OCR A Physics A-level

Topic 6.1: Capacitors

(Content in italics is not mentioned specifically in the course specification but is nevertheless topical, relevant and possibly examinable)

Definition of a Capacitor and Capacitance

A capacitor is an electrical component that stores charge on two separated metallic plates. An insulator, sometimes called a dielectric, is placed between the plates to prevent the charge from travelling across the gap.

The capacitance, C, is defined as the charge stored, Q, per unit potential difference, V, across the two plates. Therefore we can write

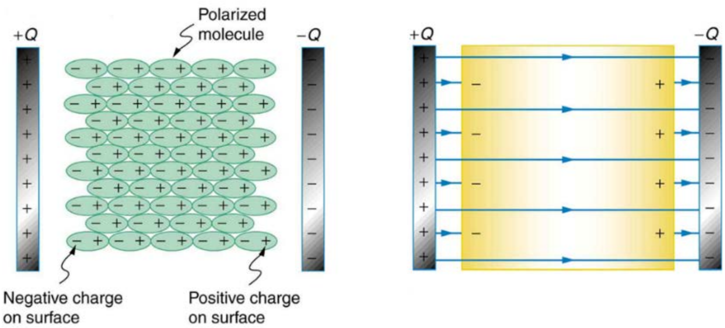
$$C = \frac{Q}{V}$$

where capacitance is measured in Farads, F (CV^{-1}).

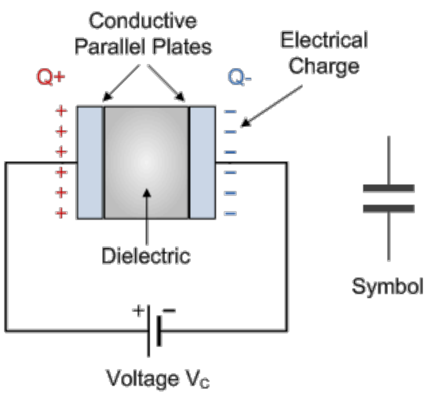
When a capacitor is connected to a DC power supply, e.g. a cell or battery, there is a brief current as the power supply draws electrons from one plate and deposits them on the other plate. This leaves the first plate with charge $+Q$ and the second with charge $-Q$. These charges will be equal and opposite due to the conservation of charge. Current will flow in the circuit until the potential difference between the plates is equal to that of the electromotive force or e.m.f. of the power supply.

Dielectric Insulators

The dielectric has another purpose: to increase the capacitance of the device by polarizing in the electric field and effectively increasing the charge stored on the plates. Dielectrics have an associated electrical permittivity (see 6.2 Electric Fields) which describes its ability to polarize and strengthen the charge storage capability of the device. This is why in reality the insulator is rarely a vacuum or just air as these materials do not polarize well (or at all in the case of the vacuum) and so are poor dielectrics.



Dielectric Insulator in the Electric Field of a Parallel Plate Capacitor



Capacitors in Series

Kirchhoff's voltage law states that the sum of the e.m.f.s in any closed loop in a circuit is equal to the sum of the potential differences in the same loop (see section 4.3).

$$V = V_1 + V_2 + V_3 + \cdots + V_N$$

From the equation $C = \frac{Q}{V}$, it is clear that $V = \frac{Q}{C}$ and so substituting this into the expression for Kirchhoff's voltage law gives

$$\frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \cdots + \frac{Q}{C_N}$$

