WJEC (Wales) Physics A-level

Topic 3.2: Vibrations

Notes

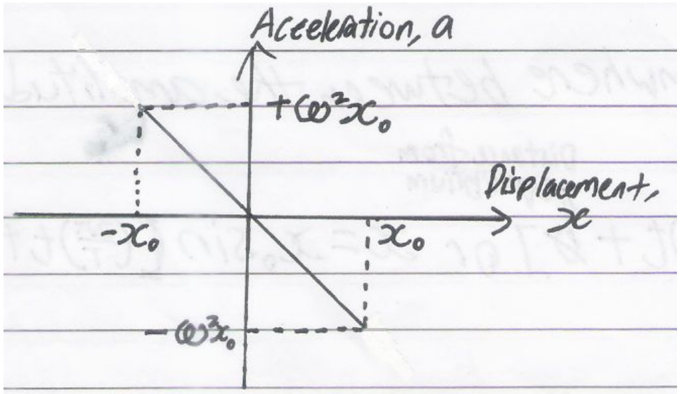
Simple Harmonic Motion

Simple harmonic motion (SHM) is a type of motion defined by a simple rule. The acceleration of a body is **directly proportional to and in the opposite direction** to its displacement from an equilibrium point. Mathematically, this is:

$$a = -\omega^2 x$$

ω^2 is the constant of proportionality and the negative sign shows the opposite direction.

Acceleration vs Displacement



https://www.miniphysics.com/simple-harmonic-motion.html

The graph shows us the definitions of SHM. We have a **straight line through the origin** meaning the acceleration is directly proportional to the displacement. For all positive values of displacement, the acceleration is negative and vice versa, hence the **acceleration is always in the opposite direction to the displacement.** Also, $a = -\omega^2 x$ by looking at the values of a for displacement values of x_0.

Solutions to the SHM Equation

The solution to the SHM equation is:

$$x = A\cos(\omega t + \varepsilon)$$

Frequency is the number of oscillations a body makes per unit time (usually the second) and is given the symbol f.

The time **period** is the time that is taken for the body to make one oscillation and is given the symbol T. The relationship between frequency and period is simple. If the body makes f oscillations within one second then the time taken to complete each one is $\frac{1}{f} = T$.

ω is the **angular velocity**. Therefore:

$$\omega = 2\pi f \text{ and } \omega = \frac{2\pi}{T} \quad \Rightarrow \quad T = \frac{2\pi}{\omega}$$

Amplitude is the maximum displacement from equilibrium which is achieved and is given the symbol A.

Phase is the part which is added in the cosine (ε) to make sure the mathematical description aligns with the physical situation. For example, if the body starts at its amplitude then $\varepsilon = 0$ but if the body starts at equilibrium then $\varepsilon = \frac{\pi}{2}$ (as $cos\left(\frac{\pi}{2}\right) = 0$).

Velocity in SHM

The velocity is given by:

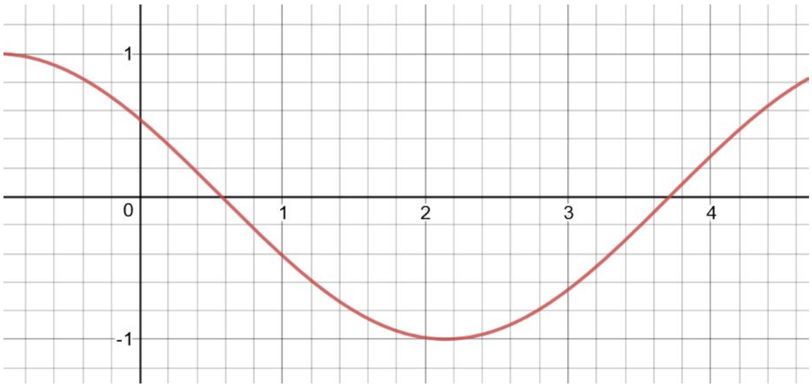
$$v = -A\omega sin(\omega t + \varepsilon)$$

(Those doing A-level maths might notice that this is the derivative of the position with respect to time).

