Circular & Gravitation
May 02

1 **(a)** The Earth may be considered to be a uniform sphere of radius 6.38×10^6 m. Its mass is assumed to be concentrated at its centre.

Given that the gravitational field strength at the Earth's surface is 9.81 N kg^{-1}, show that the mass of the Earth is 5.99×10^{24} kg.

[2]

(b) A satellite is placed in geostationary orbit around the Earth.

(i) Calculate the angular speed of the satellite in its orbit.

angular speed = ... rad s^{-1} [3]

(ii) Using the data in **(a)**, determine the radius of the orbit.

radius = ... m [3]

Nov 02

4 If an object is projected vertically upwards from the surface of a planet at a fast enough speed, it can escape the planet's gravitational field. This means that the object can arrive at infinity where it has zero kinetic energy. The speed that is just enough for this to happen is known as the escape speed.

(a) (i) By equating the kinetic energy of the object at the planet's surface to its total gain of potential energy in going to infinity, show that the escape speed v is given by

$$v^2 = \frac{2GM}{R},$$

where R is the radius of the planet and M is its mass.

(ii) Hence show that

$$v^2 = 2Rg,$$

where g is the acceleration of free fall at the planet's surface.

[3]

(b) The mean kinetic energy E_k of an atom of an ideal gas is given by

$$E_k = \frac{3}{2} kT,$$

where k is the Boltzmann constant and T is the thermodynamic temperature.

Using the equation in **(a)(ii)**, estimate the temperature at the Earth's surface such that helium atoms of mass 6.6×10^{-27} kg could escape to infinity.

You may assume that helium gas behaves as an ideal gas and that the radius of Earth is 6.4×10^{6} m.

temperature = ... K [4]

May 03

1 (a) Define gravitational potential.

....

.... [2]

(b) Explain why values of gravitational potential near to an isolated mass are all negative.

....

....

.... [3]

(c) The Earth may be assumed to be an isolated sphere of radius 6.4×10^3 km with its mass of 6.0×10^{24} kg concentrated at its centre. An object is projected vertically from the surface of the Earth so that it reaches an altitude of 1.3×10^4 km.

Calculate, for this object,

(i) the change in gravitational potential,

change in potential = ... $\mathrm{J\,kg^{-1}}$

(ii) the speed of projection from the Earth's surface, assuming air resistance is negligible.

speed = ... $\mathrm{m\,s^{-1}}$

[5]

(d) Suggest why the equation

$$v^2 = u^2 + 2as$$

is not appropriate for the calculation in **(c)(ii)**.

....

.... [1]

Nov 03

1 (a) (i) On Fig. 1.1, draw lines to represent the gravitational field outside an isolated uniform sphere.

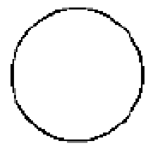

Fig. 1.1

(ii) A second sphere has the same mass but a smaller radius. Suggest what difference, if any, there is between the patterns of field lines for the two spheres.

....

....

[3]

(b) The Earth may be considered to be a uniform sphere of radius 6380 km with its mass of 5.98×10^{24} kg concentrated at its centre, as illustrated in Fig. 1.2.

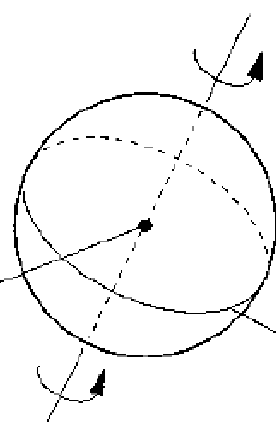

Fig. 1.2

A mass of 1.00 kg on the Equator rotates about the axis of the Earth with a period of 1.00 day (8.64×10^4 s).

Calculate, to three significant figures,

(i) the gravitational force F_G of attraction between the mass and the Earth,

F_G = ... N

(ii) the centripetal force F_C on the 1.00 kg mass,

F_C = ... N

(iii) the difference in magnitude of the forces.

difference = ... N
[6]

(c) By reference to your answers in **(b)**, suggest, with a reason, a value for the acceleration of free fall at the Equator.

....

....

