OCR B Physics A Level

Module 5.1: Creating Models

Notes

5.1.1 Creating Models

Modelling Decay

Radioactive Decay

The **activity** of a source is the number of radioactive nuclei it releases per second, or the number of decays per second. The **half life** is the time taken for the number of radioactive nuclei to halve (or for the **count rate,** or activity, to halve).

Radioactive decay is a **random** process - you cannot predict the decay of a particular nucleus, but you can predict how many nuclei will decay from a large source.

Graphing radioactive decay produces an **exponential curve**.

The **decay constant, λ,** is the fixed probability of a nucleus decaying in a given time interval, measured in s^{-1}. To find activity use the equation:

$$A = \lambda N$$

Where activity is A, number of nuclei is N, and the decay constant is λ.

This can be applied to **iterative** calculations, used to work out the decay in a set time period.

$$\Delta N = -\lambda N \Delta t \text{ or } \frac{\Delta N}{\Delta t} = -\lambda N$$

This, like any "**rates of change**" formula, is a **differential** equation, and can be rearranged to:

$$N = N_0 e^{-\lambda t}$$

Where N = number of radioactive nuclei
e = exponential constant
λ = decay constant
t = time

$$A = A_0 e^{-\lambda t}$$

Where A = activity
e = exponential constant
λ = decay constant
t = time

Capacitors

A **capacitor** is a circuit component which stores charge. It consists of two plates on which charge builds up. Electrons flow across the circuit from one plate to the other, building up opposite charges.

Capacitance, C is the charge separated/stored per volt.

$$C = \frac{Q}{V}$$

As well as charge, **energy** is stored on a capacitor.

$$E = \frac{1}{2}QV$$

On a graph of Q against V, the **capacitance** is equal to the **gradient** and the **energy** is equal to the **area under the graph**.

The **time constant,** τ , is the product of the resistance and capacitance of a circuit. In other words,

$$\tau = RC\,.$$

τ is the time (in s) taken for the charge stored on a discharging capacitor to drop to 37% of its original value, or for a charging capacitor to charge to 63%. This is the value of **1/e**

Capacitors can also be modelled with iterative methods.

$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

Charging	**Discharging**
$Q = Q_0(1 - e^{\frac{-t}{RC}})$ $V = V_0(1 - e^{\frac{-t}{RC}})$ $I = I_0 e^{\frac{-t}{RC}}$	$Q = Q_0 e^{\frac{-t}{RC}}$ $V = V_0 e^{\frac{-t}{RC}}$ $I = I_0 e^{\frac{-t}{RC}}$

Modelling Oscillators

Simple Harmonic Motion

There are 2 conditions for **simple harmonic oscillations:**

1. The force is **directly proportional** to the displacement.
2. The force is always directed towards the **equilibrium position**.

Some examples of simple harmonic oscillators are **springs** and **pendulums.**

Angular frequency, ω, is the **rate of rotation**, measured in $rads^{-1}$.

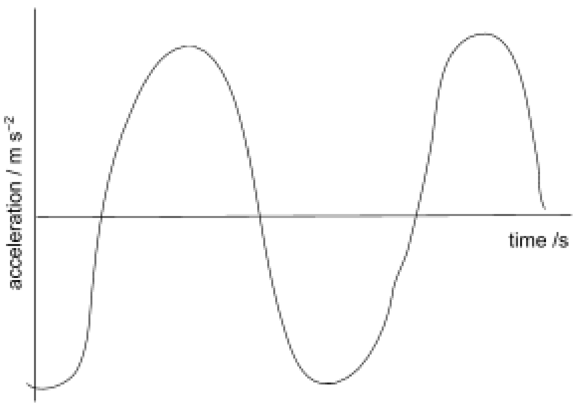
$$\omega = \frac{2\pi}{T} = 2\pi f$$

This can be applied to work out **displacement**, x (where A = **amplitude**).

$$x = A\cos(2\pi f t) = A\cos(\omega t)$$

Acceleration, a, can also be calculated:

