# OCR A Physics A-level

## Topic 5.4: Oscillations

Notes

# Simple harmonic motion

## Key definitions

- Displacement, x – the **distance** from the **equilibrium** position
- Amplitude, A – the **maximum** displacement
- Period, T – the time taken to complete **one full oscillation**
- Frequency, f – the number of oscillations **per unit time**
- Phase difference, ϕ – the **fraction** of an oscillation between the **position** of two oscillating objects (given by $\frac{\Delta t}{T} \times 2\pi$)
- Angular frequency, ω – the **rate of change** of angular position (given by $2\pi f$)

## Simple harmonic motion

Simple harmonic motion is a type of oscillation, where the **acceleration** of the oscillator is directly proportional to the **displacement** from the equilibrium position, and acts **towards** the equilibrium position. The key equation for simple harmonic motion is

$$a = -\omega^2 x$$

where $a$ is the acceleration of the oscillator, $\omega$ is the angular frequency, and $x$ is the displacement of the oscillator. The negative sign is used to show that the direction of acceleration is always towards the equilibrium position, in the opposite direction to the displacement.

An oscillator in simple harmonic motion is an **isochronous** oscillation, so the period of the oscillation is **independent** of the amplitude.

## Techniques to investigate the period and frequency of simple harmonic motion

The frequency of the oscillator is equal to the **reciprocal** of the period. The period of the oscillator, and hence the frequency, can be determined by setting the oscillator (such as a pendulum or a mass on a spring) in to motion, and using a stopwatch to measure the time taken for one oscillation.

In order to increase the accuracy of this measurement, the time for **10 oscillations** to take place can be measured, and this time divided by 10 to find the period. An oscillator in simple harmonic motion is an isochronous oscillation, so the period of the oscillation is independent of the amplitude. A **fiducial** marker is used as the point to start and stop timings, and is normally placed at the equilibrium position.

## Analysing simple harmonic motion

There are two equations which can be used to determine the displacement of a simple harmonic oscillator.

$$x = A \sin \omega t \quad x = A \cos \omega t$$

where $x$ is the displacement of the oscillator, $A$ is the amplitude, $\omega$ is the angular frequency, and $t$ is the time. The **sine** version of the equation is used if the oscillator begins at the **equilibrium** position, and the **cosine** version is used if the oscillator begins at the **amplitude** position.

Velocity and acceleration

The **velocity** of the oscillator at a given time can be determined by finding the **gradient** of the graph at that point. The maximum velocity occurs at the equilibrium position, with the oscillator being stationary at the amplitude points. The maximum acceleration occurs at the amplitude points, and is 0 when the oscillator is at equilibrium position.

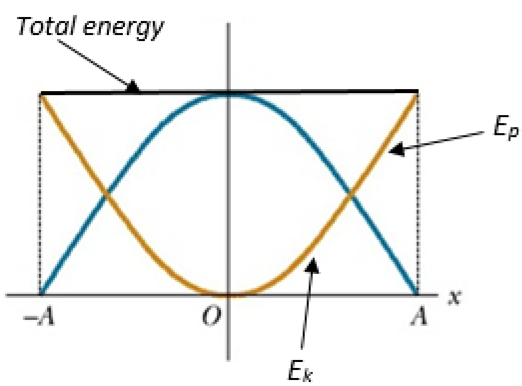
The velocity, $v$, of the oscillator is given using the equation

$$v = \pm\omega\sqrt{A^2 - x^2}$$

where $\omega$ is the angular frequency of the oscillator, $A$ is the amplitude, and $x$ is the current displacement. The **maximum velocity** occurs at the **equilibrium position**, where x = 0, so we can derive the formula $v_{max} = \omega A$ to determine the maximum velocity of the oscillator.

# Energy transfers in simple harmonic motion

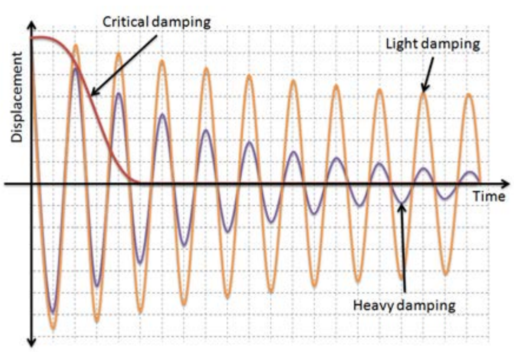
Interchange between kinetic and potential energy




During simple harmonic motion, energy is exchanged between the **kinetic** and **potential** forms. The maximum kinetic energy occurs at the equilibrium point, where the velocity is at a maximum. The maximum potential energy occurs at the amplitude positions, where displacement is at a maximum. Total energy is **always conserved**.

Damping

**Damping** is the process by which the **amplitude** of the oscillations **decreases** over time. This is due to **energy loss** to resistive forces such as drag or friction.



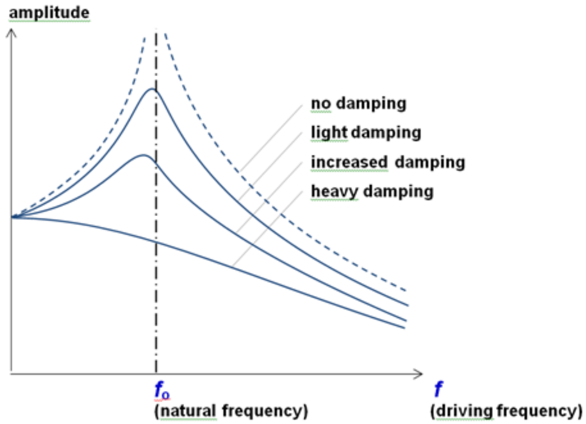

Light damping occurs naturally (e.g. pendulum oscillating in air), and the amplitude decreases exponentially. When heavy damping occurs (e.g. pendulum oscillating in water) the amplitude decreases dramatically. In critical damping (e.g. pendulum oscillating in treacle) the object stops before one oscillation is completed.

# Resonance

## Free and forced oscillations

When an object oscillates without any external forces being applied, it oscillates at its **natural frequency**. This is known as free oscillation. Forced oscillation occurs when a **periodic driving force** is applied to an object, which causes it to oscillate at a particular frequency.

## Resonance




When the **driving frequency** of the external force applied to an object is the **same** as the **natural frequency** of the object, **resonance** occurs. This is when the amplitude of oscillation rapidly increases, and if there is no damping, the amplitude will continue to increase until the system fails. As damping is increased, the amplitude will decrease at all frequencies, and the maximum amplitude occurs at a lower frequency.

## Techniques to investigate resonance

To investigate the resonance of an object experimentally, a mass can be suspended between two springs attached to an oscillation generator. A millimetre ruler can be placed parallel with the spring-mass system to record the amplitude. The driver frequency of the generator is slowly increased from zero, so the mass will oscillate with increasing amplitude, reaching maximum amplitude when the driver frequency reaches the natural frequency of the system. The amplitude of oscillation will then decrease again as frequency is increased further. The spring-mass system experiences damping from the air so the amplitude should not continue to increase until the point of system failure. To increase accuracy, the system can be filmed and the amplitude value recorded from video stills, as it can be difficult to determine this whilst the mass is oscillating.