

OCR A Physics A-level

Topic 3.1: Motion

Notes



Kinematics

Speed and velocity

Speed is defined as the **rate of change of distance**. The equation to determine the average speed v of an object is $\frac{\text{distance travelled}}{\text{time taken}} \rightarrow v = \frac{\Delta x}{\Delta t}$. The SI unit for speed is ms^{-1} .

Instantaneous speed is the speed of an object over a **very short time interval**. It can be determined by analyzing the **gradient** of a distance-time graph at a given time.

The displacement of an object is the distance it has travelled **in a given direction**, so it is a **vector quantity**, with both magnitude and direction. The velocity of an object is defined as the **rate of change of displacement**, and is given by the formula $\frac{\text{Change in Displacement}}{\text{Change in time}} \rightarrow v = \frac{\Delta s}{\Delta t}$ in ms^{-1} .

Acceleration is defined as the **rate of change of velocity**, and it can be expressed as $a = \frac{\Delta v}{\Delta t}$. Acceleration is a vector quantity. An object is accelerating when its speed is increasing, and decelerating when its speed decreases, and it will also accelerate when its direction of travel changes, such as in circular motion.

Graphical representations of speed and velocity

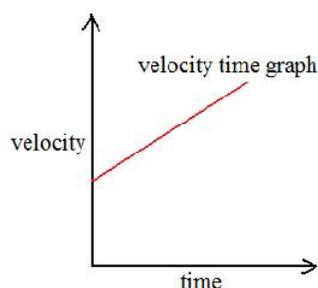
The motion of an object can be represented graphically. On a graph of displacement (y-axis) against time (x-axis), a flat line represents a stationary object. A line with constant gradient represents an object with constant velocity, and a curved line represents an object that is accelerating or decelerating.

The motion of an object can also be represented on a velocity-time graph. On these graphs, the y co-ordinate represents the instantaneous velocity of the object at that time, and the gradient of the line at that time represents the acceleration of the object. A flat line represents an object travelling with constant velocity. A line with constant gradient represents an object that is accelerating at a constant rate, with a negative gradient representing negative acceleration. A curve represents an object with changing acceleration. The area under a velocity-time graph represents the displacement of the object. This can be calculated mathematically if the graph is linear in acceleration, or if there is a curved line then estimation techniques such as counting grid squared can be used.



Linear motion

Equations of linear motion



There are four equations of motion that can be used for an object travelling with constant acceleration, and they can be derived from a velocity-time graph.

The final velocity, v , is equal to the initial velocity, u , plus the gradient a , multiplied by the change in time, t . This gives $v = u + at$.

The area under the graph is equal to the displacement, s , of the object. Splitting this in to a rectangle and a triangle, then the rectangle will have an area equal to the initial velocity multiplied by the time taken, ut . The triangle will have an area equal to $\frac{1}{2}$ multiplied by the height (final velocity minus initial velocity) multiplied by the base (the time taken), giving $\frac{1}{2}(v-u)t$ overall. The total displacement s is then equal to $ut + \frac{1}{2}(v-u)t$, which simplifies to $s = \frac{1}{2}(v+u)t$.

As the acceleration, a , of the object is equal to the gradient of the line, it is equal to the final velocity minus the initial velocity, divided by the time taken, $a = (v-u)/t$. Rearranging to get $at = (v-u)$, we can substitute in to the unsimplified version of our previous equation, $s = ut + \frac{1}{2}(v-u)t$. This gives us $s = ut + \frac{1}{2}at^2$.

Finally, we can rearrange $a = (v-u)/t$ to get $t = (v-u)/a$. Taking the equation $s = \frac{1}{2}(v+u)t$ and substituting t in gives us an equation which simplified to $v^2 = u^2 + 2as$.

This gives us our four equations of motion,

$$v = u + at$$

$$s = \left(\frac{u + v}{2}\right)t$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2$$

where s = displacement, u = initial velocity, v = final velocity, a = acceleration, and t = time.

These equations can be used in **both horizontal and vertical motion**. In vertical motion, the maximum height is reached when the velocity = 0, and acceleration vertically is usually due to gravity. One direction is taken as positive (usually downwards since gravity is an attractive force). Velocity and acceleration in the negative direction must have a **negative sign** when used in the formula or the results will not be correct.

These equations are used where the **acceleration of the object is constant**. If the acceleration of the object changes to another constant value, then the motion during this time must be **calculated separately** from the period of initial acceleration.