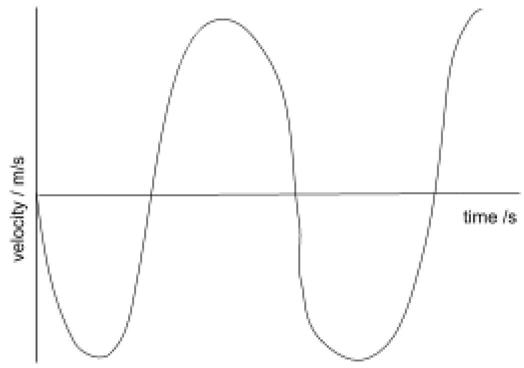
$$a = -\omega^2 x = -4\pi^2 f^2 x$$

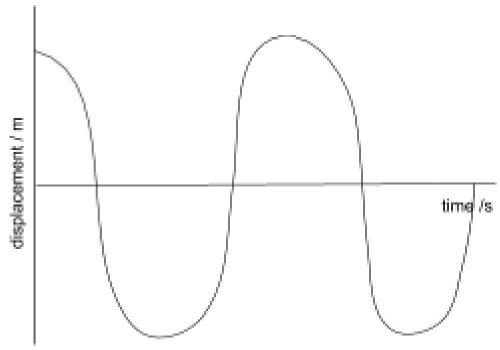


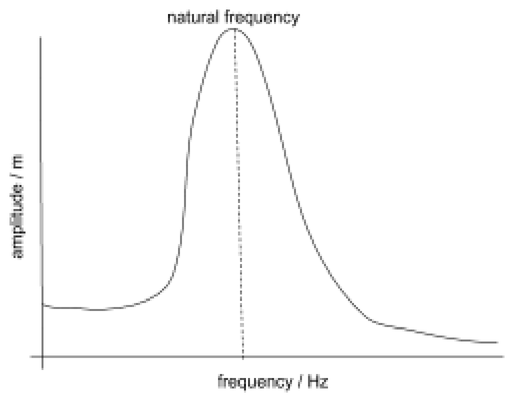
This is derived by **differentiating** the displacement formula twice.

The motion of a simple harmonic oscillator can be graphed in terms of displacement-time, velocity-time or acceleration-time graphs.

The displacement and acceleration graphs are the **inverse** of each other. The velocity graph is a **quarter of a cycle ahead** of the displacement graph.







Iterative models and calculations can be applied. The acceleration can be worked out from mass and force ($F = ma$), and then velocity and displacement can be worked out.

The **time period** of a spring oscillating in simple harmonic motion can be worked out as follows:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

This comes from:

$$a = -\frac{k}{m}x$$

and

$$a = -4\pi^2 f^2 x$$

So equating them gives:

$$\frac{k}{m} = 4\pi^2 f^2$$

Which is then rearranged to give the final equation (replacing f with 1/T). Therefore, the **time period** of a pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Resonance

There are 2 types of oscillations:

1. **Free oscillations**, which have a constant amplitude, and no resultant driving force.
2. **Forced oscillations**, which have a driving force. Without the force, the oscillations would slow to a stop.

Freely oscillating objects vibrate at their **natural frequency**.

If the **driving frequency** (the frequency of the driving force) **matches the natural frequency** of the object/system, **resonance** occurs. This will cause the **amplitude** of the oscillations to rapidly increase.

Damping is essentially the reverse of **resonance**. It occurs when energy is lost from an oscillating system, causing the amplitude to gradually decrease to zero.

Oscillators can be:

1. **Lightly** damped, where the decrease to zero is gradual, and oscillations occur around zero before reaching it.
2. **Critically** damped, where zero amplitude is reached in the shortest possible time.
3. **Heavily** damped or **overdamped**, where the decrease is more gradual than critical damping.

The energy stored by a simple harmonic oscillator is equal to $\frac{1}{2}kA^2$

5.1.2 Out into Space

Circular Motion

An object moving in a circle is **constantly accelerating towards the centre**. This acceleration, a, is given by the following equation:

$$a = \frac{v^2}{r}$$

Where v = velocity and r = radius of the circle.

The force which causes the acceleration responsible for keeping an object in circular motion is called the **centripetal force**. This force always acts at **right angles** to the motion of the object. Because F = ma,

$$F = \frac{mv^2}{r}$$

Force and acceleration can also be expressed in terms of **angular velocity**.

Angular velocity = circumference / time taken for one revolution

Therefore we find:

$$v = \frac{2\pi r}{t} = 2\pi r f$$

And remembering that because:

$$\omega = 2\pi f\ ,\ v = \omega r\ .$$

Substituting this into $a = \frac{v^2}{r}$ gives:

$$a = \omega^2 r\ .$$

$$F = ma = mr\omega^2$$

Gravity

Newton's Law of Gravitation states that all particles with mass attract all other particles by a force called gravitation.

