Revision Guide for Chapter 17

Contents
Revision Checklist

Revision Notes
Accelerators ................................................................. 4
Alpha scattering .......................................................... 7
Energy level ................................................................. 7
Model of the atom ....................................................... 8
Quark ................................................................. 9
Pair production and annihilation ................................... 9
Subatomic particles ................................................... 10
Proton ................................................................. 11
Neutron ................................................................. 11
Nucleon ................................................................. 12
Electron ................................................................. 12
Positron ................................................................. 12
Neutrino ................................................................. 13
Antimatter .............................................................. 13
Mass and energy ...................................................... 14
Relativistic calculations of energy and speed .............. 16

Summary Diagrams
The linear accelerator (from Chapter 16) ...................... 19
Principle of the synchrotron accelerator (from Chapter 16) 20
Alpha particle scattering experiment ......................... 21
Rutherford’s picture of alpha particle scattering ............. 22
Distance of closest approach ..................................... 23
Spectra and energy levels ........................................... 24
Standing waves in boxes ............................................ 25
Colours from electron guitar strings .......................... 26
Energy levels ........................................................... 27
Standing waves in atoms ............................................ 28
Size of the hydrogen atom ........................................ 29
Quarks and gluons ................................................... 30
Pair creation and annihilation .................................... 31
What the world is made of ......................................... 32
Conserved quantities in electron-positron annihilation .... 33
Relativistic momentum $p = \gamma mv$ (from Chapter 16) ... 34
Relativistic energy $E_{\text{total}} = \gamma mc^2$ (from Chapter 16) 35
Energy, momentum and mass (from Chapter 16) ............ 36
## Revision Checklist

### I can show my understanding of effects, ideas and relationships by describing and explaining cases involving:

<table>
<thead>
<tr>
<th>Description</th>
<th>Revision Notes</th>
<th>Summary Diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>the use of particle accelerators to produce beams of high energy particles for scattering (collision) experiments (knowledge of the construction details of accelerators not required)</td>
<td>accelerators</td>
<td>The linear accelerator (chapter 16), Principle of the synchrotron accelerator (chapter 16)</td>
</tr>
<tr>
<td>evidence from scattering for a small massive nucleus within the atom</td>
<td>alpha scattering</td>
<td>Alpha particle scattering experiment, Rutherford's picture of alpha particle scattering, Distance of closest approach</td>
</tr>
<tr>
<td>evidence for discrete energy levels in atoms (e.g. obtained from collisions between electrons and atoms or from line spectra)</td>
<td>energy level</td>
<td>Energy levels</td>
</tr>
<tr>
<td>a simple model of an atom based on the quantum behaviour of electrons in a confined space</td>
<td>model of the atom</td>
<td>Standing waves in boxes, Colours from electron guitar strings, Energy levels, Standing waves in atoms, Size of the hydrogen atom</td>
</tr>
<tr>
<td>a simple model of the internal structure of nucleons (protons and neutrons) as composed of up and down quarks</td>
<td>quark</td>
<td>Quarks and gluons</td>
</tr>
<tr>
<td>pair creation and annihilation using $E_{\text{rest}} = mc^2$</td>
<td>pair production and annihilation, subatomic particles</td>
<td>Pair creation and annihilation</td>
</tr>
</tbody>
</table>

### I can use the following words and phrases accurately when describing effects and observations:

<table>
<thead>
<tr>
<th>Description</th>
<th>Revision Notes</th>
<th>Summary Diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>energy level, scattering</td>
<td>energy level, alpha scattering</td>
<td>Energy levels, Rutherford's picture of alpha particle scattering</td>
</tr>
<tr>
<td>nucleus, proton, neutron, nucleon, electron, positron, neutrino, lepton, quark, gluon, hadron, antiparticle</td>
<td>subatomic particles, proton, neutron, nucleon, electron, positron, neutrino, lepton, quark, gluon, hadron, antiparticle</td>
<td></td>
</tr>
</tbody>
</table>
### 17 Probing deep into matter

**quark, antimatter**

**Summary Diagrams:** *What the world is made of*

---

#### I can sketch and interpret:

- Diagrams showing the paths of scattered particles
  
  **Summary Diagrams:** *Rutherford’s picture of alpha particle scattering, Distance of closest approach*

- Pictures of electron standing waves in simple models of an atom
  
  **Revision Notes:** *Model of the atom*
  
  **Summary Diagrams:** *Standing waves in boxes, Colours from electron guitar strings, Energy levels, Standing waves in atoms, Size of the hydrogen atom*

---

#### I can make calculations and estimates making use of:

- The kinetic and potential energy changes as a charged particle approaches and is scattered by a nucleus or other charged particle
  
  **Summary Diagrams:** *Rutherford’s picture of alpha particle scattering, Distance of closest approach*

- Changes of energy and mass in pair creation and annihilation, using $E_{\text{rest}} = mc^2$
  
  **Revision Notes:** *Mass and energy, relativistic calculations of speed and energy, Pair production and annihilation*
  
  **Summary Diagrams:** *Conserved quantities in electron–positron annihilation, Pair creation and annihilation*

- Mass, energy and speed of highly accelerated particles, using $E_{\text{rest}} = mc^2$ and relativistic factor

  $\gamma = \frac{E_{\text{total}}}{E_{\text{rest}}} = \frac{1}{\sqrt{1 - v^2/c^2}}$

  **Revision Notes:** *Mass and energy, relativistic calculations of speed and energy*
  
  **Summary Diagrams:** *Relativistic momentum $p = \gamma mv$ (chapter 16), Relativistic energy $E_{\text{total}} = \gamma mc^2$ (chapter 16), Energy, momentum and mass (chapter 16)*
Revision Notes

Accelerators
An accelerator is a linear or circular device used to accelerate charged particles. Particles are given energy by electric fields. They are steered using magnetic fields.

A Van de Graaff accelerator consists of a large isolated metal dome kept at a high potential by the accumulation of charge from a continuously moving belt. Negative ions created inside the dome in an evacuated tube are thus repelled. The work $W$ done on a particle of charge $q$ is $W = qV$, where $V$ is the potential of the dome.

The largest Van de Graaff accelerators can accelerate protons to energies of the order of 20 MeV. Although the maximum energy is low, it is stable and can be accurately controlled, allowing precision investigations of nuclear structure.

