Questions on Gravity and Orbits MS

1. Using the usual symbols write down an equation for

(i) Newton's law of gravitation

$$F = G\frac{M_1M_2}{R^2}$$ **(1)**

(ii) Coulomb's law

$$F = K\frac{Q_1Q_2}{R^2}$$ **(1)**

(2 marks)

State one difference and one similarity between gravitational and electric fields.

Difference

Gravitational fields are attractive but electric fields can be attractive or repulsive (1)

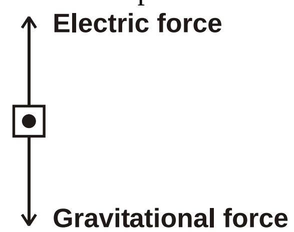
Similarity

Both have an $\propto$ range (1)

(2 marks)

A speck of dust has a mass of 1.0×10^{-18} kg and carries a charge equal to that of one electron. Near to the Earth's surface it experiences a uniform downward electric field of strength 100 N C^{-1} and a uniform gravitational field of strength 9.8 N kg^{-1}.

Draw a free-body force diagram for the speck of dust. Label the forces clearly.



Calculate the magnitude and direction of the resultant force on the speck of dust.

Electric force $= 100\ N\ C^{-1} \times 1.6 \times 10^{-19}\ C$

$= 1.6 \times 10^{-17}\ N$ **(1)**

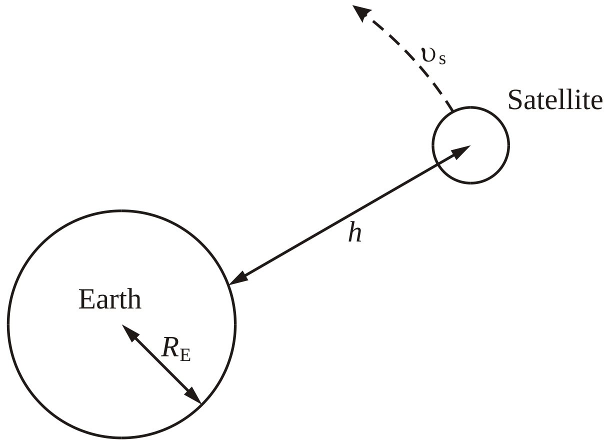
Weight $= 9.8 \times 10^{-18}\ N$ **(1)**

Net force is upward (1)

Force $= 6.2 \times 10^{-18}\ N$ **(1)**

(6 marks)
[Total 10 marks]

2. The diagram (not to scale) shows a satellite of mass *m*, in circular orbit at speed v_s around the Earth, *mass* M_E. The satellite is at a height *h* above the Earth's surface and the radius of the Earth is R_E.



Using the symbols above write down an expression for the centripetal force needed to maintain the satellite in this orbit.

$$F = \frac{m_s v_s^{2}}{R_E + h} \quad \textbf{(2)}$$

(2 marks)

Write down an expression for the gravitational field strength in the region of the satellite.

$$g = \frac{GM_E}{(R_E + h)^2} \quad \textbf{(2)}$$

State an appropriate unit for this quantity.

N kg $^{-1}$ (1)

(3 marks)

Use your two expressions to show that the greater the height of the satellite above the Earth, the smaller will be its orbital speed.

$$\frac{m_s v_s^{2}}{R_E + h} = \frac{GM_E m_s}{(R_E + h)^2} \quad \textbf{(1)}$$

$$v_S^2 = \frac{GM_E}{R_E + h} \quad \textbf{(1)}$$

Greater *h* Þ smaller v_S since *G*, M_E constant (1)

(3 marks)

Explain why, if a satellite slows down in its orbit, it nevertheless gradually spirals in towards the Earth's surface.

As it slows $\frac{GM_E m_s}{(R_E + h)^2} > \frac{m_s v_s^{2}}{R_E + h}$ **(1)**

The "spare" gravitational force not needed to provide the centripetal acceleration pulls the satellite nearer to the Earth (1)

(2 marks)
[Total 10 marks]

3. Forces

(i) $F = GM_E m/R^2$ 1

(ii) $F = GM_M m/r^2$ 1

Distance R

$$\left.\begin{array}{c}\dfrac{GM_E m}{R^2} = \dfrac{GM_m m}{r^2} \\ \text{OR} \\ \dfrac{M_E}{M_m} = \dfrac{R^2}{r^2} \text{ OR } \left(\dfrac{M_E}{M_m}\right)^{1/2} = \dfrac{R}{r}\end{array}\right\}$$

$$\frac{81}{1} = \frac{R^2}{(3.9 \times 10^7 \text{ m})^2}$$

$R = 3.5 \times 10^8$ m

Evidence that equating forces has occurred 1

Correct substitution 1

Correct answer 1

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4. Gravitational attraction of Earth on Moon