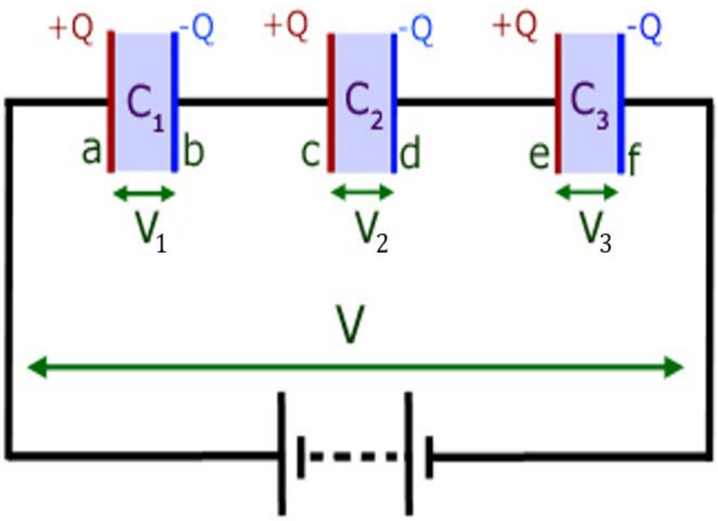
where C_T is the combined capacitance of all the series capacitors. As Q is a constant it can be factorised out to give

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

Therefore

$$C_T = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}\right)^{-1}$$

Note that this equation is similar to the equation for the total resistance of a number of resistors in parallel.



Capacitors in Parallel

Kirchhoff's current law states that the total current flowing into a node in a circuit must be equal to the total current flowing out of that node. Therefore, we can state that

$$I_T = I_1 + I_2 + I_3 + \cdots + I_N$$

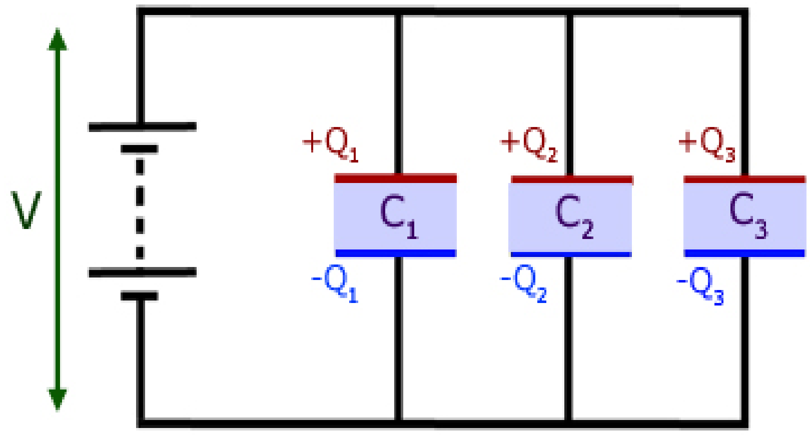
Charge can be stated as $Q = It$, so using the above and factorising out the constant time,

$$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_N$$

Finally, substituting the equation $C = \frac{Q}{V}$ and that the voltage is the same over each component in parallel we can write

$$C_T = C_1 + C_2 + C_3 + \cdots + C_N$$

Note that this equation is similar to the equation for the total resistance of a number of resistors in series.



<u>Energy Stored in a Capacitor</u>

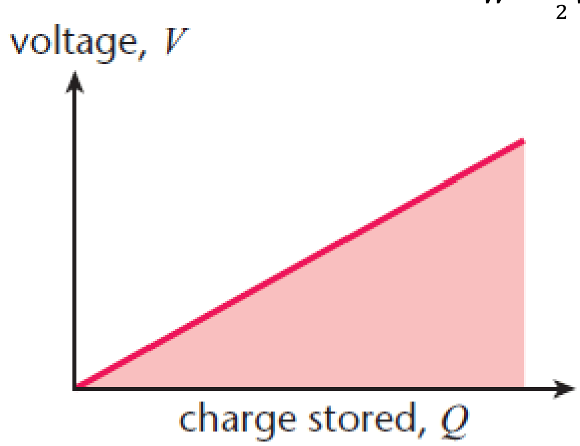
Work must be done by the power supply to deposit negatively charged electrons onto the negative plate as like charges repel according to Coulomb's law (see 6.2 Electric Fields). Equally, work is done to remove electrons from the positive plate as negative charges are attracted to positive regions.