A cyclotron consists of two hollow evacuated D-shaped metal electrodes. A uniform magnetic field is directed at right angles to the electrodes. As a result, charged particles released at the centre are forced to move round in a circular path, crossing between the electrodes every half turn. A radio-frequency alternating p.d. between the electrodes accelerates the charged particles as they cross the gap between the electrodes. The charged particles spiral out from the centre, increasing in energy every half-cycle.
The cyclotron

The following equations apply if the speed of the particles remains much less than the speed of light. The magnetic force on a charged particle \( q \) is equal to \( B q v \), where \( v \) is the particle’s speed and \( B \) is the magnetic flux density. Thus

\[
Bqv = \frac{mv^2}{r}
\]

where \( m \) is the particle's mass and \( r \) is the radius of the particle orbit.

Thus the momentum of a particle is \( m v = B q r \) and the frequency of rotation is

\[
f = \frac{v}{2\pi r} = \frac{Bq}{2\pi m}
\]

This is independent of radius \( r \) and is the constant frequency of the alternating p.d.

Relativistic effects limit the maximum energy a cyclotron can give a particle. At speeds approaching the speed of light the momentum of a particle is larger than the classical value \( mv \). The frequency of orbit in the magnetic field is no longer constant, so the alternating accelerating potential difference is no longer synchronised with the transit of a particle between the two electrodes.
The *synchrotron* makes particles travel at a fixed radius, adjusting the magnetic field as they accelerate to keep them on this fixed path. The frequency of the alternating accelerating potential difference is also adjusted as the particles accelerate, to synchronise with their time of orbit.

The machine consists of an evacuated tube in the form of a ring with a large number of electromagnets around the ring. Pairs of electrodes at several positions along the ring are used to accelerate charged particles as they pass through the electrodes. The electromagnets provide a uniform magnetic field which keeps the charged particles on a circular path of fixed radius.

In a collider, pulses of particles and antiparticles circulate in opposite directions in the synchrotron, before they are brought together to collide head-on.

A *linear accelerator* consists of a long series of electrodes connected alternately to a source of alternating p.d. The electrodes are hollow coaxial cylinders in a long evacuated tube. Charged particles released at one end of the tube are accelerated to the nearest electrode. Because the alternating p.d. reverses polarity, the particles are repelled as they leave this electrode and are now attracted to the next electrode. Thus the charged particles gain energy each time they pass between electrodes.

---

**The linear accelerator**

---

[Back to Revision Checklist](#)
Alpha scattering
Rutherford, working with Geiger and Marsden, discovered that most of the alpha particles in a narrow beam directed at a thin metal foil passed through the foil.

\[ \alpha \text{ scattering} \]

They measured the number of particles deflected through different angles and found that a small number were deflected through angles in excess of 90°. Rutherford explained these results by picturing an atom as having a small massive positively charged nucleus.

The fraction of particles scattered at different angles could be explained by assuming that the alpha particles and nucleus are positively charged and so repel one another with an electrical inverse square law force (Coulomb’s Law).

Energy level
Confined quantum objects exist in discrete quantum states, each with a definite energy. The term energy level refers to the energy of one or more such quantum states (different states can have the same energy).

The existence of discrete energy levels in atoms has been confirmed in electron collision experiments using gas-filled electron tubes. The gas atoms exchange energy with the electrons in discrete amounts corresponding to differences in energy levels of the atoms.

Evidence of discrete energy levels in atoms also comes from the existence of sharp line spectra. A line emission spectrum is seen if the light from a glowing gas or vapour is passed through a narrow slit and observed after it has been refracted by a prism or diffracted by a diffraction grating. The spectral line is just the image of the slit.

The energy of a photon \( E = h f = h c / \lambda \), where \( f \) is the frequency of the light, \( c \) is the speed of light and \( \lambda \) is its wavelength. If an electron goes from energy level \( E_2 \) to a lower energy level \( E_1 \), the emitted photon has energy \( hf = E_2 - E_1 \).
The energy levels of an atom may be deduced by measuring the wavelength of each line in the spectrum then calculating the photon energies corresponding to those lines. These energies correspond to the difference in energy between two energy levels in the atom.

**Energy levels and line spectra**

(a) Energy levels

<table>
<thead>
<tr>
<th>n</th>
<th>Description</th>
<th>Photon Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>∞</td>
<td>Ionisation level</td>
<td>hν = E₃ - E₁</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Ground state level</td>
<td></td>
</tr>
</tbody>
</table>

(b) Spectrum

**Model of the atom**

A simple model of the atom explains why the electrons have discrete energy levels.

The quantum properties of the electron are responsible for limiting its energy in the atom to certain discrete energy levels. Any quantum particle confined to a limited region of space can exist only in one of a number of distinct quantum states, each with a specific energy.

One way of thinking about this is to associate wave behaviour with the quantum particles. A particle is assigned a de Broglie wavelength, \( \lambda = h / m v \), where \( m \) is its mass, \( v \) is its velocity and \( h \) is the Planck constant.

An electron trapped in an atom can be thought of as a standing wave in a box such that the wave 'fits' into the box exactly, like standing waves fit on a vibrating string of fixed length.

Consider a model atom in which an electron is trapped in a rectangular well of width \( L \). Standing waves fit into the well if a whole number of half wavelengths fit across the well. Hence \( n \lambda = 2L \) where \( n \) is a whole number.

De Broglie's hypothesis therefore gives the electron's momentum \( m \nu = h / \lambda = nh / 2L \). Therefore, the kinetic energy of an electron in the well is:

\[
\frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{n^2h^2}{2m(2L)^2}.
\]

Thus in this model, the energy of the electron takes discrete values, varying as \( n^2 \).
This simple model explains why electrons are at well-defined energy levels in the atom, but it gets the variation of energy with number \( n \) quite wrong. Optical spectra measurements indicate that the energy levels in a hydrogen atom follow a \( 1/n^2 \) rule rather than an \( n^2 \) rule.

A much better model of the atom is obtained by considering the quantum behaviour of electrons in the correct shape of 'box', which is the \( 1/r \) potential of the charged nucleus.

The mathematics of this model, first developed by Schrödinger in 1926, generates energy levels in very good agreement with the energy levels of the hydrogen atom.

**Quark**

Quarks are the building blocks of protons and neutrons, and other fundamental particles.

The nucleons in everyday matter are built from two kinds of quark, each with an associated antiquark:

- The **up** quark \((+\frac{2}{3}e)\) and the **down** quark \((-\frac{1}{3}e)\).

A proton, charge \(+1e\), is made of two up quarks and one down quark \(uud\). A neutron, charge \(0\), is made of one up quark and two down quarks \(udd\). A **meson** consists of a quark and an antiquark. For example, a \(\pi\) meson consists of an up or a down quark and a down or up antiquark.