Graphical Representations

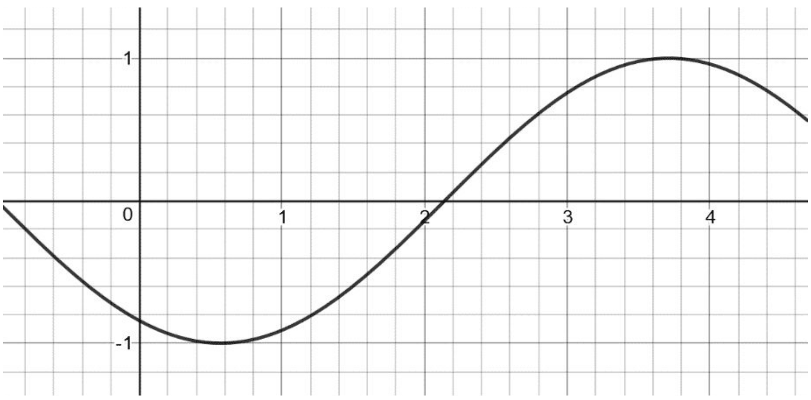
Displacement vs Time

The graph below shows the displacement on the y-axis and time on the x-axis. In this specific case, the amplitude is 1, the angular velocity 1 and phase, $\varepsilon = 1$. So when $t = 0$, $x = cos(1) = 0.54$.



Velocity vs Time

The graph below shows the velocity on the y-axis and time on the x-axis. In this specific case, the amplitude is 1, the angular velocity 1 and phase, $\varepsilon = 1$. So when $t = 0$, $v = -sin(1) = -0.84$.

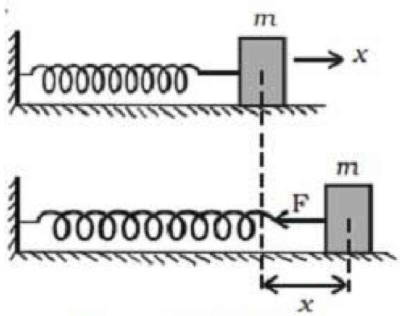


You may notice that the velocity and displacement are **always a quarter of a cycle out of phase**.

Springs

Derivation

The spring in question has a stiffness (force per unit length) k. Therefore, when the spring is extended a distance x from the equilibrium point (no extension), the **force pulling back on the block** is $F = kx$.



http://www.brainkart.com/article/Horizontal-and-Vertical-oscillations-of-spring_3139/

In the direction of the extension, this is the only force acting on the block (assuming no friction) and so by Newton's second law we can say:

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

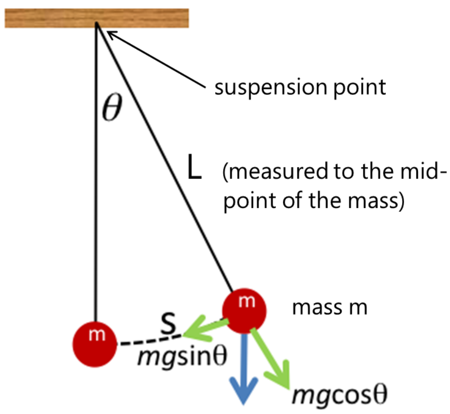
Hence:

$$\omega^2 = \frac{k}{m}$$

This means that the time period for a mass on a spring is equal to:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Pendula



http://ww25.pixshark.com/simple-harmonic-motion-pendulum-lab.htmhttps://bit.ly/2QIlcP5?subid1=20200716-2231-421f-a703-f89878696c7f

For small angles, S is approximately a straight line. Therefore we can say that:

$$ma_s = -mg\sin(\theta)$$

$$a_s = -g\theta \ (for\ small\ angles\ \sin(\theta) \approx \theta)$$

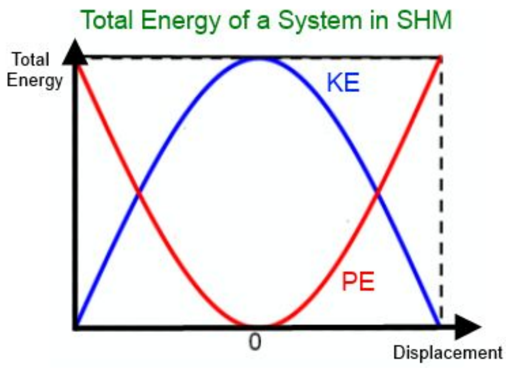
$$a_s = a_\theta L \Rightarrow a_\theta L = -g\theta$$

$$a_\theta = -\frac{g}{L}\theta \Rightarrow \omega = \sqrt{\frac{g}{L}}$$

Therefore:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Energy



https://yhoo.it/2CHQ7Un

As the body moves towards its amplitude, its potential energy increases until it has zero kinetic energy (i.e. it is stationary). However, as the body nears equilibrium (displacement = 0), its kinetic energy reaches a maximum and it has zero potential energy. The **conversion between kinetic and potential energy** is constantly happening when a body is in SHM.

A Mass on a Spring

In a mass-spring system, the kinetic energy is given by $\frac{1}{2}mv^2$. The potential energy is called elastic potential energy and it is stored in the spring when extended. When the spring is extended by x, the elastic potential energy is $\frac{1}{2}kx^2$ where k is the stiffness.