The graph below shows the charge stored on a capacitor plates against the potential difference over the device. As voltage is defined as the electrical potential energy per unit charge (see 6.2 Electric Fields), the area under the graph must therefore represent the work done in charging up the capacitor and so the energy stored in the capacitor. Therefore

$$W = \frac{1}{2}QV$$

however, $Q = CV$ and also $V = \frac{Q}{C}$ thus

$$W = \frac{1}{2}V^2C \quad W = \frac{Q^2}{2C}$$



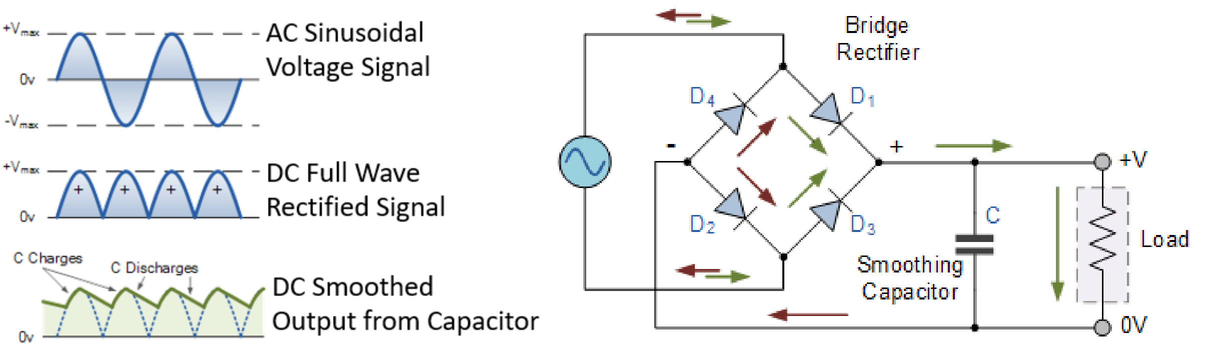
Applications of Capacitors

Capacitors are used to store and discharge large quantities of energy in a short time period. This makes them useful for short pulses of energy such as camera flashes and touch screens where a short finger press leads to a large buildup of energy in a capacitor.

They are also integral to uninterrupted power supplies or UPSs which are used as backup power supplies when the mains electricity supply fails. UPSs are commonly found in data centers to protect the hardware and in hospitals to maintain a constant power supply to life support machines.

Finally, capacitors are used in the process of converting alternating current (AC) into direct current (DC). Once a sinusoidal AC signal has passed through a full wave rectifier, the current flows in one direction but varies as shown. The current can then be passed through a smoothing circuit in which a capacitor stores energy as the p.d. rises and discharges as it falls.

This can be used maintain a more constant current. The signal can then be passed through another smoothing circuit and another until the voltage is effectively constant.



Charging and Discharging Capacitors

Once a capacitor has been charged, it can then be discharged by disconnecting the power supply and connecting up another electrical component. This can be achieved by flipping the switch from in the circuit diagram so from A to B. Often, this component is a resistor as then the resistance, and so the time constant for the fall in voltage, can be known to a high degree of accuracy.

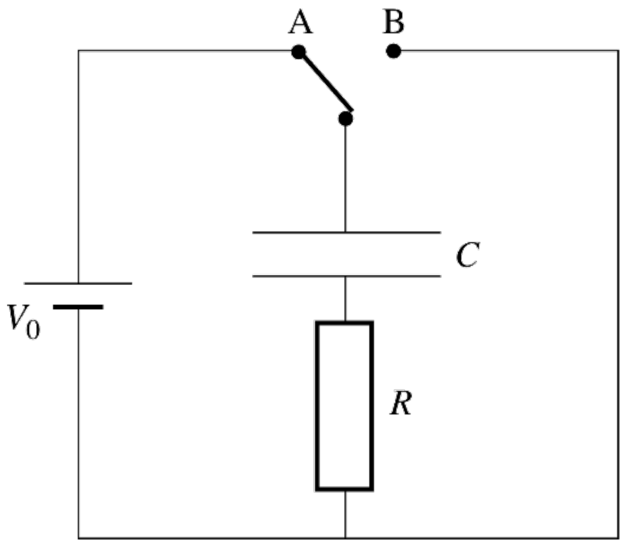
When the power supply is disconnected, the electrons packed onto the negative plate are no longer subject to the e.m.f. which held them in such close proximity. They repel one another and so flow round circuit dissipating electric energy as heat in the resistor. Once, the charges on the negative and positive plates have equilibrated, there is no longer any potential difference across the capacitor ($Q = 0$) and the electrons cease to flow resulting in the current dropping to zero.