The first direct evidence for quarks was obtained when it was discovered that very high-energy electrons in a beam were scattered from a stationary target as if there were point-like scattering centres in each proton or neutron.

Quarks do not exist in isolation.

**Beta decay**

\(\beta^-\) decay occurs in neutron-rich nuclei as a result of a down quark changing to an up quark \((udd \rightarrow uud)\) and emitting a \(W^-\), which decays into an electron (i.e. a \(\beta^-\) particle) and an antineutrino.

\(\beta^+\) decay occurs in proton-rich nuclei as a result of an up quark changing to a down quark \((uud \rightarrow udd)\) and emitting a \(W^+\), which decays into a positron (i.e. a \(\beta^+\) particle) and a neutrino.

**Pair production and annihilation**

The positron is the antiparticle of the electron. It differs from the electron in carrying an electric charge of \(+e\) instead of \(-e\). The masses of the two are identical. One point of view in quantum mechanics regards positrons as simply electrons moving backwards in time.

A gamma-ray photon of energy in excess of around 1 MeV is capable of creating an electron and a positron. Energy and momentum must always be conserved in a pair production event. The photon energy must exceed the combined rest energy \(E_{\text{rest}} = mc^2\) of the electron and of the positron, which is about 0.5 MeV for each (actual value 0.505 MeV). To conserve momentum, the creation event must take place close to a nucleus which recoils, carrying away momentum.
Pair production

\[ \gamma \text{ photon creates an electron and a positron.} \]

\[ \text{tracks curve oppositely in a magnetic field} \]

A positron and an electron annihilate each other when they collide, releasing two gamma photons to conserve momentum and energy. The energy of each gamma photon is half the total energy of the electron and positron. For example, if a positron of energy 1 MeV was annihilated by an electron at rest, the total energy would be approximately 2 MeV including the rest energy of each particle. Hence the energy of each gamma photon would be 1 MeV.

Subatomic particles

Subatomic particles divide into two main groups:

**Leptons**: particles not affected by the strong interaction, including electrons, positrons, neutrinos and antineutrinos.

**Hadrons**: composite particles, made of quarks, held together by the exchange of gluons (strong interaction).

Whereas protons and neutrons contain three quarks, intermediate particles such as pions are made of one quark and one antiquark.

A further fundamental distinction is between bosons and fermions.

A **boson** is the general name given to particles (such as photons and gluons) which carry the interaction between other particles. Bosons have integer spin, and do not obey the exclusion principle. (Some particles made of fermions – e.g. certain nuclei – also have integer spin.)

A **fermion** is the general name given to particles (such as electrons and protons) which function as particles of matter. Fermions have a half integer spin, and obey the exclusion principle (no two particles can be in the same quantum state).

The bosons such as the photon which 'carry' the forces or interactions between matter particles are called **exchange particles**. In quantum physics, all interactions are understood in terms of the exchange of such particles.

It used to be said that there are four different fundamental kinds of interaction between particles: gravity, electromagnetism, the weak interaction and the strong interaction. Electromagnetism is due to the exchange of massless virtual photons between electrically charged bodies. The weak nuclear interaction is responsible for beta decay. However, electromagnetism and the weak interaction have now been brought together into one unified theory. The strong nuclear force is the residual effect of the exchange of gluons between the quarks in a nucleon. There are hopes, not yet fulfilled in 2008, of bringing all the interactions together into one unified theory.
Proton
Protons are positively charged particles in the nucleus of every atom.

The proton is a positively charged particle of mass 1.007 28 u. Its electric charge is equal and opposite to the charge of the electron. Attempts to detect the decay of protons have so far failed, and it must be regarded as either stable or with a lifetime many times longer than the life of the Universe.

The proton and the neutron are the building blocks of the nucleus, jointly called nucleons. Neutrons and protons in the nucleus are bound together by the strong nuclear force, which is an attractive force with a range of no more than 2 to 3 fm. The strong nuclear force is strong enough to offset the electrical repulsion of the charged protons in a nucleus. Large nuclei have more neutrons than protons because additional neutrons offset the increased mutual repulsion of the large number of protons.

The atomic number of an element is equal to the number of protons in the nucleus of an atom. The symbol \( Z \) is used for proton number. An isotope \( ^{A}_{Z}X \) therefore has a nucleus which consists of \( Z \) protons and \( N = A - Z \) neutrons since \( A \) is the number of neutrons and protons in the nucleus.

Neutrons and protons and other particles which interact through the strong nuclear force are collectively referred to as hadrons. Collider experiments have confirmed the quark model of hadronic matter. The proton consists of two up quarks, each of charge \( +\frac{2}{3}e \), and a down quark which carries a charge of \( -\frac{1}{3}e \). The neutron consists of one up quark and two down quarks.

High-energy proton beams are used in collider experiments to investigate particles and antiparticles created in collisions between protons and protons or between protons and antiprotons. The antiproton is the antimatter counterpart of the proton with exactly the same properties except its electric charge which is equal and opposite to the charge of the proton.

In nuclear fusion in the Sun, protons fuse in a series of reactions to form helium. When two protons fuse, one of the protons becomes a neutron by emitting a \( \beta^{+} \) particle, releasing binding energy and leaving a \( ^{2}_{1}H \) nucleus. Further fusion takes place to form \( ^{3}_{2}He \) nuclei and then \( ^{4}_{2}He \) nuclei. In this way, hydrogen in the Sun is gradually converted to helium and energy is released. The fusion process is very slow, accounting for the long lifetime of stars such as the Sun.

Neutron
Neutrons are, with protons, the particles that make up atomic nuclei.

Neutrons are electrically uncharged particles which exist together with protons in every nucleus except the nucleus of the hydrogen atom \( ^{1}_{1}H \) which is a single proton. Neutrons and protons are the building blocks of nuclei and are collectively referred to as nucleons.

The neutron is an uncharged particle of mass 1.008 67 u. Free neutrons have a half-life of 12.8 minutes. However, in a stable nucleus, neutrons are stable.

Neutrons and protons in the nucleus are held together in the nucleus by the strong nuclear force which is an attractive force with a range of no more than 2 to 3 fm. The strong nuclear force is strong enough to offset the electrical repulsion between the positively charged protons.
**Nucleon**
Because neutrons and protons are similar in many respects they are collectively termed nucleons.

A nucleon is a neutron or a proton.

The nucleon number, also called the mass number \( A \), of an isotope, is the number of protons and neutrons in each nucleus of the isotope.

