Q1. A grinding wheel is used to sharpen chisels in a school workshop. A chisel is forced against the edge of the grinding wheel so that the tangential force on the wheel is a steady 7.0 N as the wheel rotates at 120 rad s^{-1}. The diameter of the grinding wheel is 0.15 m.

(a) (i) Calculate the torque on the grinding wheel, giving an appropriate unit.

answer = ...

(2)

(ii) Calculate the power required to keep the wheel rotating at 120 rad s^{-1}.

answer = ... W

(1)

(b) When the chisel is removed and the motor is switched off, it takes 6.2 s for the grinding wheel to come to rest.

Calculate the number of rotations the grinding wheel makes in this time.

answer = ...

(2)

(Total 5 marks)

Q2. **Figure 1** shows a human centrifuge used in pilot training to simulate the large 'g' forces experienced by pilots during aerial manoeuvres. The trainee sits in the capsule at the end of the rotating centrifuge arm, which is driven by an electric motor.

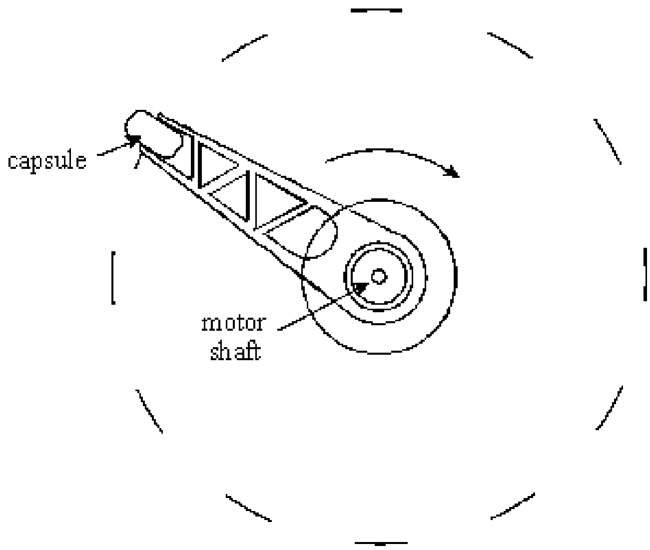

Figure 1

(a) When working at maximum power, the motor is capable of increasing the angular speed of the arm from its minimum working speed of 1.6 rad s^{-1} to its maximum speed of 7.4 rad s^{-1} in 4.4 s. The net power needed to achieve this acceleration is 150 kW.

(i) Assuming that this power remains constant during the acceleration, calculate the energy supplied to the centrifuge by the motor.

....

....

(ii) Hence estimate the moment of inertia of the rotating system.

....

....

....

....

(4)

(b) Bending stresses on the central shaft could have been reduced at the design stage by extending the arm beyond the shaft and fixing a counterweight, as shown in **Figure 2.** Its designers rejected this because savings in the manufacturing and maintenance costs of the system would be far less than the increased costs associated with a higher power motor.

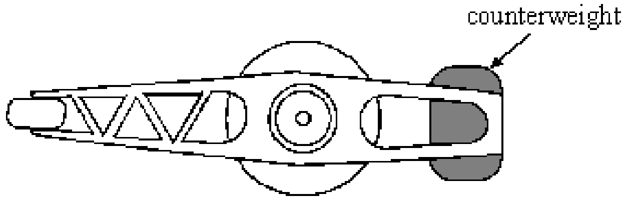

Figure 2

State and explain why, apart from increased friction associated with a heavier arm, the use of a counterweight would require greatly increased motor power. You may be awarded marks for the quality of written communication provided in your answer.

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....

....

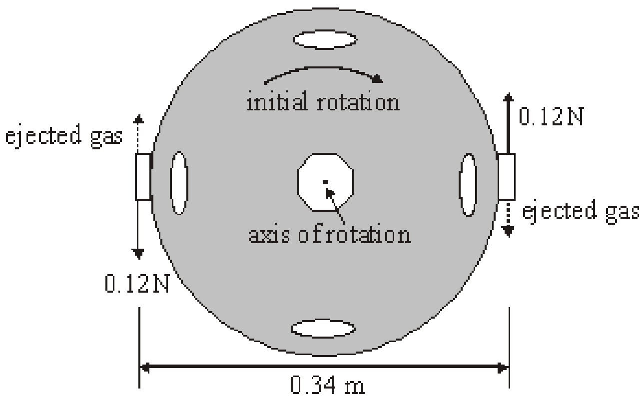
....

....

(2)
(Total 6 marks)

Q3. The figure below shows a remote-control camera used in space for inspecting space stations. The camera can be moved into position and rotated by firing 'thrusters' which eject xenon gas at high speed. The camera is spherical with a diameter of 0.34 m.

In use, the camera develops a spin about its axis of rotation. In order to bring it to rest, the thrusters on opposite ends of a diameter are fired, as shown in the figure below.



(a) When fired, each thruster provides a constant force of 0.12 N.

(i) Calculate the torque on the camera provided by the thrusters.

....

....

(ii) The moment of inertia of the camera about its axis of rotation is 0.17 kg m^2. Show that the angular deceleration of the camera whilst the thrusters are firing is 0.24 rad s^{-2}.

....

....

(3)

(b) The initial rotational speed of the camera is 0.92 rad s^{-1}. Calculate

(i) the time for which the thrusters have to be fired to bring the camera to rest,

....

....

....

(ii) the angle turned through by the camera whilst the thrusters are firing. Express your answer in degrees.

....