Techniques used to investigate motion

The terminal velocity of a falling object can be determined using **light gates** connected to a **timer or data logger**. The time when each of two light beams are broken by the object is recorded, and these times, combined with the known distance between the light-gates, can be used to calculate the velocity of the object. This technique can also be used to investigate the conservation of momentum resulting from a collision, with the objects suspended on a linear air track, to **minimise energy lost** to overcoming friction and keep the collision as elastic as possible. A small strip of cardboard can be attached to the object to interrupt the light beam.

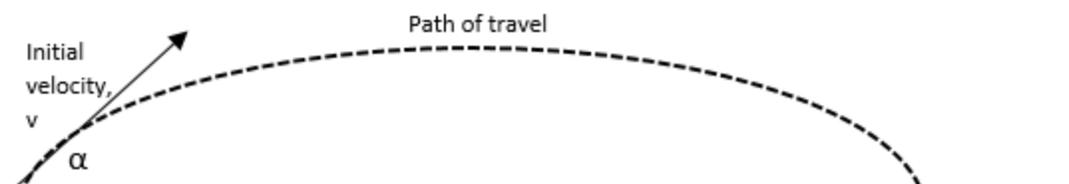
The **acceleration of free fall**, g , is the acceleration of any object in response to the **gravitational attraction** between the Earth and the object. Any object released on the Earth will accelerate downwards to the centre of the Earth (as long as there are no external forces acting on it), at the acceleration of free fall, $g = 9.81 \text{ ms}^{-2}$.

There are several methods that can be used to determine g . A commonly used method involves a trapdoor and an **electromagnet**. An electromagnet holds a small steel ball, suspended a known distance over the trapdoor. At the instant the electromagnet is deactivated a timer begins, and stops only at the instant the trapdoor is hit. As the ball was initially at rest, it's initial velocity is 0, and the distance it has fallen and the time this has taken are known, so these values can be substituted in to the equation $s = ut + \frac{1}{2} at^2$, which is rearranged to find a (as there are no other forces acting on the ball, it's acceleration is equal to g)

Another method to find the acceleration of free fall involves **filming the fall** of a small metal ball **against a ruler**. If the number of **frames per second** is known, then the time taken for the ball to move from rest a known distance down the ruler can be calculated, and used to find g .

Projectile motion

A projectile is an object that is thrown **at an angle to the horizontal**, and therefore moves in **both the horizontal and vertical planes**. The horizontal and vertical components of the object's motion are analysed **independently**, but the time is interchangeable. For this model to work (and for all projectile questions at A-level), we assume that there is **no air resistance**.



For an object projected at an initial velocity v , at an angle of α to the horizontal, the initial vertical velocity will be equal to $v \sin \alpha$, and the initial horizontal velocity is equal to $v \cos \alpha$.



There are **no forces** acting on the object in the **horizontal** direction, so the horizontal velocity remains **constant** $v \cos \alpha$ throughout the motion, and can be analysed using speed = distance / time, with the distance being the horizontal distance between the object's starting position and its current position, and the time being the time taken to reach this position.

The vertical velocity changes due to the **acceleration of free fall**, g . This means the **equations of linear motion** can be used to analyse the vertical motion. The initial velocity, u , is $v \sin \alpha$, and the acceleration is $-g$ (as the force of gravity acts in the opposite direction to the initial vertical velocity, we take it as negative). Because vertical motion is **symmetric**, if we work out how long it takes to reach maximum height (where final velocity, v , is equal to 0), then we can double this time to find the total time of motion.

Resolving the resultant velocities in both the horizontal and vertical planes, we will obtain a final horizontal and final vertical component of the motion. From these, the final velocity of the object can be calculated using **Pythagoras' theorem**.

Stopping distances

The stopping distance of a car is defined as the **total distance** travelled from when the driver of the car sees a hazard requiring them to stop, to the point they actually stop. It consists of two components, the **thinking distance** and the **braking distance**.

Thinking distance is the distance travelled between the moment the hazard is spotted, and the moment the driver applies the brake. The braking distance is the distance travelled from the time the brake is applied to the time the vehicle comes to a complete stop.

The thinking distance is proportionate to the **initial speed**, u , of the car. The thinking distance can be calculated by multiplying the initial speed of the car by the reaction time of the driver. The thinking distance is affected by the initial speed of the car, and factors that affect the **driver's reaction time**, such as tiredness, being under the influence of alcohol or drugs, and other distractions.

The braking distance of the car is proportionate to the **square of the initial speed of the car**, u^2 . This is because in order for the car to stop, all of its kinetic energy must be dissipated by the brakes, and kinetic energy = $\frac{1}{2}mv^2$. As well as the initial speed of the car, the braking distance is also increased by **poor road conditions** (e.g icy, wet), and the **car conditions** (e.g bald tyres, poor brakes, heavy load).