An isotope is characterised by the number \( Z \) of protons and the number \( N \) of neutrons in each nucleus. The nucleon number of an isotope is \( A = N + Z \). The symbol for an isotope is \( \frac{A}{Z}X \), where \( X \) is the chemical symbol of the element.

**Electron**
The electron is a fundamental particle and a constituent of every atom.

The electron carries a fixed negative charge. It is one of six fundamental particles known as leptons.

The charge of the electron, \( e \), is \( -1.60 \times 10^{-19} \) C.

The specific charge of the electron, \( e / m \), is its charge divided by its mass. The value of \( e / m \) is \( 1.76 \times 10^{11} \) C kg\(^{-1} \).

The energy gained by an electron accelerated through a potential difference \( V \) is \( eV \). If its speed \( v \) is much less than the speed of light, then \( eV = \frac{1}{2}mv^2 \).

Electrons show quantum behaviour. They have an associated de Broglie wavelength \( \lambda \), given by \( \lambda = \frac{h}{p} \), where \( h \) is the Planck constant and \( p \) the momentum. At speeds much less than the speed of light, \( p = mv \). The higher the momentum of the electrons in a beam, the shorter the associated de Broglie wavelength.

**Relationships**
The electron gun equation \( \frac{1}{2}m \nu^2 = eV \)

**Positron**
The positron is a positively charged electron.

The positron is the antiparticle of the electron. It differs from the electron in carrying a charge of \( +e \) instead of \( -e \). The masses of the two are identical. One point of view in quantum mechanics regards positrons as simply electrons moving backwards in time.

A gamma-ray photon of energy in excess of around 1 MeV is capable of creating an electron and a positron. Energy and momentum must always be conserved in a pair production event. The photon energy must exceed the combined rest energy of the electron and of the positron which is about 0.5 MeV for each.
**Pair production**

A positron and an electron annihilate each other when they collide, releasing two gamma photons to conserve momentum and energy.

Positrons are emitted by proton-rich nuclei as a result of a proton changing to a neutron in the nucleus, together with the emission of a neutrino. Positrons may be accelerated to high energies using an accelerator. Synchrotrons are used to store high-energy positrons by making them circulate round the synchrotron. In this way, high-energy positrons and electrons travelling in opposite directions can be brought into collision, annihilating each other to produce photons or other particles.

**Neutrino**

A neutrino is a neutral very nearly mass-less particle involved in beta decay.

A neutrino is a lepton, that is a particle that interacts through the weak nuclear force. It has a very low probability of interaction with any other particle. It is electrically uncharged. Its mass was at first thought to be zero; if so it travels at the speed of light. However a very small non-zero mass now seems probable.

The neutrino and its antimatter counterpart, the antineutrino, are created when a nucleus emits an electron and an antineutrino or a positron and a neutrino. Neutrinos and antineutrinos are produced in large quantities in a nuclear reactor.

Neutrinos and antineutrinos are produced in vast quantities in the core of a star as a result of the fusion reactions that take place. They leave the star at high speed, scarcely affected by the outer layers of the star, and they spread out and travel across the Universe. Finally, neutrinos and antineutrinos have been left throughout the Universe by processes in the first few moments of the Big Bang. Estimates of their number put the density of neutrinos and antineutrinos throughout the Universe at an average of over 100 000 per litre. They pass continually through your body essentially without interacting, being uncharged and because of the weakness of the weak interaction.

**Antimatter**

For each particle of matter there is an equivalent antiparticle. A few particles (e.g. photons) are their own antiparticles.

Antimatter consists of antiparticles. An antiparticle and particle pair can be produced from a photon of high-energy radiation which ceases to exist as a result. A particle and antiparticle pair have:

1. equal but opposite spin to its particle counterpart;
2. equal but opposite electric charge (or both are uncharged);
3. equal mass (rest energy).
The positron is the antiparticle of the electron.

Two key processes involving antiparticles and particles are:

1. **Pair production**, in which a high-energy photon produces a particle and its antiparticle. This can only occur if the photon energy $h\nu$ is greater than or equal to $2mc^2$, where $m$ is the mass of the particle, with rest energy $mc^2$ for each particle of the pair. More generally, particles are always created in particle–antiparticle pairs. The masses of particles and their antiparticles are identical. All other properties, such as electric charge, spin, lepton or baryon number, are equal but opposite in sign. These properties are therefore conserved in pair production.

2. **Annihilation**, in which a particle and a corresponding antiparticle collide and annihilate each other, producing two photons of total momentum and total energy equal to the initial momentum and energy of the particle and antiparticle, including their combined rest energy $2mc^2$. Because properties such as electric charge, spin, and lepton or baryon number are equal but opposite for particles and their antiparticles, these properties are conserved in particle annihilation.

In the theory of special relativity, the rest energy, that is the mass, of particles is just a part of the total energy with $E_{\text{rest}} = mc^2$. Thus energy can materialise as particle-antiparticle pairs, having rest energy (mass) greater than zero. Similarly, a particle-antiparticle pair can dematerialise, with their rest energy carried away by for example a pair of photons of zero mass.

**Mass and energy**

Mass and energy are linked together in the theory of relativity.

The theory of relativity changes the meaning of mass, making the mass a part of the total energy of a system of objects. For example, the energy of a photon can be used to create an electron-positron pair with mass 0.51 MeV / $c^2$ each.

**Mass and momentum**

In classical Newtonian mechanics, the ratio of two masses is the inverse of the ratio of the velocity changes each undergoes in any collision between the two. Mass is in this case related to the difficulty of changing the motion of objects. Another way of saying the same thing is that the momentum of an object is $p = mv$.

In the mechanics of the special theory of relativity, the fundamental relation between momentum $p$, speed $v$ and mass $m$ is different. It is:

$$p = \gamma mv$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

At low speeds, with $v << c$, where $\gamma$ is approximately equal to 1, this reduces to the Newtonian value $p = mv$.

**Energy**

The relationships between energy, mass and speed also change. The quantity

$$E_{\text{total}} = \gamma mc^2$$

gives the total energy of the moving object. This now includes energy the particle has at rest (i.e. traveling with you), since when $v = 0$, $\gamma = 1$ and:
This is the meaning of the famous equation $E = mc^2$. The mass of an object (scaled by the factor $c^2$) can be regarded as the rest energy of the object. If mass is measured in energy units, the factor $c^2$ is not needed. For example, the mass of an electron is close to 0.51 MeV.