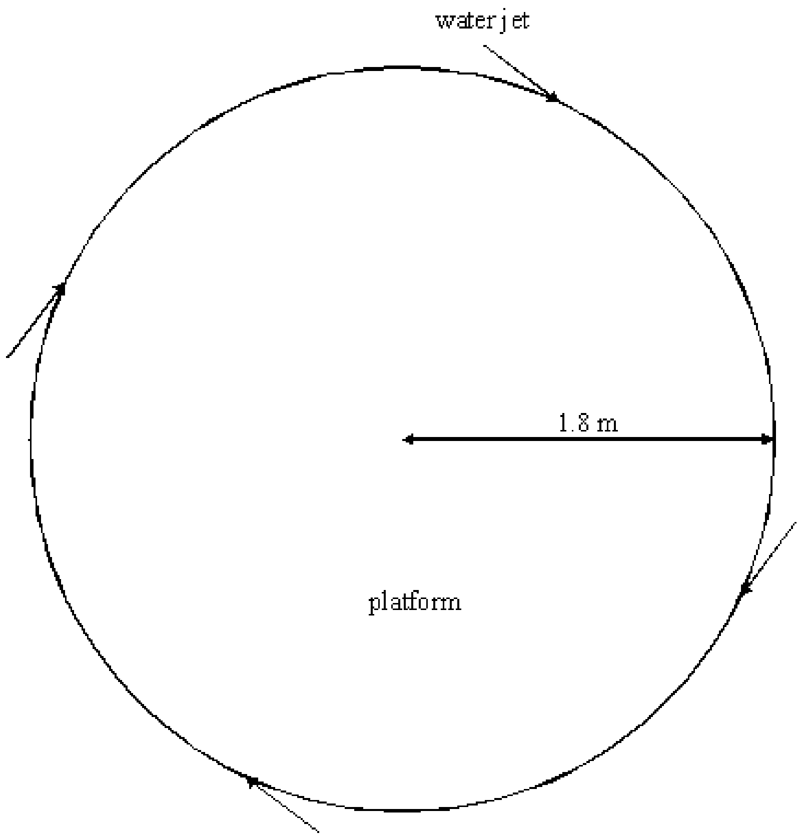
....

....

....

(3)
(Total 6 marks)

Q4. A rotating flower bed forms a novelty feature in the annual display of a horticultural society. The circular platform supporting the plants floats in a water tank and is caused to rotate by means of four water jets directed at the rim of the platform.



Each of the four jets exerts a tangential force of 0.60 N on the platform at a distance of 1.8 m from the axis of the rotation. The platform rotates at a steady angular speed, making one complete revolution in 110 s.

(a) Calculate

(i) the total torque exerted on the platform by the four jets,

....

....

(ii) the power dissipated by the frictional couple acting on the rotating platform, showing your reasoning.

....

....

....

....

(4)

(b) When the water jets are switched off, all the kinetic energy of the loaded platform is dissipated as heat by the frictional couple and the platform comes to rest from its normal steady speed in 12 s.

(i) The kinetic energy of the loaded platform when rotating at its normal steady speed is 1.5 J.
Show that this value is consistent with your answer to part (a)(ii).

....

....

....

(ii) Calculate the moment of inertia of the loaded platform.

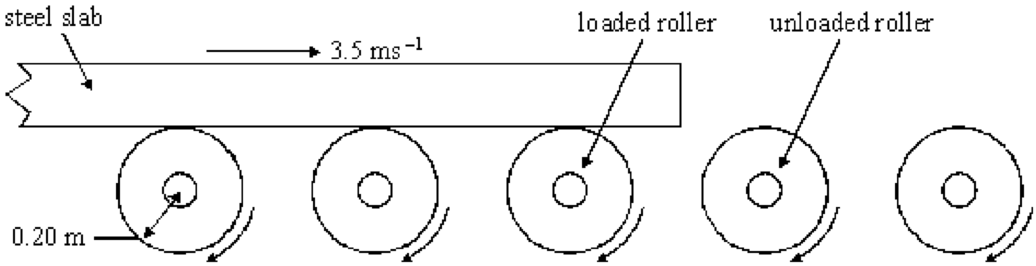
....

....

....

(3)
(Total 7 marks)

Q5. The diagram below shows a method used in a steel mill to transport steel slabs during the manufacture of steel beams. The slab rests on rollers of radius 0.20 m, each of which is driven by its own electric motor. In one operation, a slab moving at 3.5 m s^{-1} along the rollers must be brought to rest and its direction of motion reversed.



(a) Assuming that no sliding occurs between the surfaces of a loaded roller and the slab, calculate the angular speed of the roller when the slab is moving at a steady speed of 3.5 m s^{-1}.

....

....

(1)

(b) While the slab is moving at 3.5 m s^{-1}, the motor driving each roller exerts a uniform torque for 4.6 s on its roller such that the direction of motion of the slab is reversed. At the end of this time, the slab is moving at 3.5 m s^{-1} in the opposite direction.

Calculate

(i) the angular acceleration of a roller during this reversal,

....

....

....

(ii) the uniform torque that its motor must exert to produce this angular acceleration in an unloaded roller. The moment of inertia of each unloaded roller system is 40 kg m^2,

....

....

(iii) the angular impulse imparted when this torque acts on the system for 4.6 s,

....

....

(iv) the number of complete turns made by a loaded roller in bringing the slab momentarily to rest from a speed of 3.5 m s^{-1} before reversing its direction of motion.

....

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....

(6)
(Total 7 marks)

Q6. An early form of four-stroke gas engine stores kinetic energy in a large flywheel driven by the crankshaft. The engine is started from rest with its load disconnected and produces a torque which accelerates the flywheel to its off-load running speed of 110 rev min^{-1}.