Naturally, this discharging process takes time. The time constant over which this discharging process occurs depends firstly on the capacitance and also on the magnitude of the resistance in the discharging circuit. The lower the resistance in the discharging circuit, the higher the current can be as current is indirectly proportional to the resistance from Ohm's law ($I \propto \frac{1}{R}$). If the current is higher, then the charge on the plates will fall to zero in a faster time as $\Delta Q = It$. Equally, the larger the capacitance the larger the charge stored per unit potential difference. As potential difference is proportional to the current by Ohm's law, then capacitance is a measure of the charge stored per rate that charge flows from the plates that is to say

$$C = \frac{Q}{V} \propto \frac{Q}{I} \sim \tau$$

where τ is the time period over which the capacitor discharges and the symbol $\sim$ here means 'goes as' so not necessarily directly proportional but as one quantity increases so does the other. We use this symbol as the current is not constant over the time spent discharging so the relation is not as simple as $Q = I\tau$. Simply put, this means that the amount of charge that can flow before the voltage drops to zero is higher and so a longer time is needed for the discharge to take place.

Before the resistor is connected, the potential difference, V_0, across the plates is at its maximum and given by $V_0 = \frac{Q_0}{C}$ where Q_0 is the initial charge stored on the plates. At time $t = 0$, the resistor circuit is connected and the current flowing through the circuit will be V_0/R as given by Ohm's law. As the electrons flow, the charge stored will decrease as the negative plate loses electrons and the positive plate gains electrons. This in turn will decrease the potential difference over the capacitor and so current must also decrease and will eventually reach zero.



Derivation of the relationship between Charge and Time in a Discharging Capacitor

Current can be defined as the differential of charge with respect to time

$$I = -\frac{dQ}{dt}$$

where the negative sign is a result of conventional current being in the opposite direction to electron flow. Though as $V = IR$ and $Q = CV$,

$$\frac{dQ}{dt} = -\frac{Q}{CR}$$

Separating variables and integrating from $t = 0$ *when* $Q(t = 0) = Q_0$ *gives*

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{CR}$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{CR}$$

Therefore, taking the exponent

$$Q = Q_0 e^{-\frac{t}{CR}}$$

Similar equations can be written for voltage and current as they are related to the charge by $I = \frac{dQ}{dt}$ and $V = IR$.

$$V = V_0 e^{-\frac{t}{CR}} \qquad I = I_0 e^{-\frac{t}{CR}}$$

The relationship between V, I or Q and t is an exponential decay as seen in the graph on the right below.

While charging a capacitor, at any time in the circuit e.m.f. (V_0) will be equal to the sum of the p.d.s across the resistor (V_R) and the capacitor (V_C) by Kirchhoff's voltage law.