**Kinetic energy**
The total energy is the sum of rest energy and kinetic energy, so that:

\[ E_{\text{kinetic}} = E_{\text{total}} - E_{\text{rest}} \]

This means that the kinetic energy is given by:

\[ E_{\text{kinetic}} = (\gamma - 1)mc^2 \]

At low speeds, with $v \ll c$, it turns out that $\gamma - 1$ is given to a good approximation by:

\[ (\gamma - 1) = \frac{1}{2} \left( \frac{v^2}{c^2} \right) \]

so that the kinetic energy has the well-known Newtonian value:

\[ E_{\text{kinetic}} = \frac{1}{2} mv^2 \]

**High energy approximations**
Particle accelerators such as the Large Hadron Collider are capable of accelerating particles to a total energy many thousands of times larger than their rest energy. In this case, the high energy approximations to the relativistic equations become very simple.

At any energy, since $E_{\text{total}} = \gamma mc^2$ and $E_{\text{rest}} = mc^2$, the ratio of total energy to rest energy is just the relativistic factor $\gamma$:

\[ \gamma = \frac{E_{\text{total}}}{E_{\text{rest}}} \]

This gives a very simple way to find $\gamma$, and so the effect of time dilation, for particles in such an accelerator.

Since the rest energy is only a very small part of the total energy,

\[ E_{\text{kinetic}} \approx E_{\text{total}} \]

the relationship between energy and momentum also becomes very simple. Since $v \approx c$, the momentum can be written:

\[ p = \gamma mc \]

and since the total energy is given by

\[ E_{\text{total}} = \gamma mc^2 \]

their ratio is simply:

\[ \frac{E_{\text{total}}}{p} \approx c \], giving $E_{\text{total}} \approx pc$ \]

This relationship is exactly true for photons or other particles of zero rest mass, which always travel at speed $c$.

**Differences with Newtonian theory**
The relativistic equations cover a wider range of phenomena than the classical relationships do.
Change of mass equivalent to the change in rest energy is significant in nuclear reactions where extremely strong forces confine protons and neutrons to the nucleus. Nuclear rest energy changes are typically of the order of MeV per nucleon, about a million times larger than chemical energy changes. The change of mass for an energy change of 1 MeV is therefore comparable with the mass of an electron.

Changes of mass associated with change in rest energy in chemical reactions or in gravitational changes near the Earth are small and usually undetectable compared with the masses of the particles involved. For example, a 1 kg mass would need to gain 64 MJ of potential energy to leave the Earth completely. The corresponding change in mass is insignificant \((7 \times 10^{-10} \text{ kg} = 64 \text{ MJ} / c^2)\). A typical chemical reaction involves energy change of the order of an electron volt \((= 1.6 \times 10^{-19} \text{ J})\). The mass change is about \(10^{-36} \text{ kg} \approx 10^{-19} \text{ J} / c^2\), much smaller than the mass of an electron.

**Approximate and exact equations**

The table below shows the relativistic equations relating energy, momentum, mass and speed. These are valid at all speeds \(v\). It also shows the approximations which are valid at low speeds \(v << c\), at very high speeds \(v \approx c\), and in the special case where \(m = 0\) and \(v = c\).

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Relativistic factor</th>
<th>Total energy</th>
<th>Rest energy</th>
<th>Kinetic energy</th>
<th>Momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m &gt; 0)</td>
<td>(\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}})</td>
<td>(E_{\text{total}} = \gamma mc^2)</td>
<td>(E_{\text{rest}} = mc^2)</td>
<td>(E_{\text{kinetic}} = (\gamma - 1)mc^2)</td>
<td>(p = \gamma mv)</td>
</tr>
<tr>
<td>(v \text{ any value} &lt; c)</td>
<td>(\gamma = \frac{E_{\text{total}}}{E_{\text{rest}}})</td>
<td>(E_{\text{total}} \approx mc^2)</td>
<td>(E_{\text{rest}} = mc^2)</td>
<td>(E_{\text{kinetic}} \approx \frac{1}{2}mv^2)</td>
<td>(p \approx mv)</td>
</tr>
<tr>
<td>any massive particle</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m &gt; 0)</td>
<td>(\gamma = \frac{E_{\text{total}}}{E_{\text{rest}}})</td>
<td>(E_{\text{total}} = \gamma mc^2)</td>
<td>(E_{\text{rest}} = mc^2)</td>
<td>(E_{\text{kinetic}} = E_{\text{total}})</td>
<td>(p \approx \gamma mc)</td>
</tr>
<tr>
<td>(v \ll c)</td>
<td></td>
<td></td>
<td></td>
<td>(p \approx \frac{E_{\text{total}}}{c})</td>
<td></td>
</tr>
<tr>
<td>ultra-relativistic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(m = 0)</td>
<td>(\gamma \text{ is undefined})</td>
<td>(E = hf)</td>
<td>(E_{\text{rest}} = 0)</td>
<td>(E = E_{\text{kinetic}} = E_{\text{total}})</td>
<td>(p = \frac{E}{c})</td>
</tr>
<tr>
<td>(v = c)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>photons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Relativistic calculations of energy and speed**

**Calculating the speed of an accelerated particle, given its kinetic energy**

If the speed is much less than that of light, Newton’s laws are good approximations.

**Newtonian calculation:**

Since the kinetic energy \(E_K = (1/2)mv^2\), the speed \(v\) is given by:

\[
v^2 = \frac{2E_K}{m}
\]
In an accelerator in which a particle of charge \( q \) is accelerated through a potential difference \( V \), the kinetic energy is given by:

\[
E_K = qV,
\]

Thus:

\[
\nu^2 = \frac{2qV}{m}
\]

The Newtonian calculation seems to give an ‘absolute’ speed, not a ratio \( \nu/c \).

**Relativistic calculation:**

A relativistic calculation mustn’t give an ‘absolute speed’. It can only give the speed of the particle as a fraction of the speed of light. The total energy \( E_{\text{total}} \) of the particle has to be compared with its rest energy \( mc^2 \). For an electron, the rest energy corresponding to a mass of \( 9.1 \times 10^{-31} \) kg is 0.51 MeV. A convenient relativistic expression is:

\[
\frac{E_{\text{total}}}{E_{\text{rest}}} = \gamma
\]

where

\[
\gamma = \frac{1}{\sqrt{1 - \nu^2/c^2}}
\]

Since the total energy \( E_{\text{total}} = mc^2 + qV \), then:

\[
\gamma = 1 + \frac{qV}{mc^2}
\]

This expression gives a good way to see how far the relativistic calculation will depart from the Newtonian approximation. The Newtonian calculation is satisfactory only if \( \gamma \) is close to 1.

