identify coordinates (1)

Convert to time (1)

Add to reference time (1)

**(3 marks)
[Total 10 marks]**