WJEC (Eduqas) Physics A-level

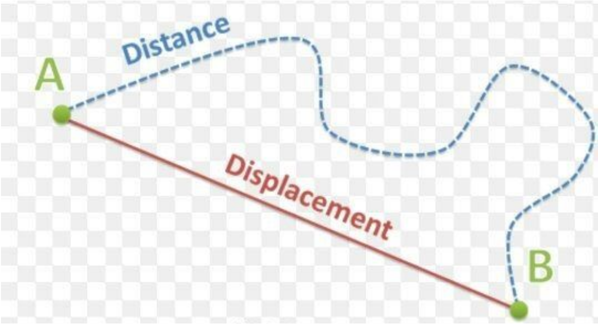
Topic 1.2: Kinematics

Notes

Key terms

Displacement

The displacement of a body is the distance of that body from a fixed **starting point** e.g. the origin of a coordinate system.



http://thescienceclassroom.org/physics/motion-in-1-d/distance-and-displacement/

Speed and velocity

The **speed** of a body is how much distance it covers in a given time i.e. the rate at which the distance a body has travelled changes. The **velocity** of a body is by how much its **displacement** changes in a given time i.e. the rate of change of displacement.

The magnitude of velocity is the speed of the body. However, the velocity is a vector, meaning that there a change in direction could also be involved.

A mean value of speed is the distance travelled divided by the time taken.

$$mean\ speed = \frac{distance\ travelled}{change\ in\ time}$$

However, the mean velocity is the change in displacement divided by the time taken.

$$mean\ velocity = \frac{change\ in\ displacement}{change\ in\ time}$$

Therefore, if an object starts from the origin, travels around and then returns to the origin, it will have some mean speed, but the mean velocity will be zero.

Instantaneous speed is the rate at which the distance a body has travelled will change if it remains at this speed. So, it is the speed of the body at a time.

The same applies for **instantaneous velocity** – it is the rate of change of displacement at a time.

Acceleration

The acceleration of a body is a **vector** quantity and is the rate of change of velocity. The mean acceleration is the **change in velocity** divided by the time taken.

$$mean\ acceleration = \frac{change\ in\ velocity}{change\ in\ time}$$

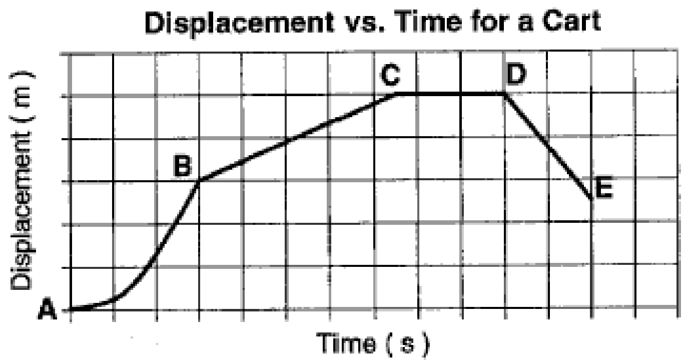
The **instantaneous acceleration** is the rate of change of velocity at a time.

For example, overall, a body could see a decrease in the magnitude of its velocity over time but at one instant during this time frame the magnitude of velocity may increase for a short time. Hence, the mean acceleration would be negative, but the instantaneous acceleration would be positive.

Graphs

The terms mentioned above can be shown in a **graphical** way as well. The following graphs show motion only in a straight line. Hence there are only two directions to apply to the vector quantities (positive or negative).

Displacement-time



http://xsienia.website.pl/2/displacement-time-graph

The **gradient** of displacement-time graphs is the **velocity** of the body over that time. If the graph has non-straight lines, then the gradient is changing so the velocity is changing and therefore the body is accelerating.
Note: we know the displacement of a body can be negative and so if you need to plot a negative displacement just place it below the time axis.

At A, the body is at its starting point. Then, it accelerates (a curved line shows changing velocity) to B. From B to C the body maintains a constant velocity (straight line). From C to D, the displacement does not change and so the velocity is zero. A constant negative velocity is shown by the line from D to E.

Velocity-time

Fig. 3.4 shows velocity–time graphs for a body moving with: (*a*) uniform acceleration, and (*b*) non-uniform acceleration.

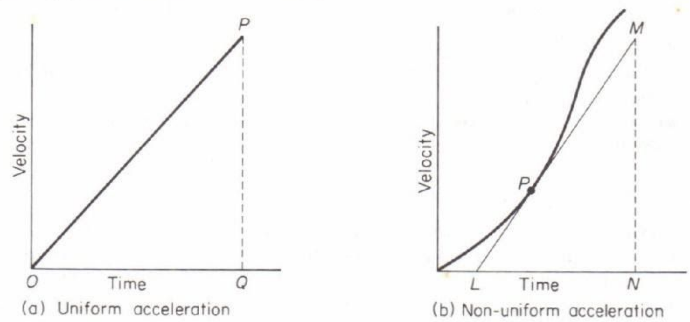


Fig. 3.4

https://physicsmax.com/acceleration-from-velocity-time-graph-5784

The **gradient** of a velocity-time graph is the **acceleration**.
Note: we know the velocity of a body can be negative and so if you need to plot a negative velocity just place it below the time axis.

The total area under the velocity-time graph (treating areas formed by positive velocity as positive, and areas formed by negative velocity as negative) is equal to the change in displacement.

In the first diagram (a) the velocity clearly increases at a **constant rate**. Therefore, we have constant (or uniform) **acceleration**.
However, in the second diagram (b) the rate of change of velocity is changing (shown by the curved lines) and so the **acceleration is changing (non-uniform)**. In this case, velocity at a given point in time can be found by drawing a tangent to the curve and finding its gradient.