(a) The flywheel has a moment of inertia of 150 kg m^2 and takes 15 s to accelerate from rest to an angular speed of 110 rev min^{-1}.

(i) Show that the rotational kinetic energy tored in the flywheel at thi peed i approximately 10 kJ.

....

....

....

(ii) Calculate the average useful power output of the engine during the acceleration.

....

....

(iii) Use your answer to part (ii) to calculate the average net torque acting on the flywheel during the acceleration.

....

....

(5)

(b) When the engine is running at 110 rev min^{-1} off-load, the gas supply to the engine is suddenly cut off and the flywheel continues to rotate for a further 35 complete turns before coming to rest. Calculate the average retarding torque acting on the flywheel.

....

....

(2)
(Total 7 marks)

Q7. Flywheels store energy very efficiently and are being considered as an alternative to battery power.

(a) A flywheel for an energy storage system has a moment of inertia of 0.60 kg m^2 and a maximum safe angular speed of 22 000 rev min^{-1}.

Show that the energy stored in the flywheel when rotating at its maximum safe speed is 1.6 MJ.

(2)

(b) In a test the flywheel was taken up to maximum safe speed then allowed to run freely until it came to rest. The average power dissipated in overcoming friction was 8.7 W.

Calculate

(i) the time taken for the flywheel to come to rest from its maximum speed,

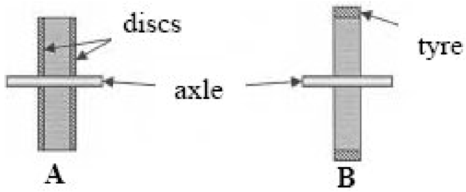
Time taken = ...

(ii) the average frictional torque acting on the flywheel

Torque = ...

(3)

(c) The energy storage capacity of the flywheel can be improved by adding solid discs to the flywheel as shown in cross-section **A** in the diagram below, or by adding a hoop or tyre to the rim of the flywheel as shown in **B** in the diagram below. The same mass of material is added in each case. State, with reasons, which arrangement stores the more energy when rotating at a given angular speed.



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(2)
(Total 7 marks)

Q8. In electrical resistance welding, two steel components are pressed together and a pulse of current passed through the junction between them. Local heating in the junction softens the metal and the components fuse together. One heavy-duty welding rig uses a rotating flywheel as the energy source for the welding operation. The flywheel drives a generator which sets up a current in the junction until the flywheel comes to rest.

(a) The flywheel is driven from rest up to its working angular speed by a motor which produces an output power of 15 kW for 3.0 minutes. The moment of inertia of the flywheel is 9.5 kg m^2.
Assuming that frictional losses are negligible, show that the working angular speed of the flywheel is about 750 rad s^{-1}.

....

....

....

....

(2)

(b) When the flywheel reaches an angular speed of 750 rad s^{-1}, it is disconnected from the motor and connected to the generator. The energy stored in the flywheel is dissipated as heat in the junction between the steel components and the flywheel comes to rest in 4.5 s. Assuming that friction can be neglected, calculate

(i) the angular impulse acting on the flywheel during the welding operation,

....

....

....

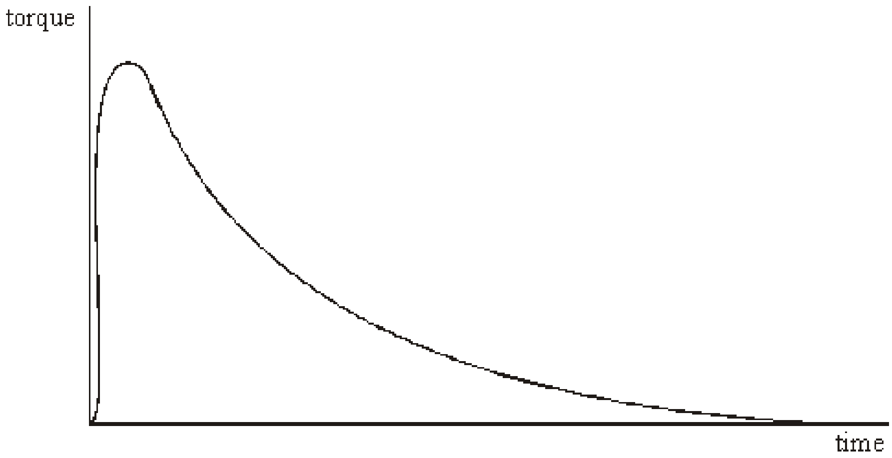
(ii) the average torque acting on the flywheel during the time it takes to come to rest.

....

....

(2)

(c) The torque is not constant during the retardation but is a maximum just after the current is established in the junction. The graph below shows the way that the torque varies with time during any welding operation.



Explain how you could use the graph, if the axes were fully calibrated, to estimate the average torque acting on the system during a welding operation.

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....

(3)
(Total 7 marks)

Q9. **Figure 1** shows a 'firewheel' used at a firework display. Thrust produced by the captive rockets creates a torque which rotates the beam about a horizontal pivot at its centre. The shower of brilliant sparks in the exhaust gases of the rapidly orbiting rockets creates the illusion of a solid wheel.