The ratio \( \nu/c \) can be calculated from \( \gamma \):

\[
\nu/c = \sqrt{1 - 1/\gamma^2}.
\]

**A rule of thumb**

As long as the accelerator energy \( qV \) is much less than the rest energy, the factor \( \gamma \) is close to 1, and \( \nu \) is much less than \( c \). The Newtonian equations are then a good approximation. To keep \( \gamma \) close to 1, say up to 1.1, the accelerator energy \( qV \) must be less than 1/10 the rest energy. So for electrons, rest energy 0.51 MeV, accelerating potential differences up to about 50 kV give speeds fairly close to the Newtonian approximation. This is a handy rule of thumb.

<table>
<thead>
<tr>
<th>Accelerating voltage / kV</th>
<th>Speed of electrons (Newton)</th>
<th>( \gamma = 1 + \frac{qV}{mc^2} )</th>
<th>Speed of electrons (Einstein)</th>
<th>Error in speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.198c</td>
<td>1.019</td>
<td>0.195c</td>
<td>1.5%</td>
</tr>
<tr>
<td>50</td>
<td>0.442c</td>
<td>1.097</td>
<td>0.412c</td>
<td>7%</td>
</tr>
<tr>
<td>100</td>
<td>0.625c</td>
<td>1.195</td>
<td>0.548c</td>
<td>14%</td>
</tr>
<tr>
<td>500</td>
<td>1.4c</td>
<td>1.97 ( \approx 2 )</td>
<td>0.86c</td>
<td>62%</td>
</tr>
<tr>
<td>5000</td>
<td>4.4c</td>
<td>10.7</td>
<td>0.99c</td>
<td>&gt;300%</td>
</tr>
</tbody>
</table>
An example: a cosmic ray crosses the Galaxy in 30 seconds

A proton of energy $10^{20} \text{ eV}$ is the highest energy cosmic ray particle yet observed (2008). How long does such a proton take to cross the entire Milky Way galaxy, diameter of the order $10^5$ light years?

The rest energy (mass) of a proton is about $1 \text{ GeV}/c^2$, or $10^9 \text{ eV}/c^2$. Then:

$$E_{\text{total}} = \gamma mc^2$$

with

$$\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$$

Inserting values: $E_{\text{total}} = 10^{20} \text{ eV}/c^2$ and $m = 10^9 \text{ eV}/c^2$ gives:

$$\gamma = \frac{10^{20} \text{ eV}}{10^9 \text{ eV}} = 10^{11}$$

The proton, travelling at very close to the speed of light, would take $10^5$ years to cross the galaxy of diameter $10^5$ light years. But to the proton, the time required will be its wristwatch time $\tau$ where:

$$t = \gamma \tau$$

$$\tau = \frac{t}{\gamma} = \frac{10^5 \text{ year}}{10^{11}} \approx \frac{3 \times 10^{12} \text{ s}}{10^{11}} = 30 \text{ s}$$

The wristwatch time for the proton to cross the whole Galaxy is half a minute. From its point of view, the diameter of the galaxy is shrunk by a factor $10^{11}$, to a mere $10^5 \text{ km}$. 

Back to Revision Checklist
### The linear accelerator (from Chapter 16)

**The principle of a linear accelerator**

![Diagram of a linear accelerator showing the accelerating field, field between electrodes, and zero field inside the tube.]

- Negative electron accelerated
- Field between electrodes
- Zero field inside tube

**Switching p.d.s to keep accelerating electrons**

- At one instant
- Electrons between electrodes are accelerated
- Alternating high-frequency p.d.
- Electrons drift through tube
- Electrons further accelerated
- Electrons getting faster

**The alternating p.d. switches back and forth, the electrons accelerating as they pass between electrodes**

---

**Summary Diagrams**

Back to list of Contents
Principle of the synchrotron accelerator (from Chapter 16)

The principle of a synchrotron accelerator

- Inject beam at \( v = c \) from smaller accelerator
- Radio frequency cavity to accelerate beam
- Magnets to bend beam
- Electrostatic deflector
to extract beam and direct it into targets
- Magnets to focus beam

Back to Revision Checklist
Alpha particle scattering experiment

Rutherford's scattering experiment

- Radium source of alpha particles
- Thin gold foil
- Alpha particle beam
- Scattered alpha particles
- Lead block to select narrow beam of alpha particles
- Microscope to view zinc sulphide screen and count alpha particles
- Zinc sulphide screen; tiny dots of light where struck by alpha particle
- Vary angle of scattering observed

Back to Revision Checklist
Rutherford’s picture of alpha particle scattering

**For calculations**

\[ F = \frac{2e^2}{4\pi \varepsilon_0 d^2} \]

**Assumptions:**
- Alpha particle is the He nucleus, charge +2e
- Gold nucleus has charge +Ze, and is much more massive than alpha particles
- Scattering force is inverse square electrical repulsion

**Careful investigation of alpha scattering supported the nuclear model of the atom**

Back to Revision Checklist
Distance of closest approach

Where does the alpha particle stop?

Initial kinetic energy

\[ \text{Initial kinetic energy} = 5 \text{ MeV} = 5 \times 10^6 \text{ eV} \times 1.6 \times 10^{-19} \text{ J eV}^{-1} = 8.0 \times 10^{-13} \text{ J} \]

Electrical potential energy

\[ V = \frac{+2 \ Z e^2}{4 \pi \epsilon_0 \ d} \]

\[ Z = 79, \ e = 1.6 \times 10^{-19} \text{ C}, \]

\[ \epsilon_0 = 8.9 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2} \]

Alpha particle stops where initial kinetic energy = electrical potential energy

\[ 8.0 \times 10^{-13} \text{ J} = \frac{+2 \ Z e^2}{4 \pi \epsilon_0 \ d} \]

Substitute values of \( Z, e, \epsilon_0 \)

\[ d = 4.5 \times 10^{-14} \text{ m} \]

The radius of a gold nucleus must be less than \( 10^{-14} \text{ m} \). Atoms are 10,000 times larger than their nuclei.

Back to Revision Checklist
Spectra and energy levels

Energy levels of an atom:

- $n = 4$
- $n = 3$
- $n = 2$
- $n = 1$

Photon emitted as electron falls from one level to a lower level.