This is summarised by the equation:

$$F = -\frac{GMm}{r^2}$$

Where F = force
G = universal constant of gravitation, $6.67 \times 10^{-11} Nm^2kg^{-2}$
M = mass of larger body
m = mass of smaller body
r = radius (or distance between the two bodies)

A **gravitational field** is the region in which a body's force of gravity is experienced by other bodies. **Gravitational field strength, g**, is measured in **N/kg**. This is the **force per unit mass** felt by the object.

$$g = -\frac{GM}{r^2}$$

g is independent of the mass of the body within the field, depending only on the mass of the body creating the field.

Gravitational potential, **V_{grav}**, is the **energy per unit mass** of the object.

$$V grav = -\frac{GM}{r}$$

The **gravitational potential energy** of an object is given by:

$$E = -\frac{GMm}{r}$$

Graphs:

All of the graphs tend towards the x axis, only reaching y=0 at infinity.

They are all below the x axis because gravity is an **attractive** force, which is signified by a negative sign.

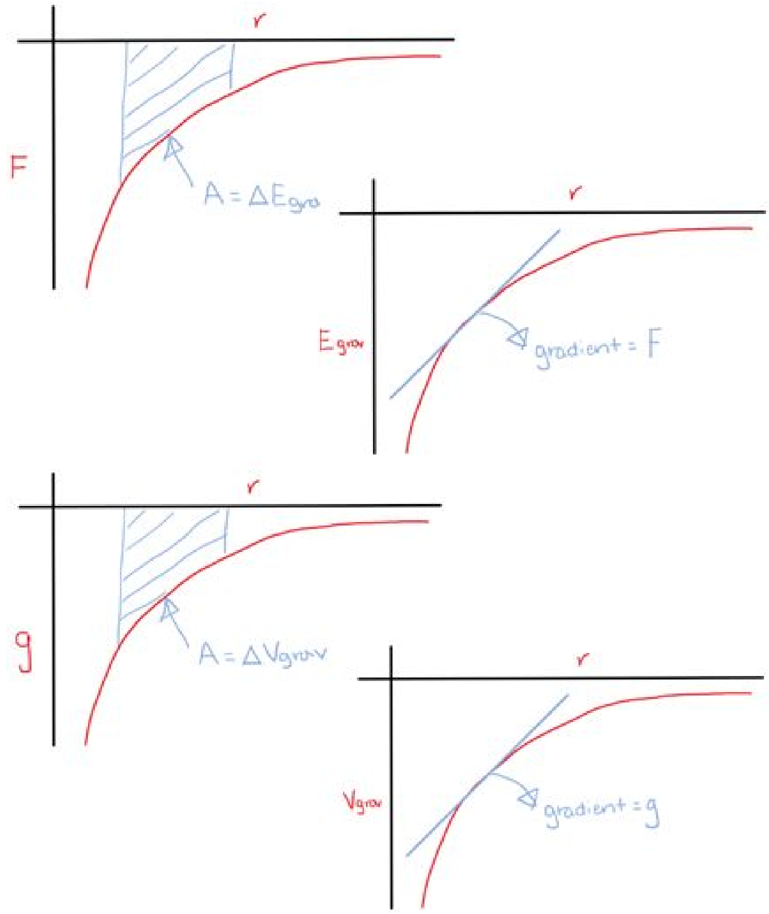
The area under a force graph is equal to the energy.

The area under the field strength graph is equal to the gravitational potential.

The gradient of the energy graph is equal to the force.

The gradient of the V_{grav} graph is equal to gravitational field strength.

This is worked out by differentiation and integration: F and g are integrated to give E and V_{grav}. V_{grav} and E are differentiated to give g and F.



Equipotentials are regions in which **field lines are parallel**, meaning there is constant gravitational potential. These are found in a **uniform field**.

Gravitational potential energy does not change when moving along field lines, only when moving up or down. Therefore, no matter what path an object takes, the energy change is the same as long as it moves the same **vertical** distance.

Escape Velocity

In order for a body to be able to escape the **gravitational potential well** around a planet, it must have enough kinetic energy to outweigh its gravitational potential energy.

Total energy = gravitational potential energy + kinetic energy

Remember that gravitational potential energy is negative.

An object must have a **positive** total energy in order to escape.