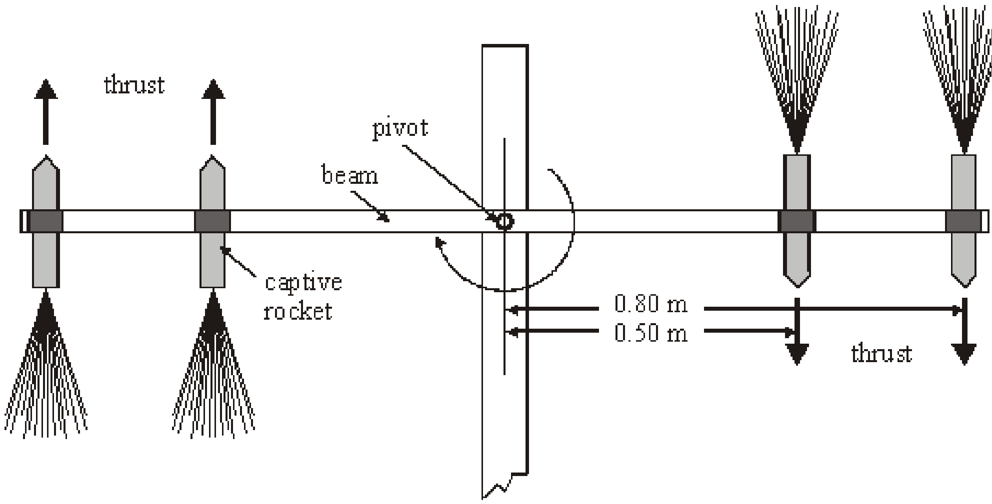


Figure 1

(a) The rockets are fixed symmetrically about the pivot at distances of 0.50 m and 0.80 m from the pivot. The initial mass of each rocket is 0.54 kg and the moment of inertia of the beam about the pivot is 0.14 kg m^2.

Show that the initial moment of inertia of the firewheel about the pivot is 1.10 kg m^2.

....

....

....

(2)

(b) The rockets are ignited simultaneously and each produces a constant thrust of 3.5 N. The frictional torque at the pivot is negligible. Calculate

(i) the total torque about the pivot when all the rockets are producing thrust,

....

....

(ii) the initial angular acceleration of the firewheel,

....

....

(iii) the time taken for the firewheel to make its first complete turn, starting from rest.

....

....

....

(c) The total thrust exerted by the rockets remains constant as the firewheel accelerates. Explain why, after a short time, the firewheel is rotating at a constant angular speed which is maintained until the rocket fuel is exhausted.

....

....

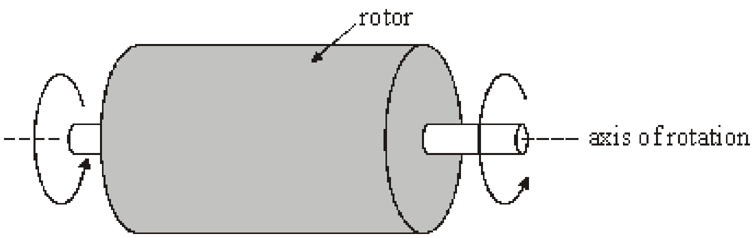
....

....

....

(2)
(Total 8 marks)

Q10. ‘Low inertia’ motors are used in applications requiring rapid changes of speed and direction of rotation. These motors are designed so that the rotor has a very low moment of inertia about its axis of rotation.



(a) (i) Explain why a low moment of inertia is desirable when the speed and direction of rotation must be changed quickly.

....

....

....

....

(ii) State, giving a reason in each case, **two** features of rotor design which would lead to a low moment of inertia about the axis of rotation.

....

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....

....

(4)

(b) In one application, a rotor of moment of inertia 4.4×10^{-5} kg m^2 about its axis of rotation is required to reverse direction from an angular speed of 120 rad s^{-1} to the same speed in the opposite direction in a time of 50 ms. Assuming that the torque acting is constant throughout the change, calculate

(i) the angular acceleration of the rotor,

....

....

(ii) the torque needed to achieve this acceleration,

....

....

(iii) the angular impulse given to the rotor during the time the torque is acting,

....

....

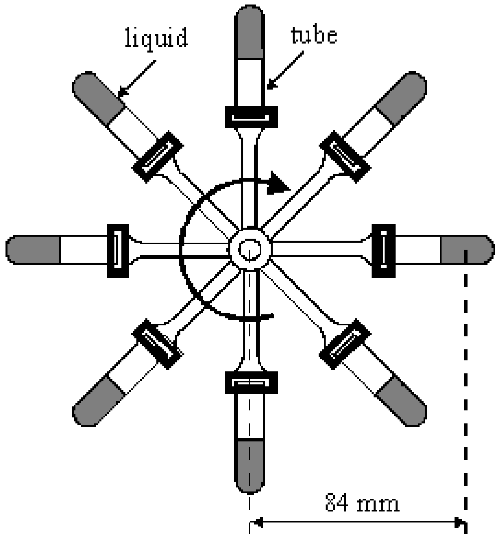
(iv) the angle turned through by the rotor in coming to rest momentarily before reversing direction.

....

....

(4)
(Total 8 marks)

Q11. The diagram shows an overhead view of the load carrier of a spinning centrifuge, used to separate solid particles from the liquid in which they are suspended.



(a) When the centrifuge is operated with empty tubes, it reaches its working angular speed of 1100 rad s^{-1} in a time of 4.2 s, starting from rest. The moment of inertia of this system is 7.6×10^{-4} kg m^2. Calculate

(i) the angular acceleration of the system,

....

....

(ii) the torque required to produce this angular acceleration.

(2)

(b) In normal operation, each of the eight tubes contains 3.0×10^{-3} kg of liquid, whose centre of mass, when spinning, is 84 mm from the axis of rotation. The torque produced by the motor is the same as when the tubes are empty.

Show that this system takes approximately 5 s to reach its working speed of 1100 rad s^{-1}, starting from rest.

....

....

....

....

....