.... [2]

May 04

3 A binary star consists of two stars that orbit about a fixed point C, as shown in Fig. 3.1.

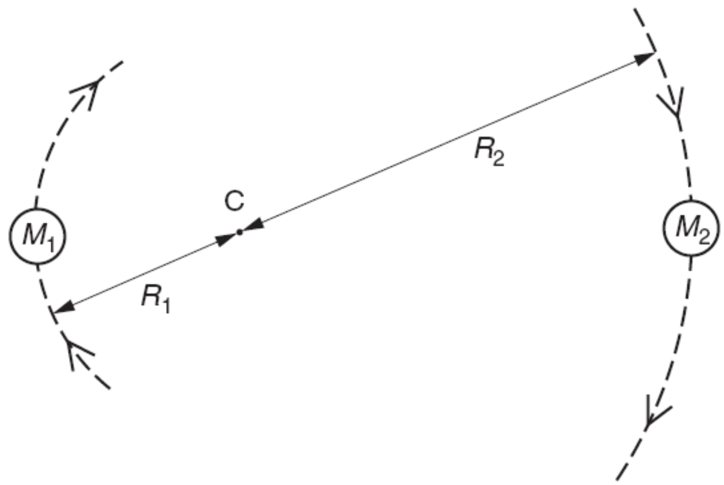

Fig. 3.1

The star of mass M_1 has a circular orbit of radius R_1 and the star of mass M_2 has a circular orbit of radius R_2. Both stars have the same angular speed ω, about C.

(a) State the formula, in terms of G, M_1, M_2, R_1, R_2 and ω for

(i) the gravitational force between the two stars,

....

(ii) the centripetal force on the star of mass M_1.

....
[2]

(b) The stars orbit each other in a time of 1.26×10^8 s (4.0 years). Calculate the angular speed ω for each star.

angular speed = ... rad s^{-1} [2]

(c) **(i)** Show that the ratio of the masses of the stars is given by the expression

$$\frac{M_1}{M_2} = \frac{R_2}{R_1}.$$

[2]

(ii) The ratio $\frac{M_1}{M_2}$ is equal to 3.0 and the separation of the stars is 3.2×10^{11} m.

Calculate the radii R_1 and R_2.

R_1 = ... m

R_2 = ... m
[2]

(d) **(i)** By equating the expressions you have given in **(a)** and using the data calculated in **(b)** and **(c)**, determine the mass of one of the stars.

mass of star = ... kg

(ii) State whether the answer in **(i)** is for the more massive or for the less massive star.

....
[4]

Nov 04

1 A particle is following a circular path and is observed to have an angular displacement of 10.3°.

(a) Express this angle in radians (rad). Show your working and give your answer to three significant figures.

angle = ...rad [2]

(b) (i) Determine tan10.3° to three significant figures.

tan10.3° = ...

(ii) Hence calculate the percentage error that is made when the angle 10.3°, as measured in radians, is assumed to be equal to tan10.3°.

percentage error = ...
[3]

May 05

1 The orbit of the Earth, mass 6.0×10^{24} kg, may be assumed to be a circle of radius 1.5×10^{11} m with the Sun at its centre, as illustrated in Fig. 1.1.

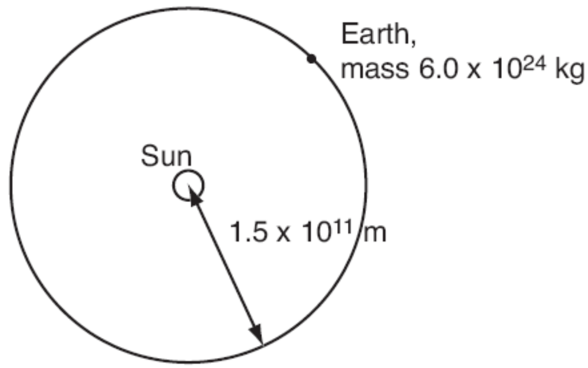

Fig. 1.1

The time taken for one orbit is 3.2×10^{7} s.