Because the total energy must be the same we can say:

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$v^2 = \frac{k}{m}\left(A^2 - x^2\right)$$

$$v = \pm\omega\sqrt{A^2 - x^2}$$

You can use this to calculate its speed at certain points during its motion but you do not have to know this equation.

Pendulum

In a pendulum system, we have **gravitational potential energy** instead of elastic potential energy.

$$L - Lcos\,(\theta_{max}) = h_{max}$$

$$mgh_{max} = \frac{1}{2}mv^2 + mgh$$

$$gh_{max} = \frac{v^2}{2} + gh$$

$$v^2 = 2g\,(h_{max} - h) = 2gL\,((1 - cos\,(\theta_{max})) - (1 - cos\,(\theta)))$$

$$v^2 = 2gL(cos(\theta) - cos(\theta_{max}))$$

Note: you do not need to know these equations but they may be helpful because you may have to work out the speed of the pendulum bob at a certain angle.

Free Oscillations and Damping

Free oscillations are oscillations which are **unaffected by external forces or actions** – they oscillate like the cases we have looked at above.

However, in real systems oscillations are not free. They are affected by other forces such as friction which cause an effect known as damping. **The damping means that the total energy of the oscillations is being reduced.**

Examples of Damped Oscillators

Damping can be useful. In car suspension, damping is used to reduce the energy of the vibrations which would be caused by a car going over bumps in the road. This allows for a safer and more comfortable ride for passengers.

Speedometers on cars are damped such that when the car accelerates the needle doesn't oscillate which would lead to confusion for the driver. Sometimes, doors have a mechanism which means when they are closed they close gently and slowly. This mechanism uses damping to slow down the door and to stop it from oscillating.

Critical Damping

Critical damping is a form of damping where the oscillations **are brought quickly to rest**. This is the type used in vehicle suspensions because you do not want any oscillation (because of safety concerns) and you want the suspension to **return to its normal level** as soon as possible.

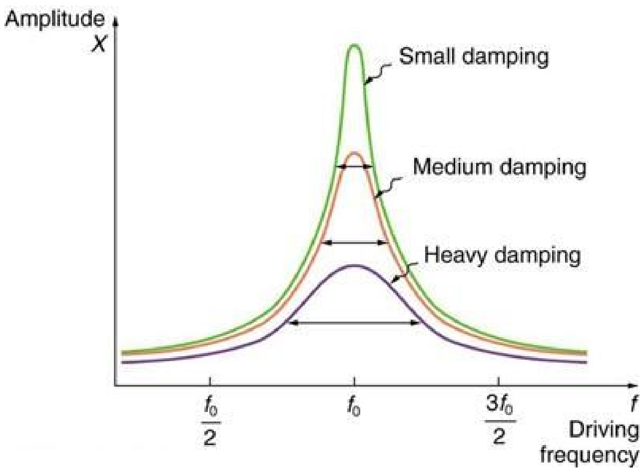
Many different instruments (such as the speedometer mentioned above) are critically damped because the **needle or display needs to move quickly to the right position** without going over it and oscillating.

Forced Oscillations

Forced oscillations occur when an **external force(s)** is applied to a system trying to cause an oscillation with a **frequency determined by an external vibrator**.

The **amplitude of the oscillations will vary** depending on how the driving frequency (the frequency at which the driving oscillations are applied) compares to the **natural frequency** of the system (the frequency it would oscillate at if it were in a free oscillation).

When the driving frequency matches the natural frequency a phenomenon called resonance occurs. At this point, the **amplitude of the oscillations increases significantly**. Increasing or decreasing the driving frequency from this value will cause a reduction in amplitude.The diagram below shows this effect.



https://opentextbc.ca/physicstestbook2/chapter/forced-oscillations-and-resonance/

When you **push somebody on a swing**, if you stand behind them and push every time they come back to you, your driving frequency is equal to the natural frequency and there is some resonance – the swing goes higher.

To tune a **radio to a particular frequency**, resonance is required. In **microwave** ovens, the natural vibrational frequency of water molecules in food is matched by the driving frequency of the microwaves which causes the amplitude to increase and their energy increases causing your food to cook faster. Resonance can be used in medical physics such as **MRI** (magnetic resonance imaging) scans

However, **resonance should be avoided in some cases**. For example, **in bridge design**. It should be avoided here because if there are strong winds or many pedestrians crossing, the frequency of the wind or of the pedestrians walking could cause resonance which can damage or break the bridge. It is very unsafe.

Damping on Resonance

If you observe the graph above, you will notice that as you increase the amount of damping, the **resonance curve widens**. Therefore small changes in the driving frequency have less of an effect for more highly damped systems compared to those with little damping.