(3)

(c) The normal operating cycle of the centrifuge takes a total time of 1 min. The centrifuge accelerates uniformly during the first 5.0 s to a speed of 1100 rad s^{-1}, after which the speed remains constant until the final 6.0 s of the cycle, during which it is brought to rest uniformly.
Calculate the angle turned by a tube during one complete operating cycle.

....

....

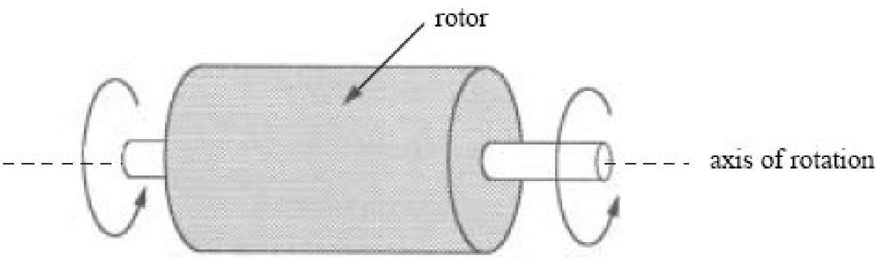
....

....

....

(3)
(Total 8 marks)

Q12. 'Low inertia' motors are used in applications requiring rapid changes of speed and direction of rotation. These motors are designed so that the rotor has a very low moment of inertia about its axis of rotation.



(a) (i) Explain why a low moment of inertia is desirable when the speed and direction of rotation must be changed quickly.

....

....

....

....

(ii) State, giving a reason in each case, **two** features of rotor design which would lead to a low moment of inertia about the axis of rotation.

....

....

....

....

(4)

(b) In one application, a rotor of moment of inertia 4.4×10^{-5} kg m^2 about its axis of rotation is required to reverse direction from an angular speed of 120 rad s^{-1} to the same speed in the opposite direction in a time of 50 ms. Assuming that the torque acting is constant throughout the change, calculate

(i) the angular acceleration of the rotor,

....

....

(ii) the torque needed to achieve this acceleration,

....

....

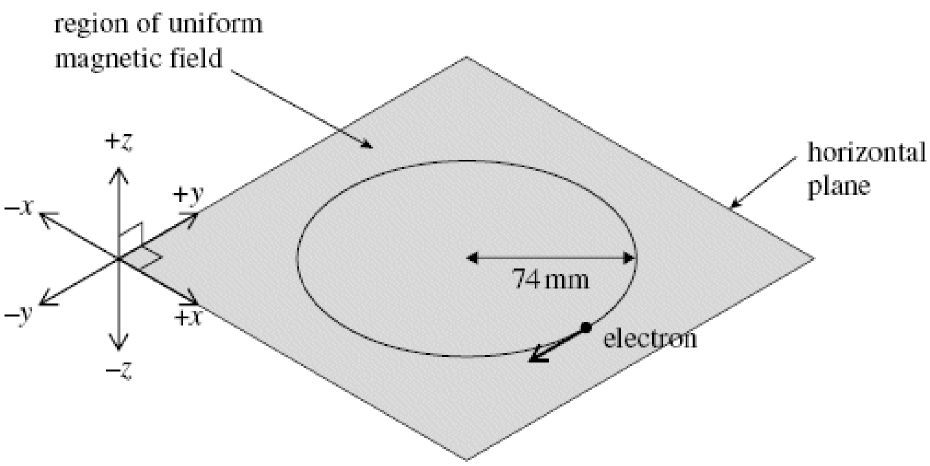
(iii) the angle turned through by the rotor in coming to rest momentarily before reversing direction.

....

....

(3)
(Total 7 marks)

Q13. When travelling in a vacuum through a uniform magnetic field of flux density 0.43 m T, an electron moves at constant speed in a horizontal circle of radius 74 mm, as shown in the figure below.



(a) When viewed from vertically above, the electron moves clockwise around the horizontal circle. In which one of the six directions shown on the figure above, +*x*, –*x*, +*y*, –*y*, +*z* or –*z*, is the magnetic field directed?

direction of magnetic field ....

(1)

(b) Explain why the electron is accelerating even though it is travelling at constant speed.

....

....

....

....

....

....

(2)

(c) (i) By considering the centripetal force acting on the electron, show that its speed is 5.6×10^6 m s^{-1}.

(2)

(ii) Calculate the angular speed of the electron, giving an appropriate unit.

answer = ...

(2)

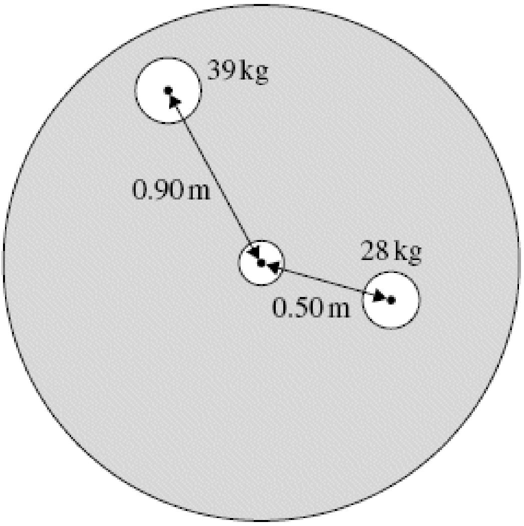
(iii) How many times does the electron travel around the circle in one minute?

answer = ...

(2)

(Total 9 mark)

Q14. (a) A playground roundabout has a moment of inertia about its vertical axis of rotation of 82 kg m^2. Two children are standing on the roundabout which is rotating freely at 35 revolutions per minute. The children can be considered to be point masses of 39 kg and 28 kg and their distances from the centre are as shown in the figure below.



