

Mark Scheme SHM Paper Questions Jan 2002—Jan 2010 (old spec)

- 1(a) forced vibrations or resonance ✓ (1)
- (b) reference to natural frequency (or frequencies) of structure ✓ **Q1 Jun 2002**
 driving force is at same frequency as natural frequency of structure ✓
 resonance ✓
 large amplitude vibrations produced or large energy transfer to structure ✓
 could cause damage to structure [or bridge to fail] ✓ max(4)
- (c) stiffen the structure (by reinforcement) ✓
 install dampers or shock absorbers ✓
 [or other acceptable measure e.g. redesign to change natural frequency
 or increase mass of bridge or restrict number of pedestrians] (2)
 (7)
- 2(a) use of $mg = ke$ gives $k = \frac{0.20 \times 9.81}{3.5 \times 10^{-2}}$ ✓ **Q2 Jan 2002**
 $= 56 \text{ N m}^{-1}$ ✓ [or kg s^{-2}] (2)
- (b)(i) 28 (N m^{-1}) ✓ (unit to be given in either (a) or (b))
 (allow C.E. from (a))
- (ii) (use of $T = 2\pi\sqrt{\frac{m}{k}}$ gives) $T = 2\pi\sqrt{\frac{0.50}{28}} = 0.84 \text{ (s)}$ ✓
 (allow C.E. for value of k from (b)(i))
 number of oscillations per minute = $\frac{60}{0.84} = 71$ ✓
 (allow C.E. from (b)(ii)) (3)
 (5)

1(a) (use of $T = 2\pi\sqrt{\frac{l}{g}}$ gives) $T = 2\pi\sqrt{\frac{0.80}{9.81}}$ ✓
 $= 1.8 \text{ s}$ ✓

Q1 Jan 2003

(2)

(b) $mgh = \frac{1}{2}mv^2$ ✓
 $v = \sqrt{(2 \times 9.81 \times 20 \times 10^{-3})}$ ✓ (= 0.63 m s⁻¹)

$$v_{\max} = 2\pi fA = \frac{2\pi A}{T}$$
 ✓

$$A = \frac{0.63 \times 1.8}{2\pi}$$
 ✓ (= 0.18 m)

[or by Pythagoras $A^2 + 780^2 = 800^2$ ✓

gives $A = \sqrt{(800^2 - 780^2)}$ ✓ (= 180 mm)

(or equivalent solution by trigonometry ✓ ✓)

$$v_{\max} = 2\pi fA \text{ or } = \frac{2\pi A}{T}$$
 ✓

$$= \frac{2\pi \times 0.18}{1.8}$$
 ✓ (= 0.63 m s⁻¹)] (4)

(c) tension given by F , where $F - mg = \frac{mv^2}{l}$ ✓

$$F = 25 \times 10^{-3} \left(9.81 + \frac{0.63^2}{0.8} \right) = 0.26 \text{ N}$$
 ✓ (2)

(8)

Question 1

Q1 Jan 2005

(a) acceleration is proportional to displacement ✓
 acceleration is in opposite direction to displacement, or
 towards a fixed point, or towards the centre of oscillation ✓ (2)

(b)(i) $f = \frac{25}{23} = 1.1 \text{ Hz (or s}^{-1}\text{)} \checkmark \quad (1.09 \text{ Hz})$

(ii) (use of $a = (2\pi f)^2 A$ gives) $a = (2\pi \times 1.09)^2 \times 76 \times 10^{-3} \checkmark$
 $= 3.6 \text{ m s}^{-2} \checkmark \quad (3.56 \text{ m s}^{-2})$
 (use of $f = 1.1 \text{ Hz}$ gives $a = 3.63 \text{ m s}^{-2}$)
 (allow C.E. for incorrect value of f from (i))

(iii) (use of $x = A \cos(2\pi ft)$ gives) $x = 76 \times 10^{-3} \cos(2\pi \times 1.09 \times 0.60) \checkmark$
 $= (-)4.3(1) \times 10^{-2} \text{ m} \checkmark \quad (43 \text{ mm})$
 (use of $f = 1.1 \text{ Hz}$ gives $x = (-)4.0(7) \times 10^{-2} \text{ m} \quad (41 \text{ mm})$)
 direction: above equilibrium position or upwards ✓ (6)

(c)(i) graph to show:
 correct shape, i.e. cos curve ✓
 correct phase i.e. $-(\cos)$ ✓

(ii) graph to show:
 two cycles per oscillation ✓
 correct shape (even if phase is wrong) ✓
 correct starting point (i.e. full amplitude) ✓

max(4)
(12)