Speed-time

For uniform acceleration and positive velocity, a speed-time graph will look just like the first velocity-time graph shown above. For non-uniform acceleration, with positive velocity, a speed-time graph will look just like the second velocity-time graph shown above.

The difference between speed-time and velocity-time graphs is visible when the velocity becomes negative and dips below the time axis. On a velocity-time graph the line will pass below the time axis but on a speed-time graph it will never do so. It will take the magnitude of the velocity (i.e. the velocity without the negative sign) and place it above the time-axis.

The gradient of speed-time graphs is the magnitude of the acceleration.

Constant acceleration formulae

The constant acceleration formulae (often called SUVAT, UVAST) are the equations which represent uniformly accelerated motion in a straight line.

From our definitions of velocity

$$a = \Delta v / t$$

$$\Delta v = at$$

$$v_1 = v_0 + \Delta v$$

$$v_1 = v_0 + at \quad [1]$$

Deriving the SUVAT equations

This (1) is the first equation of motion. v_0 is the initial velocity and v_1 is the final velocity. We often replace these we the letters u and v respectively.

Secondly, we have our average velocity, $\overline{v}$, is

$$\overline{v} = \frac{u+v}{2}$$

Then, from our definition of average velocity we have

$$x = x_0 + \overline{v}t$$

$$x = x_0 + \frac{u+v}{2}\, t$$

Letting x_0 our initial displacement be zero i.e. defining the starting point to be here

$$x = \frac{u+v}{2}t \quad [2]$$

This is our next equation.

And now using (1) and (2) together.

$$x = \frac{u+(u+at)}{2}\,t$$

$$x = \frac{2u+at}{2}\,t$$

$$x = ut + \tfrac{1}{2}at^2 \quad [3]$$

This is our next equation.

Finally, starting with (1)

$$v = u + at$$

$$v^2 = (u + at)^2$$

$$v^2 = u^2 + 2uat + a^2t^2$$

Then using (3) we have

$$a^2t^2 = 2a\,(x - ut)$$

So,

$$v^2 = u^2 + 2uat + 2a\,(x - ut)$$

$$v^2 = u^2 + 2ax \quad [4]$$

This is the fourth and final equation.

Extension derivation

We can derive the final equation (4) using an energy method. (It may be useful to read the notes on topic 1.4 first.)

We are given a body is accelerating with acceleration a and we will say the body's mass is m.

Therefore, we know the resultant force on the body (using Newton's second law) is

$$F = ma$$

By using the work-energy relationship

$$Fx = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

$$max = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

$$ax = \tfrac{1}{2}v^2 - \tfrac{1}{2}u^2$$

$$v^2 - u^2 = 2ax$$

$$v^2 = u^2 + 2ax$$

And then we have our formula.

To use these formulae, it is sometimes easier to write down the values **you do know** out of $x,\ u,\ v,\ a,\ and\ t$. Then you can select the formula which contains the values you know and will give you the value you **wish to find**.

Bodies falling in a gravitational field

There are two cases of bodies falling in gravitational fields which are useful to know about. One of them is when there is **no air resistance** acting on the body. When this is the case, the resultant force on the body is simply its weight. Therefore, the acceleration of the body is the gravitational field strength.

So, without air resistance, all objects would fall at the same rate – near the Earth's surface, this is about 9.8 metres per second per second, a value known as '*g*'.
Note: This acceleration will be constant as there will be no changes to the force acting on the body (as long as it remains near the Earth's surface) and so we can use the **constant acceleration formulae**.

However, when we do consider air resistance. Air resistance **increases with speed** and so as the body accelerates, the resistive forces acting on it increase. Therefore, the acceleration of the body will decrease as the body gets faster. Eventually, the air resistance forces will match the weight forces and the body will no longer accelerate. It will maintain a maximum speed – called **terminal velocity**. In this case, the acceleration is not constant and so the **constant acceleration formulae cannot be applied**.

Vertical and horizontal motion

In many situations, it may be easier to **isolate the vertical and the horizontal motion** of a body moving under gravity.

In a gravitational field, if a body is moving freely, there are no resistive forces – the only force acting is the weight. **Weight acts vertically**. Therefore, we can consider motion in only the vertical direction when looking at the weight. In the horizontal direction there are no forces acting and so the body has horizontal equilibrium.

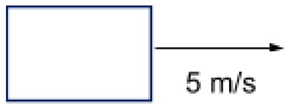
If any forces arose which acted horizontally on the body, they **would not affect the vertical motion**. This is because these forces do not change the fact that the only force acting vertically in the weight.

If a body had a uniform velocity in one direction and then there was a uniform acceleration in a direction perpendicular to the uniform velocity, we can apply a **similar principle** to the one above. As the directions are perpendicular they **can be treated separately**. In the direction of uniform velocity there is no acceleration and so its velocity component in that direction remains constant.

However, in the perpendicular direction, the body will accelerate and so the velocity component in this direction will increase.

An example:

Let us take a block which is currently moving at 5 m/s to the right.



Now we will apply an acceleration perpendicular of $10\ ms^{-2}$ and after 3 seconds the velocity in that direction will be $30\ ms^{-1}$. In this time, in the other direction, the block will have travelled 15 metres.

If you have a constant acceleration you can use the constant acceleration formula in one direction. In the above example to work out the final velocity of the block in the accelerating direction we use:

$$v = u + at$$

$$\Rightarrow v = 0 + (10 \times 3) = 30\ ms^{-1}$$