$E = hf$

$E = hf$ is the energy difference between two levels.
Standing waves in boxes

Waves on a string
fixed end
fixed end

Waves on a circular diaphragm
rigid edge

Only certain field patterns are possible because the waves must fit inside the box.
Colours from electron guitar strings

Molecular guitar strings

Carotene molecule $C_{40}H_{56}$

Electrons spread along the molecule

Analogy with guitar string

Electrons make standing waves along the molecule

Length $L$

Electron wavelength proportional to $L$

Long molecules absorb visible wavelengths of light
Energy levels

Standing waves and energy levels

Waves and energy levels

Standing waves allow only discrete values of wavelength \( \lambda \).

\[
L = \frac{\lambda}{2}
\]

\[
L = \lambda
\]

...etc

For each wavelength there is a discrete value of momentum \( p \):

\[
p = \frac{h}{\lambda}
\]

If motion is non-relativistic:

momentum: \( p = mv \)

kinetic energy:

\[
E_K = \frac{1}{2} mv^2
\]

Kinetic energy and momentum are related by:

\[
E_K = \frac{p^2}{2m}
\]

For each wavelength there is a discrete value of kinetic energy:

\[
p^2 = \frac{h^2}{\lambda^2}
\]

so

\[
E_K = \frac{h^2}{2m\lambda^2}
\]

Discrete wavelengths imply discrete levels of energy

small \( \lambda \), high energy

large \( \lambda \), low energy

Back to Revision Checklist
Standing waves in atoms

Standing waves and energy levels

A guitar–string atom

Simplify:
Change the 1/r potential well of the nucleus into a pair of fixed high walls

\[ E_4 = \frac{\hbar^2}{2m\lambda_4} = 4^2E_1 \]
\[ E_3 = \frac{\hbar^2}{2m\lambda_3} = 3^2E_1 \]
\[ E_2 = \frac{\hbar^2}{2m\lambda_2} = 2^2E_1 \]
\[ E_1 = \frac{\hbar^2}{2m\lambda_1} \]

Energy levels increase as \( n^2 \)

Each level has a quantum number \( n \). The energy depends on the quantum number.

Note that in a real one-electron atom (hydrogen) the energy of the levels varies as \( 1/n^2 \), not as \( n^2 \).

Back to Revision Checklist
Size of the hydrogen atom

How small could a hydrogen atom be?

Replace 1/r potential by a box of width d = 2r

Calculate kinetic energy for waves λ = 2d = 4r

Calculate potential energy at r

potential energy \( E_p = -\frac{e^2}{4\pi\varepsilon_0 r} \)

kinetic energy \( E_k = \frac{\hbar^2}{2m\lambda^2} \)

Find the minimum radius of an atom, for total energy < 0

If size is too small, the kinetic energy is too large for electrical potential energy to bind the electron

Back to Revision Checklist
Quarks and gluons

Quark–gluon interaction

Quarks interact by exchanging gluons, which change the quark colours. Here a red quark and a blue quark exchange a red–blue gluon. The red quark becomes blue and the blue quark becomes red. The quarks exchange energy and momentum.

Quarks pulled apart make more quarks

Two quarks held together by the gluon field...

...pull the quarks apart. The gluon field increases in energy...

...a quark–antiquark pair materialises from the gluon field
**Pair creation and annihilation**

**Pair annihilation and creation**

**Annihilation**

Gamma energy  
\[= 2 \times 0.511 \text{ MeV} \text{ plus kinetic energy of electrons} \]

**Creation?**

Extremely rare  
(cannot bring two identical photons together)

**Pair creation**

Gamma energy  
\[= 2 \times 0.511 \text{ MeV} \text{ (minimum) nucleus carries away momentum, to conserve momentum and energy} \]

---

[Back to Revision Checklist](#)
What the world is made of

The particles of the everyday world
Everything you touch around you is made of just these particles:

<table>
<thead>
<tr>
<th>The world around you</th>
<th>Leptons</th>
<th>charge</th>
<th>rest energy / MeV</th>
<th>Quarks</th>
<th>charge</th>
<th>rest energy / MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(^{-}) electron</td>
<td>-1</td>
<td>0.511</td>
<td>u up</td>
<td>+2/3</td>
<td>6?</td>
<td></td>
</tr>
<tr>
<td>(\nu) neutrino</td>
<td>0</td>
<td>0?</td>
<td>d down</td>
<td>-1/3</td>
<td>10?</td>
<td></td>
</tr>
</tbody>
</table>

A complete picture of your world should include their antiparticles too:

<table>
<thead>
<tr>
<th>The world around you</th>
<th>Leptons</th>
<th>charge</th>
<th>rest energy / MeV</th>
<th>Quarks</th>
<th>charge</th>
<th>rest energy / MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(^{+}) positron</td>
<td>+1</td>
<td>0.511</td>
<td>(\bar{u}) anti-up</td>
<td>-1/3</td>
<td>6?</td>
<td></td>
</tr>
<tr>
<td>(\bar{\nu}) antineutrino</td>
<td>0</td>
<td>0?</td>
<td>(\bar{d}) anti-down</td>
<td>+1/3</td>
<td>10?</td>
<td></td>
</tr>
</tbody>
</table>

To account for all known matter, the pattern of a pair of leptons and a pair of quarks repeats three times:

<table>
<thead>
<tr>
<th>Generation</th>
<th>Leptons</th>
<th>charge</th>
<th>rest energy / MeV</th>
<th>Quarks</th>
<th>charge</th>
<th>rest energy / MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 The world around you</td>
<td>e(^{-}) electron</td>
<td>-1</td>
<td>0.511</td>
<td>u up</td>
<td>+2/3</td>
<td>6?</td>
</tr>
<tr>
<td></td>
<td>(\nu) neutrino</td>
<td>0</td>
<td>0?</td>
<td>d down</td>
<td>-1/3</td>
<td>10?</td>
</tr>
<tr>
<td>2</td>
<td>(\mu)– muon</td>
<td>-1</td>
<td>106</td>
<td>(s) strange</td>
<td>-1/3</td>
<td>200?</td>
</tr>
<tr>
<td></td>
<td>(\nu) muon-neutrino</td>
<td>0</td>
<td>0?</td>
<td>(c) charmed</td>
<td>+2/3</td>
<td>1500?</td>
</tr>
<tr>
<td>3</td>
<td>(\tau)– tau</td>
<td>-1</td>
<td>1780</td>
<td>(b) bottom</td>
<td>+1/3</td>
<td>5000?</td>
</tr>
<tr>
<td></td>
<td>(\nu) tau-neutrino</td>
<td>0</td>
<td>0?</td>
<td>(t) top</td>
<td>+2/3</td>
<td>90 000?</td>
</tr>
</tbody>
</table>

The other particles that make up the world are the bosons, the carriers of interactions:

<table>
<thead>
<tr>
<th>interaction</th>
<th>force carrier</th>
<th>electric charge</th>
<th>rest energy / GeV</th>
<th>explains</th>
</tr>
</thead>
<tbody>
<tr>
<td>electromagnetism</td>
<td>photon</td>
<td>0</td>
<td>0</td>
<td>Everyday interactions including all chemistry</td>
</tr>
<tr>
<td>weak interaction</td>
<td>(Z^0)</td>
<td>0</td>
<td>93</td>
<td>Radioactive decays; changing particle nature</td>
</tr>
<tr>
<td></td>
<td>(W^+)</td>
<td>+1</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(W^-)</td>
<td>-1</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>strong interaction</td>
<td>8 different ‘colour combinations’ of gluons</td>
<td>0</td>
<td>0</td>
<td>What holds nucleons and mesons together</td>
</tr>
<tr>
<td>gravity</td>
<td>‘graviton’</td>
<td>0</td>
<td>0</td>
<td>Conjectured, but not detected</td>
</tr>
</tbody>
</table>

The hunt is on at the LHC (Large Hadron Collider) for another particle, the Higgs boson, which is thought to be responsible for particles having mass.

Back to Revision Checklist
Conserved quantities in electron-positron annihilation

### Energy is conserved

<table>
<thead>
<tr>
<th>total energy before</th>
<th>=</th>
<th>total energy after</th>
</tr>
</thead>
<tbody>
<tr>
<td>kinetic energy of particles + rest energy of particles</td>
<td>=</td>
<td>energy after is energy of gamma photons</td>
</tr>
<tr>
<td>minimum value of energy before is rest energy:</td>
<td>=</td>
<td>= 2 × 0.511 MeV</td>
</tr>
<tr>
<td>$2 , mc^2 = 2 \times 0.511 \text{ MeV}$</td>
<td>=</td>
<td>$2 \times 0.511 \text{ MeV}$</td>
</tr>
</tbody>
</table>

### Momentum is conserved

<table>
<thead>
<tr>
<th>total linear momentum before</th>
<th>=</th>
<th>total linear momentum after</th>
</tr>
</thead>
<tbody>
<tr>
<td>same mass; equal and opposite velocities</td>
<td>=</td>
<td>energy $E$, momentum $p = E/c$</td>
</tr>
<tr>
<td>total momentum before = 0</td>
<td>=</td>
<td>photons identical, momentums opposite</td>
</tr>
<tr>
<td>total momentum = 0</td>
<td>=</td>
<td>total momentum = 0</td>
</tr>
</tbody>
</table>

### Electric charge is conserved

<table>
<thead>
<tr>
<th>total charge before</th>
<th>=</th>
<th>total charge after</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-e) + (+e) = 0$</td>
<td>=</td>
<td>charge</td>
</tr>
<tr>
<td>$0 + 0 = 0$</td>
<td>=</td>
<td>= 0</td>
</tr>
</tbody>
</table>

Energy, momentum and electric charge are always conserved in electron-positron annihilation
Relativistic momentum \( p = \gamma mv \) (from Chapter 16)

Einstein redefines momentum

Problem:
time \( t \) depends on relative motion because of time dilation (chapter 12)

Newton’s definition of momentum
\[
\begin{align*}
   p &= mv \\
   p &= m \Delta x / \Delta t
\end{align*}
\]

Einstein’s solution:
Replace \( \Delta t \) by \( \Delta \tau \), the change in wristwatch time \( \tau \), which does not depend on relative motion

from time dilation:
\[
\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}
\]
\[
\Delta t = \gamma \Delta \tau
\]
substitute for \( \Delta \tau \)
\[
\Delta x = v \Delta t
\]

Einstein’s new definition of momentum
\[
\begin{align*}
   p &= m \Delta x / \Delta \tau \\
   p &= \gamma m \Delta x / \Delta \tau \\
   p &= \gamma^2 m v
\end{align*}
\]

Relativistic momentum \( p = \gamma mv \) increases faster than Newtonian momentum \( mv \) as \( v \) increases towards \( c \)

Back to Revision Checklist
Relativistic energy $E_{\text{total}} = \gamma mc^2$ (from Chapter 16)

**Einstein rethinks energy**

**Relativistic idea:**
Space and time are related. Treat variables $x$ and $ct$ similarly. Being at rest means moving in wristwatch time $\tau$.

So invent relativistic momentum for "movement in wristwatch time".

From time dilation $\frac{\Delta x}{\Delta t} = \gamma$

Multiply $p_0$ by $c$, getting a quantity $E$ having units of energy (momentum $\times$ speed).

Interpret energy $E = \gamma mc^2$

1. Particle at rest: $v = 0$ and $\gamma = 1$  
   \[ E_{\text{rest}} = mc^2 \]  
   Particle has rest energy

2. Particle moving at speed $v$  
   Energy $\gamma mc^2$ greater than rest energy  
   \[ E_{\text{kinetic}} = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2 \]  
   \[ E_{\text{kinetic}} = (\gamma - 1)mc^2 \approx \frac{1}{2}mv^2 \] (see graph)

3. At low speeds $v \ll c$ kinetic energy has same value as for Newton

Interpret $E$ as total energy = kinetic energy + rest energy

Total energy = $\gamma mc^2$. Rest energy = $mc^2$. Total energy = kinetic energy + rest energy.

**Back to Revision Checklist**
**Energy, momentum and mass (from Chapter 16)**

### Key relationships

<table>
<thead>
<tr>
<th>Low speeds ($v \ll c$)</th>
<th>High speeds ($v \approx c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>$E_{\text{total}} = mc^2$</td>
</tr>
<tr>
<td>Kinetic energy small compared to total energy</td>
<td>$E_{\text{kinetic}} = \frac{1}{2}mv^2$</td>
</tr>
<tr>
<td>Momentum</td>
<td>$p = \gamma mv$</td>
</tr>
<tr>
<td>$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$</td>
<td>$\gamma = \frac{E_{\text{total}}}{E_{\text{rest}}}$</td>
</tr>
</tbody>
</table>

### Kinetic energy small compared to rest energy

- Low speeds: $E_{\text{total}} = E_{\text{rest}} = mc^2$
- High speeds: $E_{\text{kinetic}} = (\gamma - 1)mc^2$

### Kinetic energy large compared to rest energy

- Low speeds: $E_{\text{total}} = mc^2$
- High speeds: $E_{\text{kinetic}} = (\gamma - 1)mc^2$

---

Back to Revision Checklist

Back to list of Contents