Use $\dfrac{Gm_1 m_S}{r^2}$, ie $\dfrac{GMm}{(60R)^2}$ **(1)** 1

Orbital speed of the Moon

$\dfrac{m\upsilon^2}{r} = \dfrac{GMm}{r^2}$ **(1)**

Use of $r = 60R$, ie $60 \times 6.4 \times 10^6$ **(1)**

Rearrangement ie $\upsilon = \sqrt{\dfrac{6.67 \times 10^{-11}\,\text{Nm}^2\text{kg}^{-2} \times 6 \times 10^{24}\,\text{kg}}{60 \times 6.4 \times 10^6\,\text{m}}}$ **(1)**

= 1020 m s^{-1} **(1)** 4

Orbit period

Time = $\dfrac{2\pi r}{\upsilon}$ /ω = $\dfrac{2\pi}{T}$ **(1)**

Calculation: $\dfrac{2 \times \pi \times 60 \times 6.4 \times 10^6}{1020\,\text{m}\,\text{s}^{-1}}$ **(1)**

Divide by 3600 × 24 **(1)**
Using 1020m/s: (27 – 27.4) days
Using 1000m/s: (27.8 – 28) days **(1)** 4

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5. Word equation:

Force proportional to <u>product of masses</u> and inversely proportional to (<u>distance / separation) squared</u> **(1)(1)**

2

[No force 0/2]

OR

$$F = \frac{G \times \text{mass}_1 \times \text{mass}_2}{(\text{distance})^2}$$ **(1)**

[or $(\text{separation})^2$ instead of bottom line]

Calculation of force:

From Newton's law OR idea that force = weight = mg_{planet} **(1)**

$$F = \frac{6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 6.42 \times 10^{23}\ \text{kg} \times 1\ \text{kg}}{(3.40 \times 10^6)^2\ \text{m}^2}$$

[Substitution in correct equation only]

OR

$$g_{\text{mars}} = \frac{\text{G} \times 6.42 \times 10^{23}\ \text{kg}}{(3.4 \times 10^6)^2\ \text{m}^2}$$ **(1)**

= 3.7 N **(1)**

Smaller **(1)**

3

Explanation of reasoning:

g is less, but ρ is similar/same [so R is less] **(1)**

[2nd mark is consequential on first mark]

2

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6. Base units of G: $\text{kg}^{-1}\ \text{m}^3\ \text{s}^{-2}$ **(1)**
Equation homogeneity:

Correct substitution of units of G, r^3 (m^3), M (kg) 1

leading to S^2 and linked to T^2 **(1)**
[Allow e.c.f. of their base unit answer into substitution mark] 3

<u>Use of relationship to find mass of the Earth</u>

Any two from:
Adding 20 000 + 6400 **(1)**
Converting km to m **(1)**
h to s (× 43 200) **(1)** 2
Answer $M = 5.8\ (4) \times 10^{24}$ kg **(1)** 1

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7. Expression for gravitational force

$F = GMm/r^2$ **(1)** 1

Derived expression

Reasoning step must be clear, e,g, $mg = GMm/r^2$ **(1)**
so $g = GM/r^2$ **(1)** 2

Sun's gravitational field strength

$g = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $\times 1.99 \times 10^{30}$ kg / $(1.50 \times 10^{11}$ m$)^2$ **(1)**

$= 5.9 \times 10^{-3}$ (N kg^{-1}) [no u.e.] **(1)** 2

Diagram

(i) Jupiter marked closest to Earth **(1)**

(ii) (Labelled) arrows towards Jupiter and Sun (radially) **(1)** 2

Maximum percentage change

$3.2 \times 10^{-7} / 5.9 \times 10^{-3} \times 100\% = 0.005\%$ **(1)** 1

Maximum value of the ratio

Use of $g \propto M/r^2$ **(1)**

Hence $g_{\text{Venus}}/g_{\text{jupiter}} = 15^2/400 = 0.56$ [OR 9/16] **(1)** 2

Comment

E.g. reference to % change caused by Jupiter (combined with effect caused by Venus)
OR g of (all) planets is **very** small compared with g of Sun **(1)** 1

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8. Show that:

$F = GMm/r^2$ **(1)**

$= 6.9 \times 10^{24}$ (N) **(1)** 2

Calculation:

$a = F/M = 3.1 \times 10^{-6}$ m s^{-2} **(1)** 1

Explanation:

Planet exerts gravitational force on star **(1)**

Planet revolves around star, so direction of force changes with time **(1)**

Diagram showing force (or effect of force on star due to planet) **(1)** 3

Speed of star:

Using $\upsilon = 2\pi r / T$ and $a = \upsilon / r$ **(1)**

$r = \upsilon T/2\pi$ so $a = 2\pi\upsilon / T$

so $\upsilon = aT / 2\pi$ **(1)**

$= 3.1 \times 10^{-6} \times 9.2 \times 10^{7} \div 2\pi$ [allow ecf for *a*]

$= 45.4$ m s^{-1} 3

Calculation:

$\Delta\lambda = \lambda\, \upsilon/c = 656 \times 10^{-9} \times 45 / 3.0 \times 10^{8}$ **(1)**

$= 9.8 \times 10^{-14}$ m

[Accept 2 × Δλ for maximum marks] **(1)** 2

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9. <u>Formula</u>

$F = GMm/r^2$ **(1)** 1

<u>Show that $\tan\theta = MR^2 / M_e r^2$</u>

Horizontally $T\sin\theta = F_{mountain}$ and vertically $T\cos\theta = mg$ **(1)**
[OR vector diagram showing forces and θ]

Dividing equations [OR from vector diag.]: $\tan\theta = F_{mountain} \div mg$ **(1)**
and $F_{mountain} = GMm \div r^2$ and $mg = GM_e m \div R^2$ **(1)**
so $\tan\theta = GMm/r^2 \div GM_e m/R^2$ **(1)** 4

<u>Value for gravitational constant, *G*</u>

Volume = $4/3\pi R^3 = 4/3\pi\,(6.4 \times 10^6 \text{ m})^3 = 1.1 \times 10^{21}$ (m^3) **(1)**

$M_e = V\rho = 1.1 \times 10^{21}$ m^3 $\times 4.5 \times 10^3$ kg m^{-3} $= 4.9 \times 10^{24}$ (kg) **(1)**

$$G = \frac{gR^2}{M_e} = \frac{3gR^2}{4\pi R^3 \rho} = \frac{3g}{4\pi R\rho} = \frac{3 \times 9.8\,\text{m s}^{-2}}{4\pi \times 6.4 \times 10^6\,\text{m} \times 4.5 \times 10^3\,\text{kg m}^{-3}}$$

$= 8.1 \times 10^{-11}$ N m^2 kg^{-2} **(1)** 3

<u>Reason for inaccuracy</u>

Any one from:

- Maskelyne's density is incorrect
- Earth/mountain not uniform density
- (centre of) mass of mountain not known
- mountain is not spherical
- difficult to determine vertical / measure very small θ
- Earth not a perfect sphere / point mass **(1)** 1

<u>Earth's core</u>

Much denser than mountain **(1)** 1

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10. <u>Minimum mass for comet</u>

Volume = $4/3\pi\ (9 \times 10^3/\ 2)^3$ m^3 (= 3.8×10^{11} m^3) **(1)**

Use of mass = density × volume

= 500 kg m^{-3} × $4/3\ \pi\ (9 \times 10^3\ /\ 2)^3$ m^3 **(1)**

1.91×10^{14} kg **(1)** 3

<u>Jupiter's gravitational field strength</u>

$g = GM/r^2$ **(1)**

= 6.6720×10^{-11} N m^{-2} kg,$^{-2}$ × 1.8987×10^{27} kg / $(96\ 009 \times 10^3)^2$ **(1)**

= 13.7432 N kg^{-1} **(1)** 3

<u>Explanation</u>

Any two from:

- Jupiter's force/field strength different on the two sides of the comet **(1)**
- difference sufficient to pull comet apart **(1)**
- Jupiter's force larger than (cohesive) force between particles of comet **(1)** Max 2

<u>Difference and similarity between gravitational and electric fields</u>

Difference: e.g. gravitational fields only attractive, electrical can be attractive or repulsive, gravitational fields due to mass, electrical due to charge, cannot shield **(1)** g-field, can shield E field

Similarity: e.g. follow equivalent mathematical formulae, both obey inverse square law **(1)** 2

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11. Definitions:
An electric field is a region where charged objects experience a force (E = F/Q) **(1)** 1
A gravitational field is a region where masses experience a force(g = F/m) **(1)** 1

Similarities:

- Both fields obey an inverse square law OR inverse square equations quoted **(1)**
- Both fields are radial for point objects / spherical distributions **(1)**
- Both fields have an infinite range / field strength approaches zero a long way from source **(1)** max 2

Differences:

- Electric forces can be attractive or repulsive but gravitational forces are always attractive **(1)**
- Electric forces are (much) stronger than gravitational forces OR comparison of size of coupling constants in the two force equations **(1)**
- Electric forces only act on charged particles but gravitational forces act on all matter **(1)**
- Electric forces can be shielded (e.g. by use of a Faraday cage) but gravitational forces cannot **(1)** max 3

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12. Magnitude of gravitational force on Cassini

$F = GMm/r^2$ 1

Expression

$g = F/m$ **(1)**

so $g = GM/r^2$ **(1)** 2

Maximum acceleration

Appreciation that acceleration = g-field **(1)**

Addition of orbital height to radius of Venus **(1)**

$g = G \times 4.87 \times 10^{24}$ kg / $(6384 \times 10^3)^2$

= 7.97 m s^{-2} 3

Effect of acceleration on velocity of Cassini

Any 2 from:

- Acceleration is at right angles to direction of motion
- Speed unchanged

(Velocity changed since) direction changed Max 2

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