$$V_0 = V_R + V_C$$

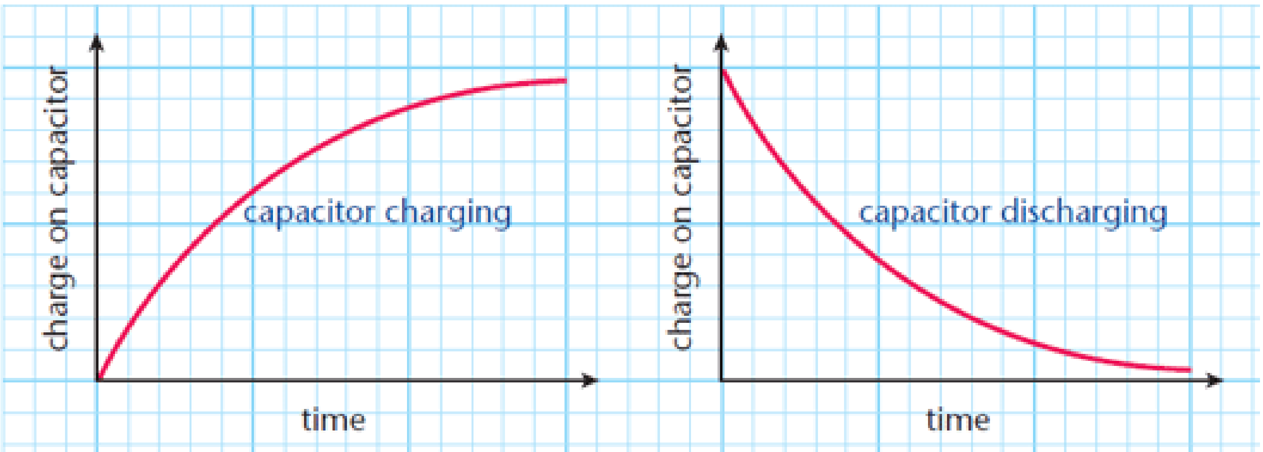
by substituting in $V_R = IR$, and $I = I_0 e^{-\frac{t}{CR}}$

$$V_C = V_0 - I_0 R e^{-\frac{t}{CR}}$$

or

$$V_C = V_0 \left(1 - e^{-\frac{t}{CR}}\right)$$

A similar expression can be written for the charge on the capacitor. This relationship is shown in the graph on the left below.



The charge on a capacitor during charging and discharging

The value of the time constant of the circuit is seen in the exponents of the equations above and is often give the symbol $\tau = CR$. For a discharging capacitor, when $t = \tau$ the charge on the capacitor will have decreased to approximately 37% of its original value.

Graphing Variables in Capacitor-Resistor Circuits

The change in charge with time can be graphed iteratively for a capacitor-resistor circuit. First, the time constant is calculated from the known values of the capacitance and the resistance. Then, using the equation

$$\frac{\Delta Q}{\Delta t} = -\frac{Q}{CR}$$

it can be seen that, in a small time interval Δt compared with τ, the change in charge stored, ΔQ, can be calculated. From this a new charge stored can be calculated at the new time.

$$t_{i+1} = t_i + \Delta t \qquad \Delta Q_{i+1} = -\frac{Q_i}{CR}\Delta t \qquad Q(t_{i+1}) = Q(t_i) + \Delta Q_i$$

This can be repeated for each new charge to give a value for the charge at each moment in time. These could be graphed to give the approximate behaviour of the charge stored with time.

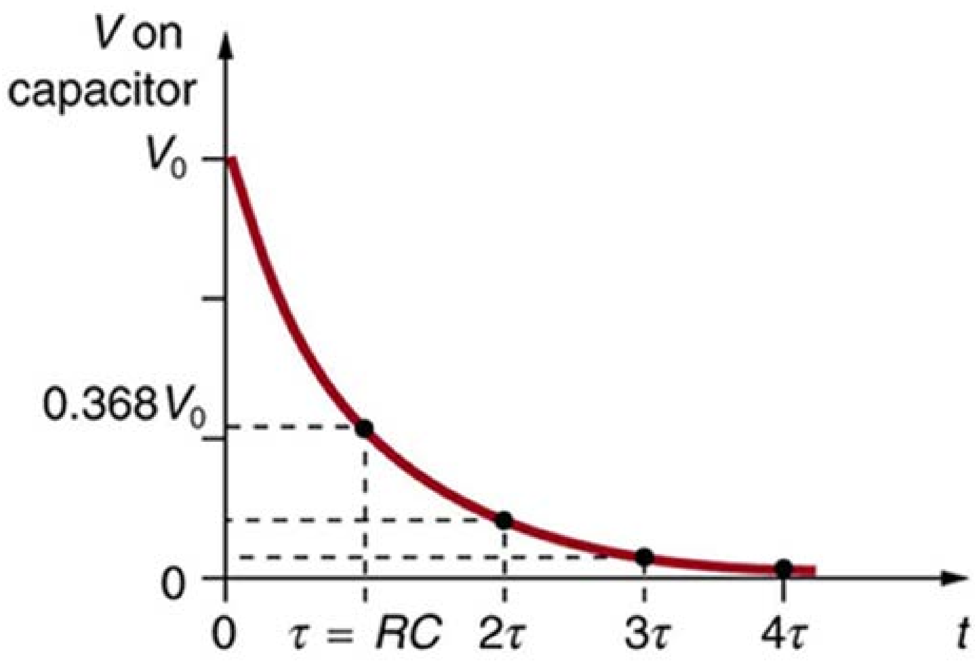
These exponential graphs also show that if V, I or Q are measured at set time intervals that