(i) Calculate the total moment of inertia of the roundabout and children about the axis of rotation. Give your answer to an appropriate number of significant figures.

answer = ... kg m^2

(3)

(ii) Calculate the total rotational kinetic energy of the roundabout and children.

answer = ... J

(2)

(b) The children move closer to the centre of the roundabout so that they are both at a distance of 0.36 m from the centre. This changes the total moment of inertia to 91 kg m^2.

(i) Explain why the roundabout speeds up as the children move to the centre of the roundabout.

....

....

....

....

(2)

(ii) Calculate the new angular speed of the roundabout. You may assume that the frictional torque at the roundabout bearing is negligible.

answer = ... rad s^{-1}

(2)

(iii) Calculate the new rotational kinetic energy of the roundabout and children.

answer = ... J

(1)

(c) Explain where the increase of rotational kinetic energy of the roundabout and children has come from.

....

....

....

(1)
(Total 11 marks)

M1. (a) (i) $T = Fr = 7.0 \times 0.075$
$= 0.53$ **(1)** N m **(1)**

2

(ii) $P = T\omega$

$= 0.53 \times 120 = 64$ W **(1)**

1

(b) use of equation(s) of motion:

$\theta = ½(120 + 0) \times 6.2 = 370$ rad **(1)**

$370/2\pi = 59$ rotations **(1)**

2

[5]

M2. (a) (i) energy = power × time = $150 \times 10^3 \times 4.4 = 6.6 \times 10^5$ J **(1)**

(ii) increase in kinetic energy = energy supplied during acceleration **(1)**

$6.6 \times 10^5 = 0.5\, I\, (\omega_2^{\,2} - \omega_1^{\,2})$ **(1)**

(allow C.E. for energy value from (i))

$I = \frac{6.6 \times 10^5 \times 2}{7.4^2 - 1.6^2} = 2.5(3) \times 10^4$ kg m^2 **(1)**

4

(b) greater moment of inertia (I) **(1)**
($E_k \propto I$) more kinetic energy must be given in the <u>same time</u> **(1)**

2
QWC 2

[6]

M3. (a) (i) torque = force × diameter **(1)**
$= 0.12 \times 0.34 = 4.1 \times 10^{-2}$ Nm **(1)**

(ii) (use T = $I\alpha$ gives) $\alpha = \frac{4.1 \times 10^{-2}}{0.17}$ **(1)**

(= 0.24 rad s^{-2})

3

(b) (i) (use of $w_2 = w_1 + \alpha t$ gives) 0 = 0.92 – 0.24 t and t = 3.8(3)s **(1)**

(ii) (use of $w_2^2 = w_1^2 - 2a\theta$ gives) $0 = 0.92^2 - (2 \times 0.24 \times \theta)$ **(1)**

θ = (1.7(6) rad) = $\left(1.76 \times \dfrac{360}{2\pi}\right) 101^{(o)}$

[or $\theta = w_1 t - \dfrac{1}{2}\alpha t^2$}

3

[6]

M4. (a) (i) torque = 4 × 0.60 × 1.8 = 4.3(2) N m **(1)**

(ii) $\omega = \dfrac{2\pi}{110}$ = 5.7(1) × 10^{-2} (rad s^{-1}) **(1)**

at steady speed, frictional torque = applied torque **(1)**

(use of $P = T\omega$ gives) P = 4.32 × 5.71 × 10^{-2} = 0.25 W **(1)**

(allow C.E. for value of T from (i))

4

(b) (i) average power = 0.5 × 0.25 = 0.125 (W) **(1)**

energy = average power × time = 0.125 × 12 **(1)** (= 1.5 J)

(allow C.E. for value of P from (a)(ii))

(ii) (use of *kinetic energy* = ½$I\omega^2$ = 1.5 gives)

$I = \dfrac{2 \times 1.5}{(5.71 \times 10^{-2})^2}$ = 910 kg m^2 **(1)**

(allow C.E. for value of ω from (a)(ii))

3

[7]

M5. (a) (use of $v = \omega r$ gives $\omega = \dfrac{3.5}{0.2}$ = 18 rad s^{-1} **(1)**

1

(b) (i) $\alpha = \frac{\omega_1 - \omega_2}{t} = (-)\frac{(17.5 + 17.5)}{4.6} = (-)7.6 \text{ rad s}^{-2}$ **(1)**

(ii) (use of $T = I\alpha$ gives) $T = 40 \times 7.6 = 300$ N m **(1)**
(allow C.E. for value of α from (i))

(iii) (use of *angular impulse* = *Tt* gives)
angular impulse = $300 \times 4.6 = 1.4 \times 10^3 \text{ kg m}^2 \text{ rad s}^{-1}$ **(1)**
(allow C.E. for value of *T* from (ii))

(iv) uniform torque therefore uniform acceleration, ∴ $t = 2.3$ s **(1)**

$\theta = \frac{(\omega_1 + \omega_2)}{2} t = \frac{17.5}{2} 2.3 = 20(.13)$ (rad) **(1)**

number of turns = $\frac{20.13}{2\pi} = 3.2$ (so 3 complete turns) **(1)**

6

[7]

M6. (a) (i) 110 rpm = $\left(\frac{110 \times 2\pi}{60}\right) = 11.5 \text{ (rad s}^{-1})$ **(1)**

kinetic energy = $\frac{1}{2}I\omega^2 = 0.5 \times 150 \times 11.5^2 = 9.9(2)$ kJ **(1)**

(use of 12 for conversion above gives 10.8 kJ)

(ii) average useful $P_{out} = \frac{9.92 \times 10^3}{15} = 660$ W **(1)** (661 W)