Question 1			
(a)	(i)	$mg = ke \checkmark$ $k = \left(\frac{0.25 \times 9.81}{40 \times 10^{-3}} \right) = 61(.3) \text{ N m}^{-1} \checkmark$	<p style="text-align: center;">Q1 Jan 2006</p> <p style="text-align: center;">4</p>
	(ii)	$T \left(= 2\pi \sqrt{\frac{m}{k}} \right) = 2\pi \sqrt{\frac{0.69}{61.3}} \checkmark (= 0.667 \text{ s})$ $f \left(= \frac{1}{T} \right) = \frac{1}{0.667} \checkmark (= 1.50 \text{ Hz})$	
(b)	(i)	forced vibrations (at 0.2 Hz) ✓ amplitude less than resonance ($\approx 30 \text{ mm}$) ✓ (almost) in phase with driver ✓	Max 6
	(ii)	resonance [or oscillates at 1.5 Hz] ✓ amplitude very large ($> 30 \text{ mm}$) ✓ oscillations may appear violent ✓ phase difference is 90° ✓	
	(iii)	forced vibrations (at 10 Hz) ✓ small amplitude ✓ out of phase with driver [or phase lag of (almost) π on driver] ✓	
Total			10

Question 1		
(a)	shm is defined by acceleration \propto displacement (from mean position) ✓ explanation of – sign ✓ (e.g. acceleration is in opposite direction to displacement, or is always directed towards a fixed point, or towards equilibrium position)	2
(b) (i)	$T \left(= 2\pi \sqrt{\frac{l}{g}} \right) = 2\pi \sqrt{\frac{0.64}{9.81}} \checkmark$ gives $T = 1.60 \text{ (s)} \checkmark$ time for bob to travel from A to C = $T + 4 = 0.40 \text{ s} \checkmark$ Q1 Jan 2009	7
(ii)	max speed of bob $v_{\text{max}} = 2\pi fA \checkmark$ $= \frac{2\pi \times 44 \times 10^{-3}}{1.60} \checkmark (= 0.173 \text{ ms}^{-1})$ max E_K of bob ($= \frac{1}{2} m v_{\text{max}}^2$) = $\frac{1}{2} \times 1.5 \times 10^{-2} \times 0.173^2 \checkmark$ $= 2.2(4) \times 10^{-4} \text{ J} \checkmark$ [or max E_K of bob = E_p gained in moving from C to B ✓ (1280 - Δh) $\Delta h = 442$ [or $6402 = 442 + (640 - \Delta h)^2$] gives $\Delta h = 1.52 \text{ (mm)} \checkmark$ max E_K of bob ($= mg\Delta h$) = $1.5 \times 10^{-2} \times 9.81 \times 1.52 \times 10^{-3} \checkmark$ $= 2.2(4) \times 10^{-4} \text{ J} \checkmark$	
Total		9

Question 1		
(a)	$mg = ke \checkmark$ Q1 Jan 2010 $k = \frac{0.20 \times 9.81}{43 \times 10^{-3}} = 46 \text{ N m}^{-1} \checkmark (45.6)$	2
(b) (i)	new spring constant = $23 \text{ N m}^{-1} \checkmark (22.8)$ [unit of N m^{-1} or kg s^{-2} to appear in either (a) or (b) (i)]	5
(ii)	period $T \left(= 2\pi \sqrt{\frac{m}{k}} \right) = 2\pi \sqrt{\frac{0.50}{22.8}} = 0.930 \text{ (s)} \checkmark$ number of oscillations per minute = $\frac{60}{0.930} = 65 \checkmark (64.5)$ [In (b) (ii), answer should be consistent with use of 0.50 kg and candidate's answer to (b) (i)]	
Total		5

Question 1	Q1 Jun 2005	
(a)	reference to resonance ✓ air set into vibration at frequency of loudspeaker ✓ resonance when driving frequency = natural frequency of air column ✓ more than one mode of vibration ✓ stationary wave (in air column) ✓ (or reference to nodes and antinodes) maximum amplitude vibration (or max energy transfer) at resonance ✓ [alternative answer to (a): first two marks as above, remaining four marks for wave reflected from surface (of water) ✓ interference/superposition (between transmitted and reflected waves) ✓ maximum intensity when path difference is $n\lambda$ ✓ maxima (or minima) observed when l changes by $\lambda/2$ ✓]	Max 4
(b) (i)	$\frac{\lambda}{2} = 523 - 168 \text{ ✓ (= 355 mm)}$ $\lambda = 710 \text{ mm ✓}$ [if $\frac{\lambda}{4} = 168$, giving $\lambda = 670 \text{ mm, ✓ (1 max) (672 mm)}$]	4
(ii)	$c(=f\lambda) = 480 \times 0.71 \text{ ✓}$ $= 341 \text{ m s}^{-1} \text{ ✓}$ (allow C.E. for incorrect λ from (i)) [allow $480 \times 0.67 = 320 \text{ m s}^{-1} \text{ ✓ (1max) (322 m s}^{-1})$]	