(a) Calculate

(i) the magnitude of the angular velocity of the Earth about the Sun,

angular velocity = ... rad s^{-1} [2]

(ii) the magnitude of the centripetal force acting on the Earth.

force = ... N [2]

(b) **(i)** State the origin of the centripetal force calculated in **(a)(ii)**.

....

....[1]

(ii) Determine the mass of the Sun.

mass = ... kg [3]

Nov 05

1 The Earth may be considered to be a sphere of radius 6.4×10^6 m with its mass of 6.0×10^{24} kg concentrated at its centre.
A satellite of mass 650 kg is to be launched from the Equator and put into geostationary orbit.

(a) Show that the radius of the geostationary orbit is 4.2×10^7 m.

[3]

(b) Determine the increase in gravitational potential energy of the satellite during its launch from the Earth's surface to the geostationary orbit.

energy = ………………………………….. J [4]

(c) Suggest one advantage of launching satellites from the Equator in the direction of rotation of the Earth.

....

....[1]

May 06

1 The Earth may be considered to be a uniform sphere with its mass *M* concentrated at its centre.

A satellite of mass *m* orbits the Earth such that the radius of the circular orbit is *r*.

(a) Show that the linear speed *v* of the satellite is given by the expression

$$v = \sqrt{\left(\frac{GM}{r}\right)}.$$

[2]

(b) For this satellite, write down expressions, in terms of *G*, *M*, *m* and *r*, for

(i) its kinetic energy,

kinetic energy = ... [1]

(ii) its gravitational potential energy,

potential energy = ... [1]

(iii) its total energy.

total energy = ... [2]

(c) The total energy of the satellite gradually decreases.

State and explain the effect of this decrease on

(i) the radius r of the orbit,

....

....

.... [2]

(ii) the linear speed v of the satellite.

....

....

.... [2]

Nov 06

1 The definitions of electric potential and of gravitational potential at a point have some similarity.

(a) State one similarity between these two definitions.

....

.... [1]

(b) Explain why values of gravitational potential are always negative whereas values of electric potential may be positive or negative.

....

....

....

....

.... [4]

Nov 06

4 A rocket is launched from the surface of the Earth.

Fig. 4.1 gives data for the speed of the rocket at two heights above the Earth's surface, after the rocket engine has been switched off.

height / m	speed / m s^{-1}
$h_1 = 19.9 \times 10^6$	$v_1 = 5370$
$h_2 = 22.7 \times 10^6$	$v_2 = 5090$

Fig. 4.1

The Earth may be assumed to be a uniform sphere of radius $R = 6.38 \times 10^6$ m, with its mass M concentrated at its centre. The rocket, after the engine has been switched off, has mass m.

(a) Write down an expression in terms of

(i) G, M, m, h_1, h_2 and R for the change in gravitational potential energy of the rocket,

.... [1]

(ii) m, v_1 and v_2 for the change in kinetic energy of the rocket.

.... [1]

(b) Using the expressions in **(a)**, determine a value for the mass M of the Earth.

M = ... kg [3]

May 07

1 **(a)** Explain what is meant by a *gravitational field*.

....

.... [1]

(b) A spherical planet has mass M and radius R. The planet may be considered to have all its mass concentrated at its centre.
A rocket is launched from the surface of the planet such that the rocket moves radially away from the planet. The rocket engines are stopped when the rocket is at a height R above the surface of the planet, as shown in Fig. 1.1.

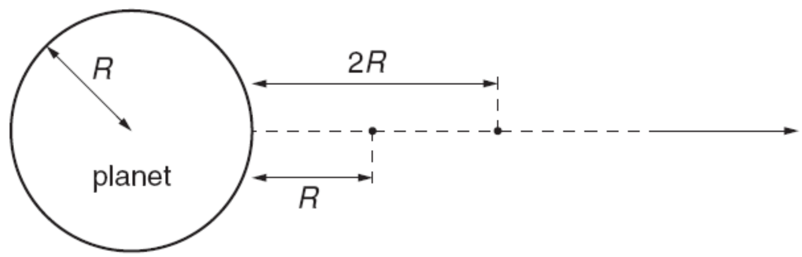

Fig. 1.1

The mass of the rocket, after its engines have been stopped, is m.