(use of k.e. = 10.8 kJ gives 720 W)

(iii) $P_{av} = T_{acc}\omega_{av}$ gives $T = \left(\frac{661}{11.5/2}\right) = 115$ N m **(1)** **(1)** (for ω_{av})

(use of P_{out} = 720 W gives 125 N m)

5

(b) work done against friction = $T_r\theta$ and $T_r = \dfrac{9.95 \times 10^3}{35 \times 2\pi}$ **(1)**

= 45(.2) N m **(1)**

[or use of $\omega_2^2 = \omega_1^2 + 2\alpha\theta$, $T = I\alpha$

i.e. $0 = 11.5^2 + (2\alpha \times 2\pi \times 35$ gives $\alpha = 0.301$ (rad s^{-2})

T (= $I\alpha$) = 150 × 0.301 = 45(.1) N m]

2

[7]

M7. (a) 22 000 (rev min^{-1}) × 2π/60 **(1)** (= 2300 rad s^{-1})

energy stored (= ½ $I\omega^2$) = ½ × 0.60 × 2300^2 **(1)** (= 1.6 MJ)

2

(b) (i) $t = E/P = \dfrac{1.6 \times 10^6}{8.7}$ **(1)** = 1 84 × 10^5 s **(1)** (51 hours)

(ii) torque = $\dfrac{\text{power}}{\text{average speed}} = \dfrac{8.7}{(2300/2)}$ = 7.5(6) × 10^{-3} **(1)** N m **(1)**

or $T = I\,a$ = 0.60 × 2300/(1.84 × 10^5) = 7.5 × 10^{-3} **(1)** N m **(1)**

max 3

(c) in **B** more of the mass is at a greater radius than in **A** **(1)**

so I greater and so energy stored greater (for same speed) **(1)**

2

[7]

M8. (a) energy supplied = 15 × 10^3 × 3 × 60 = 2.7 MJ **(1)**

(use of $E_k = \frac{1}{2} I \omega^2$ gives) 2.7 × 10^6 = 0.5 × 9.5 × ω^2 **(1)**

(gives ω = 754 ≈ 750 rad s^{-1})

2

(b) (i) impulse (= $\Delta I\omega$) = (–)9.5 × 754
=(–)7.2 × 10^3N m s (or kg m^2 rad s^{-1}) **(1)**(7.16 × 10^3N m s)
(use of ω = 750 gives impulse = 7.1(3) × 10^3N m s)

(ii) average torque $\left(= \frac{\text{angular impulse}}{\text{time}}\right) = \frac{716 \times 10^3}{4.5}$
= 1600 N m **(1)**

[or $T_{av} = I\alpha$, where $\alpha = \frac{750}{4.5}$ = 167 (rad s^{-2})
T_{av} = 9.5 × 167 = 1600 N m **(1)**] 2

(c) area under curve = angular impulse = T_{av} × t **(1)**
area found by counting squares (or correct alternatives) **(1)**

$T_{av} = \frac{\text{angular impulse}}{t}$, where *t* is obtained from graph **(1)**
3
[7]

M9. (a) moment of inertia of the rockets

= (2 × 0.54 × $(0.80)^2$) + (2 × 0.54 × $(0.50)^2$) = 0.96 (kg m^2) **(1)**

total moment of inertia = 0.96 + 0.14 (kg m^2) **(1)** (= 1.10 kg m^2)
2

(b) (i) torque = (2 × 3.5 × 0.80) + (2 × 3.5 × 0.50) = 9.1 N m **(1)**

(ii) $\alpha \left(= \frac{T}{I}\right) = \frac{9.1}{1.1}$ = 8.3 rad s^{-2} **(1)** (8.27 rad s^{-2})
(allow C.E. for value of torque from (i))

(iii) one turn = 6.28 rad **(1)**
$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ gives 6.28 = 0.5 × 8.3 ×t^2 and *t* = 1.2(3) s **(1)**
(allow C.E. for value of α from (ii))
4

(c) frictional couple (due to air resistance) increases as angular speed increases **(1)**
when frictional couple = driving torque [or when no resultant torque], then no acceleration **(1)**

2

[8]

M10. (a) (i) energy: kinetic energy = $\frac{1}{2}I\omega^2$, → small stored energy
[or less work/energy needed to produce change]
power = rate of energy change, fast change → high power
torque: $T = I\alpha$, α large so large torques needed unless I small,
momentum , impulse: $L = I\omega$, impulse = ΔL so unless I small, large angular impulses are needed
marking: for any **one** of the above:
for correct consideration **(1)**
for mathematical justification **(1)**

4

(ii) explanations based on $I = mr^2$ **(1)**
low **mass**, small diameter **(1)**

(b) (i) $\alpha = \frac{\omega_1 - \omega_2}{t} = \frac{120 + 120}{50 \times 10^{-3}}$ = 4.8×10^3 rad s^{-2} **(1)**

(ii) $T = I\alpha = 4.4 \times 10^{-5} \times 4.8 \times 10^3 = 0.21(1)$ N m **(1)**

(allow C.E. from incorrect value of α from (i))

(iii) impulse = torque × time = $0.21 \times 50 \times 10^{-3} = 1.1 \times 10^{-2}$ N m s **(1)**
(1.05×10^{-2} N m s)
(allow C.E. for value of T from (ii))
[or $\Delta L = I(\omega_2 - \omega_1) = 4.4 \times 10^{-5} \times 240 = 1.1 \times 10^{-2}$ N m s]

(iv) $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t = \left(\frac{120 + 0}{2}\right)25 \times 10^{-3} = 1.5$ rad **(1)**