The minimum velocity required to achieve this kinetic energy is called the **escape velocity**.

$$Ekinetic\ +\ Epotential\ \geq 0$$
$$\frac{1}{2}mv^2\ +\ -\frac{GMm}{r^2}\ \geq 0$$
$$\frac{1}{2}mv^2\ \geq \frac{GMm}{r^2}$$
$$V\,esc\ = \sqrt{\frac{2GM}{r}}$$

5.1.3 Our place in the universe

Distances

Light years are used to measure large distances. One light year is the **distance travelled by light in a vacuum in one year.**

1 light year = $3 \times 10^8 \times 60 \times 60 \times 24 \times 365 \approx 10^{16} ms^{-1}$

Radar is used to measure distances in the solar system. A pulse is transmitted and detected from the same place, after reflecting off a body. The pulse travels at the same speed as light, so **s = ct** can be used to work out the distance travelled by the pulse. This is halved to work out the distance to the body (it is assumed that reflection occurs in the middle of the journey).

This can be used to work out velocity, by repeating a known time apart. The distance moved between the two pulses, divided by the time between readings will give the relative velocity.

Large distances can also be measured by **parallax**. This involves measuring the angle of an object in the sky at different times of year and using **Pythagoras' Theorem** to work out the distance.

Standard candles and **Hertzsprung-Russel diagrams** use the light from stars to estimate distances. These compare the apparent and absolute brightness of light to work out how far the light has had to travel.

The Doppler Effect and Red Shift

Red shift, or the **Doppler effect**, is the perceived increase in wavelength (eg. of light), when a source of waves is moving away from an observer. This is observed in light from **distant galaxies**, providing evidence that their relative velocity is directed away from us. The further away a galaxy is, the more red-shifted the light is. This provides evidence for the **Big Bang Theory**. You can calculate the proportional change in wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

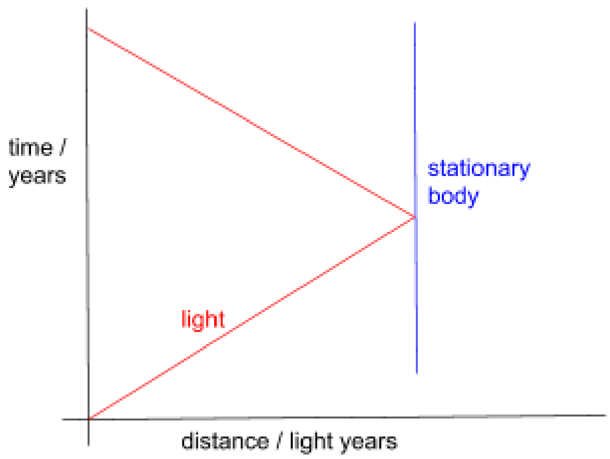
Hubble's Law states that more distant galaxies have a higher **recessional velocity**. H_0 is the Hubble constant, with units 1/time. 1/ H_0 is **Hubble Time**, an estimation of the **age of the universe.**

Cosmic Microwave Background Radiation is also evidence of the Big Bang - it is radiation found in all directions. It was believed to have originated as gamma rays produced in the Big Bang. As the Universe cooled and expanded, the waves lost energy and their frequency decreased to form microwaves.

Space-Time Diagrams

These plot time, in years, against distance, in light years.

A ray of light always makes a path at **45 degrees**, because it travels 1 light year in 1 year.



The lines representing paths are called **worldlines**.

Special Relativity

Einstein's First Postulate states that physics works in the same way for all observers, regardless of their relative motion. This means that when there is relative motion between two bodies, in an ideal system, it is impossible to tell/prove which one is moving and which one is stationary.

Einstein's Second Postulate states that the speed of light, c, is a **universal constant**. This means that every observer, moving or stationary, would perceive the speed of light to be 3×10^8 m/s.

Time dilation is when two observers with different relative velocities record a different time taken for an event to occur.

The usual model for this is a **light clock** - a ray of light bounces/reflects off two mirrors. For a stationary observer (relative to the clock), the light only has to travel up and down. If the clock is moving relative to the observer, the light will seem to travel along the hypotenuse, bouncing diagonally as the mirrors move. The speed of light is constant, so if the distance is increased, the time taken must also increase.

Time dilation is summarised by the following equation.

$$t = \gamma\tau$$

Where t = time perceived by a moving observer (relative to the clock)

τ = time perceived by a stationary observer (relative to the clock)

γ = the relativistic factor

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \text{ so } t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}$$