(i) Show that, for the rocket to travel from a height R to a height $2R$ above the planet's surface, the change ΔE_P in the magnitude of the gravitational potential energy of the rocket is given by the expression

$$\Delta E_P = \frac{GMm}{6R}.$$

[2]

(ii) During the ascent from a height R to a height $2R$, the speed of the rocket changes from $7600\,\text{m}\,\text{s}^{-1}$ to $7320\,\text{m}\,\text{s}^{-1}$. Show that, in SI units, the change ΔE_K in the kinetic energy of the rocket is given by the expression

$$\Delta E_K = (2.09 \times 10^6)m.$$

[1]

(c) The planet has a radius of $3.40 \times 10^6\,\text{m}$.

(i) Use the expressions in **(b)** to determine a value for the mass M of the planet.

M = ... kg [2]

(ii) State one assumption made in the determination in **(i).**

....

.... [1]

Nov 07

1 (a) Explain

(i) what is meant by a *radian*,

....

....

....[2]

(ii) why one complete revolution is equivalent to an angular displacement of 2π rad.

....

....[1]

(b) An elastic cord has an unextended length of 13.0 cm. One end of the cord is attached to a fixed point C. A small mass of weight 5.0 N is hung from the free end of the cord. The cord extends to a length of 14.8 cm, as shown in Fig. 1.1.

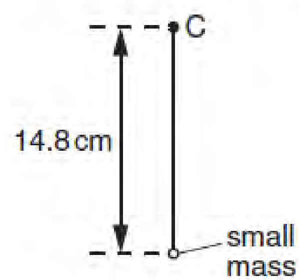

Fig. 1.1

The cord and mass are now made to rotate at constant angular speed ω in a vertical plane about point C. When the cord is vertical and above C, its length is the unextended length of 13.0 cm, as shown in Fig. 1.2.

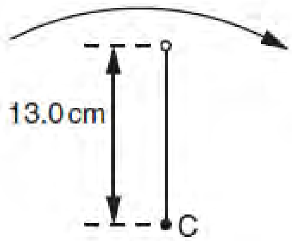

Fig. 1.2

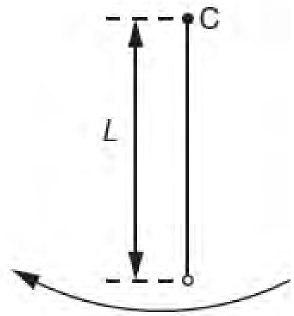

Fig. 1.3

(i) Show that the angular speed ω of the cord and mass is 8.7 rad s^{-1}.

[2]

(ii) The cord and mass rotate so that the cord is vertically below C, as shown in Fig. 1.3.

Calculate the length L of the cord, assuming it obeys Hooke's law.

L = ... cm [4]

May 08

1 (a) (i) Define the *radian*.

....

....

....[2]

(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed, as shown in Fig. 1.1.

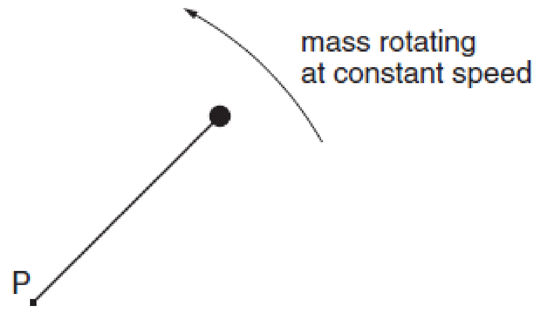

Fig. 1.1

Explain what is meant by the *angular* speed about point P of the mass.

....

....

....[2]

(b) A horizontal flat plate is free to rotate about a vertical axis through its centre, as shown in Fig. 1.2.

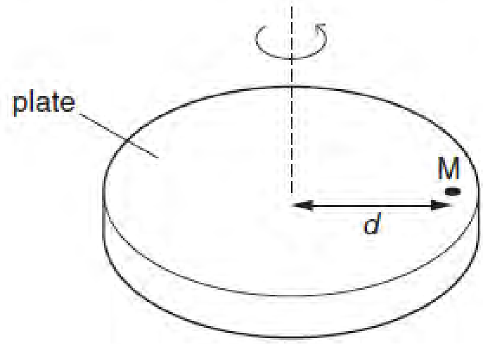

Fig. 1.2

A small mass M is placed on the plate, a distance d from the axis of rotation.
The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.