4

[8]

M11. (a) (i) $\alpha\left(= \frac{\omega_2 - \omega_1}{t}\right) = \frac{1100 - 0}{4.2}$ = 260 rad s^{-2} **(1)** (262 rad s^{-2})

(ii) T (= $I\alpha$) = $7.6 \times 10^{-4} \times 262 = 0.20$ N m **(1)**

2

(b) $I_{liquid} = 8 \times (3.0 \times 10^{-3}\ (84 \times 10^{-3})^2 = 1.7 \times 10^{-4}$ (kg m^2) **(1)**

$I_{total} = 7.6 \times 10^{-4} + 1.7 \times 10^{-4} = 9.3 \times 10^{-4}$ (kg m^2) **(1)**

$\alpha \left(= \frac{T}{I}\right) = \frac{0.20}{9.3 \times 10^{-4}}$ = 215 (rad s^2) **(1)**

$t \left(= \frac{\omega_2 - \omega_1}{\alpha}\right) = \frac{1100}{215}$ **(1)** (= 5.1 s)

(allow C.E for value of I_{liquid})

3

(c) $\theta_1 \left(= \frac{(\omega_1 + \omega_2)}{2} t\right) = \frac{1100}{2} \times 5.0 = 2750$ (rad)

$\theta_3 = \frac{1100}{2} \times 6.0 = 3300$ (rad) **(1)** **(**for both θ_1 and θ_3)

θ_2 (= $\omega_2 t$) = 1100 × (60 – 11) = 53900 (rad) **(1)**

total angle turned = $\theta_1 + \theta_2 + \theta_3 = 60 \times 10^3$ rad **(1)**

3

[8]

M12. (a) (i) energy: kinetic energy = ½$I\omega^2$, → small stored energy
[or less work/energy needed to produce change]
power = rate of energy change, fast change → high power
torque: $T = I\alpha$, α large so large torques needed unless I small,
momentum , impulse: $L = I\omega$, impulse = ΔL so unless I small, large angular impulses are needed
marking: for any **one** of the above:
for correct consideration **(1)**
for mathematical justification **(1)**

(ii) explanations based on $I = mr^2$ **(1)**
low **mass**, small diameter **(1)**

4

(b) (i) $\alpha = \frac{\omega_1 - \omega_2}{t} = \frac{120 + 120}{50 \times 10^{-3}}$ = 4.8 × 10³ rad s⁻²

(ii) $T = I\alpha = 4.4 \times 10^{-5} \times 4.8 \times 10^{3} = 0.21(1)$ N m **(1)**
(allow C.E. from incorrect value of *α* from (i))

(iii) $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t = \left(\frac{120 + 0}{2}\right)25 \times 10^{-3} = 1.5$ rad **(1)**

3

[7]

M13. (a) magnetic field direction: −*z* **(1)**

1

(b) direction changes meaning that velocity is not constant **(1)**

acceleration involves change in velocity
(or acceleration is rate of change of velocity) **(1)**

[**alternatively**

magnetic force on electron acts perpendicular to its velocity **(1)**
∴ force changes direction of movement causing acceleration **(1)**]

2

(c) (i) $BQv = \frac{mv^2}{r}$ **(1)** gives $v \left(= \frac{BQr}{m}\right)$

$= \frac{0.43 \times 10^{-3} \times 1.60 \times 10^{-19} \times 74 \times 10^{-3}}{9.11 \times 10^{-31}}$ **(1)** (= 5.59×10^{6} m s^{-1})

2

(ii) angular speed $\omega \left(= \frac{v}{r}\right) = \frac{5.59 \times 10^{6}}{74 \times 10^{-3}} = 7.5(5) \times 10^{7}$ **(1)**

unit: rad s^{-1} **(1)** (accept s^{-1})

2

(iii) frequency of electron's orbit $f \left(= \frac{\omega}{2\pi}\right) = \frac{7.55 \times 10^7}{2\pi}$ **(1)**
(= 1.20×10^7 s^{-1})

number of transits min^{-1} = $1.20 \times 10^7 \times 60 = 7.2 \times 10^8$ **(1)**

[**alternatively**

orbital period $\left(= \frac{2\pi r}{v}\right) = \frac{2\pi \times 74 \times 10^{-3}}{5.59 \times 10^6}$ [**or** $\left(= \frac{2\pi}{\omega}\right) = \frac{2\pi}{7.55 \times 10^{-7}}$]
(= 8.32×10^{-8} s)

number of transits min^{-1} = $\frac{60}{8.32 \times 10^{-8}} = 7.2 \times 10^8$ **(1)**] 2

[9]

M14. (a) (i) $I = 82 + 39 \times 0.90^2 + 28 \times 0.50^2$ **(1)**

= 120 kg m^2 **(1)** to 2 sig figs **(1)** 3

(ii) $\omega = 35 \times 2\pi/60$ **(1)** = 3.7 rad s^{-1}

$E = ½I\omega^2 = 0.5 \times 120 \times 3.7^2 = 820$ J **(1)**
(accept 800 to 821 J depending on sf carried through) 2

(b) (i) angular momentum must be conserved **(1)**

so if *I* decreases ω must increase **(1)** 2

(ii) $120 \times 3.7 = 91 \times \omega_2$ **(1)**

$\omega_2 = 4.9$ rad s^{-1} **(1)** 2

(iii) $E = 0.5 \times 91 \times 4.9^2 = 1100$ J (1090 J) **(1)**

(give CE only if correct *I* value used)
accept 1050 1100 J 1

(c) work done or energy transferred as children move towards the centre **(1)**

or work done as centripetal force moves inwards **(1)**

1

[11]