$$\frac{V_1}{V_0} = \frac{V_2}{V_1} = \frac{V_3}{V_2} \dots$$

or more generally

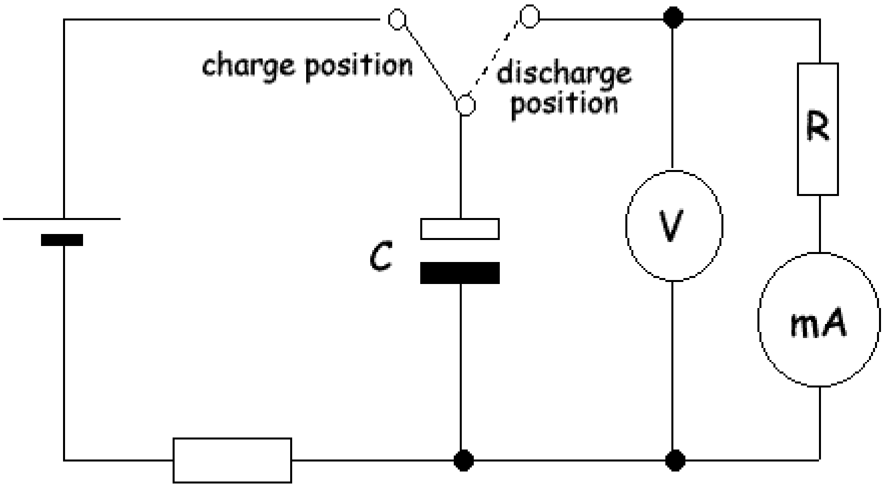
$$\frac{V_{i+1}}{V_i} = \frac{V_{i+2}}{V_{i+1}}$$

This relationship is always true for exponential decays.



Experimental Techniques to investigate Capacitor-Resistor Circuits

To investigate the charge or discharge of a capacitor a circuit with a DC power supply, a capacitor, a resistor in series, an ammeter in series and a voltmeter in parallel are needed. Data loggers can be used to collect the data in time as capacitors often discharge very quickly. Plotting current and voltage with time in charging and discharging circuits can be used to investigate of the exponential relationships between the variables current and p.d. with time. The readings for the voltage and current should be taken at set intervals which should be small compared to the time constant. This can then allow for an experimental determination of the time constant. The experimental value could then be compared to the theoretical value based on the values of the resistance and capacitance.



Dependence of Capacitance on Dimensions of the Capacitor

The capacitance of a parallel plate capacitor depends on the number of electrons that can be stored on the negative plate and so is directly proportional to the area of the plates, A. The attraction between charges on the negative plate and the positive plate depends on the separation of the plates, d. Therefore the capacitance is indirectly proportional to d so

$$C \propto A \qquad C \propto \frac{1}{d}$$

With a vacuum between the two plates capacitance is then defined as

$$C = \frac{\varepsilon_0 A}{d}$$

For non-vacuum insulators this permittivity changes, such that $\varepsilon = \varepsilon_r \varepsilon_0$ *where* ε_r *is the relative permittivity of the dielectric medium. Hence, for a general parallel plate capacitor*

$$C = \frac{\varepsilon A}{d}$$