The maximum frictional force F between the plate and the mass is given by the expression

$$F = 0.72W,$$

where W is the weight of the mass M.
The distance d is 35 cm.

Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate. Explain your working.

number = ... [5]

(c) The plate in **(b)** is covered, when stationary, with mud.
Suggest and explain whether mud near the edge of the plate or near the centre will first leave the plate as the angular speed of the plate is slowly increased.

....

....

.... [2]

Nov 08

1 A spherical planet has mass *M* and radius *R*.
The planet may be assumed to be isolated in space and to have its mass concentrated at its centre.
The planet spins on its axis with angular speed ω, as illustrated in Fig. 1.1.

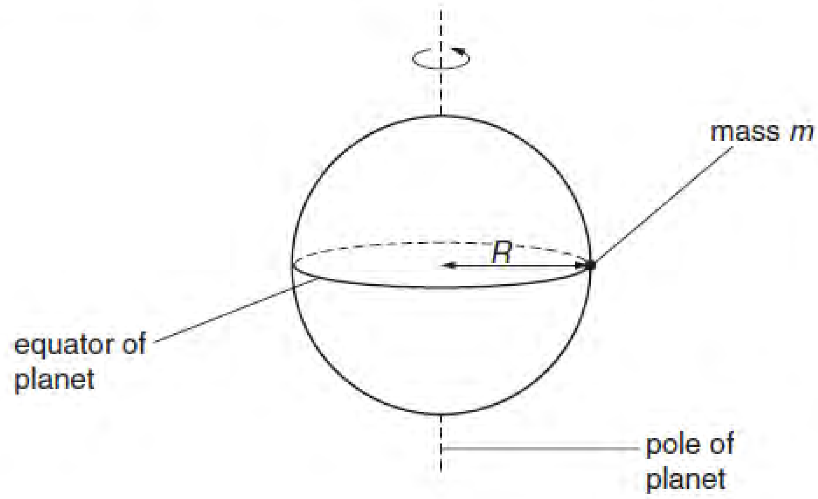

Fig. 1.1

A small object of mass *m* rests on the equator of the planet. The surface of the planet exerts a normal reaction force on the mass.

(a) State formulae, in terms of *M*, *m*, *R* and ω, for

(i) the gravitational force between the planet and the object,

.... [1]

(ii) the centripetal force required for circular motion of the small mass,

.... [1]

(iii) the normal reaction exerted by the planet on the mass.

.... [1]

(b) **(i)** Explain why the normal reaction on the mass will have different values at the equator and at the poles.

....

....

.... [2]

(ii) The radius of the planet is 6.4×10^6 m. It completes one revolution in 8.6×10^4 s. Calculate the magnitude of the centripetal acceleration at

1. the equator,

acceleration = ...m s^{-2} [2]

2. one of the poles.

acceleration = ...m s^{-2} [1]

(c) Suggest two factors that could, in the case of a real planet, cause variations in the acceleration of free fall at its surface.

1. ....

....

2. ....

....
[2]

May 09

1 (a) Define *gravitational field strength*.

....

.... [1]

(b) A spherical planet has diameter 1.2×10^4 km. The gravitational field strength at the surface of the planet is $8.6\,\text{N kg}^{-1}$.
The planet may be assumed to be isolated in space and to have its mass concentrated at its centre.
Calculate the mass of the planet.

mass = ... kg [3]

(c) The gravitational potential at a point X above the surface of the planet in **(b)** is $-5.3 \times 10^7\,\text{J kg}^{-1}$.
For point Y above the surface of the planet, the gravitational potential is $-6.8 \times 10^7\,\text{J kg}^{-1}$.

(i) State, with a reason, whether point X or point Y is nearer to the planet.

....

....

.... [2]

(ii) A rock falls radially from rest towards the planet from one point to the other.
Calculate the final speed of the rock.

speed = ... m s^{-1} [2]