Question 1	Q1 Jun 2006	
(a)	$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \text{ ✓}$ oscillations must be of small amplitude ✓	2
(b) (i)	$f = \frac{25}{46.5} = 0.53(8) \text{ (s}^{-1}) \text{ ✓ [or } T = \frac{46.5}{25} = 1.8(6) \text{ (s)]}$ $l \left(= \frac{g}{4\pi^2 f^2} \right) = \frac{9.81}{4\pi^2 \cdot 0.538^2} \text{ [or } l \left(= \frac{T^2 g}{4\pi^2} \right) = \frac{1.86^2 \times 9.81}{4\pi^2} \text{] ✓}$ $l = 0.85(9) \text{ m ✓}$ (allow C.E. for values of f or T)	6
(ii)	$a_{\max} \{ = (-)(2\pi f)^2 A \} = (2\pi \times 0.538)^2 \times 51 \times 10^{-3} \text{ ✓ (= } 0.583 \text{ m s}^{-2})$ (allow C.E. for value of f from (i)) $F_{\max}(= ma_{\max}) = 1.2 \times 10^{-2} \times 0.583 \text{ ✓}$ $= 7.0 \times 10^{-3} \text{ N ✓ (6.99} \times 10^{-3} \text{ N)}$ [or $F_{\max}(= mg \sin \theta_{\max})$ where $\sin \theta_{\max} = \frac{51}{859} \text{ ✓}$ $= 1.2 \times 10^{-2} \times 9.81 \times \frac{51}{859} \text{ ✓}$ $= 6.99 \times 10^{-3} \text{ N ✓}]$	
	Total	8

Question 2	Q2 Jun 2006	
(a)	vibrates or oscillates or moves in shm ✓ vibration/oscillation is vertical/perpendicular to wave propagation direction ✓ frequency ($=c/\lambda$) = 3.0 (Hz) ✓ (or same as P) amplitude = 90 (mm) ✓ (or same as P) Q has a phase lag on P ✓ (or vice versa) phase difference of $\left(\frac{0.4}{1.2} \times 2\pi\right) = \frac{2\pi}{3}$ (rad) or 120° ✓	max 5
(b)	use of $f = 3.0$ (Hz) ✓ $v_{\max} (= 2\pi f A) = 2\pi \times 3.0 \times 90 \times 10^{-3}$ ✓ $= 1.7(0) \text{ m s}^{-1}$ ✓	3
	Total	8

Question 1	Q1 Jun 2007	
(a) (i)	P at any peak or trough ✓ Q at any point where velocity is zero and slope is negative ✓	4
(ii)	R at any point where velocity is zero ✓ acceleration is gradient of v/t graph which is a maximum at R [or in SHM acceleration is greatest when velocity is zero (or equivalent statement)] ✓	
(b) (i)	$mg = ke \therefore \text{static extension } e = \frac{0.40 \times 9.81}{28} = 0.14(0) \text{ m}$ ✓	4
(ii)	total extension = $0.140 + 0.060 = 0.200 \text{ m}$ ✓ energy stored ($= \frac{1}{2} F e$) = $\frac{1}{2} \times (28 \times 0.200) \times 0.200$ ✓ $= 0.56 \text{ J}$ ✓ (allow ✓ for use of $\frac{1}{2} F e$ if incorrect value is taken for e) or E_P stored at equilibrium ($= \frac{1}{2} F e$) = $\frac{1}{2} \times (28 \times 0.14) \times 0.14$ $= 0.274 \text{ (J)}$ ✓ maximum E_K of oscillating mass ($= \frac{1}{2} m (2 \pi f A)^2$) $= 0.050 \text{ (J)}$ ✓ total E_P stored = $0.274 + 0.050 + m g A$ $= 0.324 + (0.40 \times 9.81 \times 60 \times 10^{-3})$ $= 0.324 + 0.235 = 0.56 \text{ J}$ ✓	
	Total	8

Question 1			
(a)		$T \left(= 2\pi \sqrt{\frac{m}{k}} \right) \text{ gives } \frac{1}{0.92} = 2\pi \sqrt{\frac{400}{k}} \checkmark$ <p style="text-align: right;">Q1 Jun 2008</p> <p>from which $k = 1.3(4) \times 10^4 \text{ N m}^{-1} \checkmark$</p> <p>[or by use of effective spring constant for all four springs: springs in parallel so $k' = 4k$ for a total mass of 1600 kg]</p>	2
(b)	(i)	<p>when $t = 0.20 \text{ s}$</p> $x (= A \cos 2\pi ft) = 90 \cos 2\pi (0.92 \times 0.20) \checkmark$ <p style="text-align: center;">gives $x = 36(.3) \text{ mm} \checkmark$</p> <p style="text-align: center;">downwards \checkmark</p>	max 4
	(ii)	<p>vertical speed $v (= 2\pi f \sqrt{A^2 - x^2})$</p> $= 2\pi \times 0.92 \sqrt{(90 \times 10^{-3})^2 - (36.3 \times 10^{-3})^2} \checkmark$ <p>[or $v = (-2\pi fA \sin 2\pi ft)$</p> $= (-) 2\pi \times 0.92 \times 90 \times 10^{-3} \sin (2\pi \times 0.92 \times 0.20) \checkmark]$ <p>gives $v = 0.47(6) \text{ m s}^{-1} \checkmark$</p>	
(c)		<p>same period maintained throughout graph \checkmark</p> <p>exponential decay of amplitude \checkmark</p>	2
(d)	(i)	resonance \checkmark	2
	(ii)	3300 (rev min ⁻¹) \checkmark	
Total			10