WJEC (Wales) Physics A-level

Topic 3.6: Nuclear Energy

Notes

Mass-Energy Equivalence

Arguably the most well known equation in physics is:

$$E = mc^2$$

The c^2 here is the speed of light squared. But what does this equation really tell us? Essentially it expresses the fact that **mass and energy are in some way equivalent,** in that if an object has mass it also has an associated energy equal to its mass multiplied by the speed of light squared. This equation was proposed by Einstein in 1905.

The speed of light is a huge number, so the value of it squared is incredibly large. This has the consequence that the energy associated with mass is incredibly large. To illustrate, just 1kg of matter corresponds to roughy $9 \times 10^{16}\ J$ of energy.

Most of the time this equation is used to study very small objects with masses roughly equal to that of protons and neutrons. Since these masses are so tiny, even the associated energies are very small which makes using the Joule an inconvenient unit as it is relatively large. Instead Mega-electron volts are used, the relation between these and Joules is: $1\ MeV = 1.60 \times 10^{-13}\ J$.

The kilogram is also quite a large unit, and so instead **atomic mass units** are used. **The atomic mass unit 'u' is defined to be exactly 1/2th the mass of one atom of Carbon-12**. It has the following value: $u = 1.660539 \times 10^{-27}\ kg$ (to 6 decimal places).

We can express the mass of protons, neutrons and electrons in terms of u, where they have the following values:

Proton mass m_p = 1.007276 u
Neutron mass m_n = 1.008665 u
Electron mass m_e = 0.000549 u

Mass Defect

Let's say we found the mass of a helium nucleus using the above masses of neutrons and protons: $(2 \times 1.007276\ u) + (2 \times 1.008665\ u) = 4.031882\ u$. However, experimentally we can show that the mass of a helium nucleus is actually $4.001508\ u$. This gives a difference in mass between what we would expect and the real mass of $0.030374\ u$. This is a reasonably large mass so it can't be due to the resolution of whatever equipment might have been used to measure it.

The reason there is a difference in mass is because we haven't accounted for the energy of the helium nucleus. To understand this, let's imagine taking our helium nucleus and slowly pulling it apart so that all of the 4 nucleons are completely separated. **The strong nuclear force** acts as an attractive force between the all 4 nucleons when they form the nucleus. Pulling the nucleus apart **requires work to be done against this force**, thus giving the nucleus a form of potential energy. From Einstein's equation, **if the nucleons have gained energy they must have gained some mass!** This mass which they gain will be equal to the difference in mass we previously found, which is referred to as the **mass defect.**

The mass defect is the difference in mass between a nucleus and the sum of the constituent nucleons in the nucleus.

To summarise, the mass defect exists because the nucleons in a nucleus have less energy (and thus mass) than they would if they didn't exist inside a nucleus.

Binding Energy

The amount of energy which must be used to completely separate a nucleus into its constituent parts is called the binding energy. The binding energy causes the mass defect, and so the relation between these two is:

$$Binding\ energy = E_{bind} = \Delta mc^2$$

Where Δm is the **mass defect.**

Another useful quantity is called the **binding energy per nucleon,** which as the name suggests is given by:

$$\frac{E_{bind}}{A} = \frac{\Delta mc^2}{A}$$

Here A is the **nucleon number** of the nucleus.

Example question: What is the binding energy per nucleon of Uranium-235 if the mass of a U-235 nucleus is $235.043\ u$?

Firstly we have to find the mass defect. Uranium-235 has 92 protons and 143 neutrons. Finding the total mass of these individually gives:

$$(92\times1.00727u)+(143\times1.00866u) = 236.907u$$

Subtracting the given actual mass of the uranium nucleus gives the **mass defect:**

$$\Delta m = 236.907u - 235.043u = 1.86u$$

Using Einstein's equation we can then find the total binding energy the uranium nucleus:

$$\Delta mc^2 = 1.86u \times c^2 = 1740\ MeV$$

And finally, dividing this by the nucleon number $A = 235$, gives the answer:

$$\frac{1740\ MeV}{235} = 7.40\ MeV\ per\ nucleon$$

Nuclear Fusion

Nuclear fusion is a process by which two or more nuclei are forced so closely together that they merge to form a single nucleus, releasing energy and subatomic particles.

