M1. (a) (i) d_0 =(speed × time = 1.8 × 10^8 × 95 × 10^{-9}) = 17(.1) m ✓ 1

(ii) d (= d_0 $(1 - v^2/c^2)^{½}$)

= 17.1 × $(1 - (1.8 \times 10^8/3.0 \times 10^8)^2))^{½}$ ✓

= 14 m ✓ (or 13.7 m or 13.68 m)

or

$t = t_0 (1 - v^2/c^2)^{-½}$

95 = t_0 × $(1 - (1.8 \times 10^8/3.0 \times 10^8)^2)^{-½}$ gives t_0 = 76 ns ✓

$d = vt_0$ = 1.8 × 10^8 × 76 × 10^{-9} = 14 m ✓ (or 13.7 m or 13.68 m) 2

(b) m (= m_0 $(1 - v^2/c^2)^{-½}$)

= 1.67(3) × 10^{-27} × $(1 - (1.8 \times 10^8/3.0 \times 10^8)^2)^{-½}$) ✓

= 2.09 × 10^{-27} kg ✓

kinetic energy = $(m - m_0)\ c^2$

or correct calculation of $E = mc^2$ (= 1.88 × 10^{-10} J)

or correct calculation of $E_0 = m_0c^2$ (= 1.50 × 10^{-10} J) ✓

$$\frac{\text{kinetic energy}}{\text{rest energy}} = \frac{(m - m_0)c^2}{m_0c^2} = \frac{(2.09 - 1.67) \times 10^{-27}}{1.67 \times 10^{-27}}$$ ✓

= 0.25 (allow 0.245 to 0.255 or ¼ or 1:4) ✓ 5

[8]

M2.(a) (i) (use of $v = \frac{d}{t}$ gives) $v = \frac{240}{0.84 \times 10^{-6}}$ = 2.8(6) × 10^8 m s^{-1} **(1)**

(ii) actual length = 240 m **(1)**

(use of $l = l_0\left(1-\frac{v^2}{c^2}\right)^{1/2}$ gives)

length in particle frame, $l = 240\left(1-\frac{2.86^2}{3^2}\right)^{1/2}$ **(1)**

(allow C.E. for value of v)

l = (240 × 0.30) = 72(.5) m **(1)**

4

(b) time between two events depends on speed of observer

[or $t = t_0\left(1-\frac{v^2}{c^2}\right)^{1/2}$ or rocket time depends on speed of traveller] **(1)**

traveller's journey time is the proper time between start and stop
[or t_0 is the proper time or t is the time on Earth] **(1)**
journey time measured on Earth > journey time measured by traveller
[or $t > t_0$ or rocket time slower / less than Earth time] **(1)**
traveller younger than twin on return to Earth **(1)**

4

[8]

M3.(a) (i) speed of light (in free space) independent of motion of source **(1)**
and of motion of observer **(1)**
[*alternative (i)*
speed of light is same in all frames of reference **(1)**]

(ii) laws of physics have same form in all inertial frames **(1)**
inertial frame is one in which Newton's 1st law of motion obeyed **(1)**
laws of physics unchanged in coordinate transformation
from one inertial frame of reference to any other inertial frame **(1)**

(max 4)

(b) (i) $m\left(= m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right) = 1.88 \times 10^{-28}\,(1-(0.996)^2)^{-\frac{1}{2}}$ **(1)**

$= 2.10 \times 10^{-27}$ kg **(1)**

(ii) $t_0 = 2.2 \times 10^{-6}$ s **(1)**

$t\left(= t_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}\right) = 2.2 \times 10^{-6}\,(1-(0.996)^2)^{-\frac{1}{2}}$ (s) **(1)**

$= 2.46 \times 10^{-5}$ (s) **(1)**

$s(= vt = 3.00 \times 10^{8} \times 0.996 \times 2.46 \times 10^{-5}) = 7360$ m **(1)**

[*alternative (ii)*

$l\,(= vt = 0.996 \times 3.0 \times 10^{8} \times 2.2 \times 10^{6}) = 657$ (m) **(1)**

correct substitution of l in $l = l_0\sqrt{1-\frac{v^2}{c^2}}$ **(1)**

$l_0\left(= \frac{l}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{657}{\sqrt{1-0.996^2}}$ **(1)**

$l_0 = 7360$ m **(1)**

(6)

[10]

M4.(a) (i) $l = (vt = 1.00 \times 10^8 \times 15 \times 10^{-9}) = 1.50$m **(1)**

(ii) $\left(l = l_0\sqrt{1 - \frac{v^2}{c^2}}\right)$

$1.50 = l_0 \sqrt{1 - \frac{(1.00 \times 10^8)^2}{(3.00 \times 10^8)^2}}$ **(1)**

$l_0 \left(= \frac{1.50}{0.943}\right) = 1.59$ m **(1)**

3

(b) (i) $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ **(1)** $\left[\text{or } \frac{m_0}{\sqrt{1 - \frac{(1.00 \times 10^8)^2}{(3.00 \times 10^8)^2}}}\right]$

$m \left(\text{or } \frac{m_0}{\sqrt{1 - \frac{(1.00 \times 10^8)^2}{(3.00 \times 10^8)^2}}}\right) = 1.06m_0$

[or = $1.06 \times 1.67 \times 10^{-27}$ or 1.77×10^{-27} kg] **(1)**
kinetic energy = $(m - m_0)c^2$ **(1)**
[or = $0.06m_0c^2$ or $0.06 \times 1.67 \times 10^{-27} \times (3 \times 10^8)^2$]
= 9.1×10^{-12} (J) **(1)**

(ii) total k.e. = $(10^7 \times 9.1 \times 10^{-12}) = 9.1 \times 10^{-5}$ (J) **(1)**

k.e. per second $\left(= \frac{9.1 \times 10^{-5}}{1.5 \times 10^{-9}}\right)$ = 6080W

max 5

[8]