For example a nuclear fusion reaction which is common within stars is the fusion of Deuterium and Tritium, which are **two isotopes of hydrogen**. It can be expressed in the following reaction form:

$$^3_1H\ +\ ^2_1H \rightarrow\ ^4_2He +\ ^1_0n\ +\ 17.50MeV$$

Fusing these two isotopes produces a helium nucleus, along with an emitted neutron and an amount of energy. **This energy released can be in the form of a high-energy photon, or as the kinetic energy of the products.**

The energy released is the result of the **change in binding energy** between the reactants and the products. For this example, the combined binding energy of the deuterium and tritium is smaller than the binding energy of the helium nucleus and the neutron, where this change in energy is $17.50\ MeV$.

This means the energy released in a fusion reaction can be found by calculating the mass defect Δm between the products and reactants, then finding the energy as Δmc^2.

Nuclear fusion is the process by which the sun makes energy; using its large mass to confine the reactants at a high enough pressure that they undergo fusion.

Nuclear Fission

In nuclear fission reactions a large unstable nucleus is split into multiple smaller, more stable nuclei, releasing energy and subatomic particles as a result.

Most of the time, fission is caused by a large nucleus that has been bombarded with a high energy neutron. This neutron enters the nucleus and causes it to become **extremely unstable,** causing it to split apart. Sometimes if a nuclei is unstable enough it can undergo fission without another particle colliding with it, this is called **spontaneous fission.**

For example, a Uranium-235 undergoes fission when hit by a neutron:

$$^{235}_{92}U + \,^{1}_{0}n \rightarrow \,^{141}_{56}Ba \,+\, ^{92}_{36}Kr + 3\,^{1}_{0}n \,+\, 202.5MeV$$

The Uranium nucleus splits to form Barium and Krypton nuclei, along with three further neutrons and $202.5\ MeV$ of energy. **Once again this energy is released as either a photon or as the kinetic energy of the products.**

Note that the energy released from a single fission reaction is **generally much larger** than the energy released from a fusion reaction.

Just like with fusion, **energy is released from fission reactions because the binding energy of the reactants is larger than the products.**

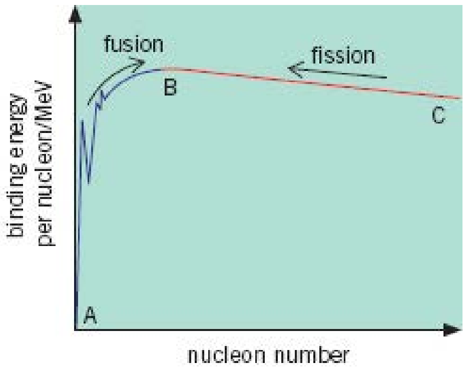
This means the energy released in a fission reaction can be found by calculating the **mass defect** Δm between the products and reactants, then finding the energy as Δmc^2.

Nuclear fission is the process by which nuclear power plants make energy. Scientists are yet to build a sustainable way of generating nuclear power from nuclear fusion, but this would solve many of the problems that make nuclear fission power plants controversial.

For example, **the products of nuclear fission are often unstable themselves**, and generally have long half-lives. This means that fission fuel which has been used is very dangerous to humans, and has to be buried deep underground in sealed containers to ensure it is kept far from humans and other animals.

In the example above with the fission of Uranium, **3 neutrons are produced**. These neutrons can go on to hit other Uranium nuclei, causing further fission reactions. This can produce a **chain reaction** which if not controlled can be extremely dangerous. Many nuclear weapons function by purposely causing a chain of fission reactions, releasing significant amounts of energy and radiation.

Binding Energy Curve



Here is a plot of the **binding energy per nucleon** vs **nucleon number** for all the isotopes which we know to exist.

Nuclei which have **higher binding energies** per nucleon require more energy per nucleon to separate, so they are **more stable.**

The nuclei at the peak of this curve labelled 'B' is Iron-52, and so is the **most stable known nuclei.**

Energy is released in nuclear reactions because the products have a larger binding energy than the reactants. This means that isotopes which are between **'A' and B' must undergo fusion reactions to release energy,** whilst on the other side between '**B' and 'C' to release energy the nucleon number has to become smaller which requires fission reactions.**