

- Candidates should be able to :

- State the principle of conservation of momentum.
- Apply the principle of conservation of momentum to solve problems when bodies interact in one dimension.
- Define a perfectly elastic collision and an inelastic collision.
- Explain that whilst the **momentum** of a system is **always conserved** in the interaction between bodies, some change in **kinetic energy** usually occurs.

- **CONSERVATION OF MOMENTUM**

### THE PRINCIPLE OF CONSERVATION OF MOMENTUM

When bodies in a system interact, the **TOTAL MOMENTUM REMAINS CONSTANT** provided no external force acts on the system.

- FOR A COLLISION

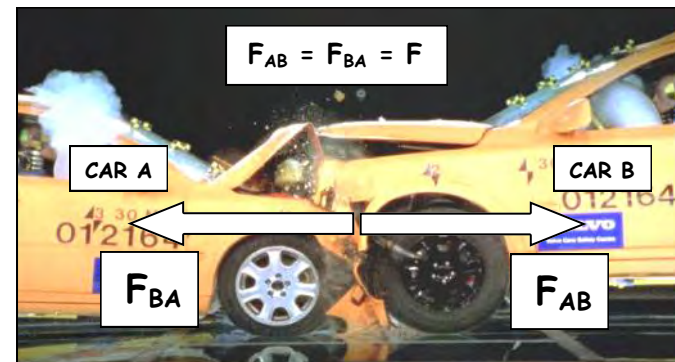
momentum before collision = momentum after collision

- FOR AN EXPLOSION

momentum after explosion = momentum before explosion

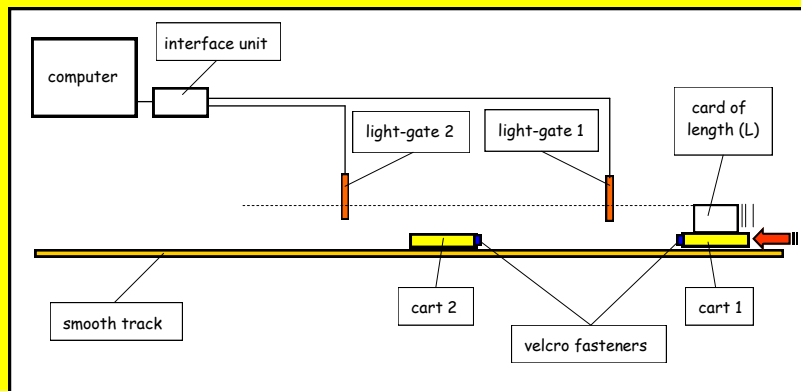
- The principle of conservation of momentum is deduced from **NEWTON'S second and third laws.**

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- If two bodies collide, then by **NEWTON III** each body exerts an equal and opposite force ( $F$ ) on the other.
- Since force ( $F$ ) acts on each body for the same time ( $\Delta t$ ), the two bodies will experience an equal and opposite impulse ( $F\Delta t$ ).
- Each body therefore experiences an equal and opposite change of momentum, which means that the total change of momentum of the two bodies is zero (i.e. **TOTAL MOMENTUM REMAINS CONSTANT**).
- The principle of conservation of momentum is universally true and it applies as much to the motion of galaxies as to the interaction of sub-atomic particles.

**EXPERIMENTAL VERIFICATION OF THE PRINCIPLE OF CONSERVATION OF MOMENTUM**



- The apparatus shown above can be used to test the validity of the principle of conservation of momentum.
- Cart 1** is pushed towards **stationary cart 2** and as it passes through **light-gate 1** the computer automatically calculates its speed ( $v_1$ ).  

$$v_1 = \frac{\text{length of card passing through the light-gate}}{\text{time taken for card to pass through light-gate}} = \frac{L}{t}$$
- When **cart 1** collides with **cart 2**, the velcro fasteners cause the carts to stick together.
- The two carts then pass through **light-gate 2** and their common speed ( $v_2$ ) is also calculated by the computer.
- The mass of each cart is measured using an electronic balance.

**RESULTS**

Mass of cart 1,  $m_1$  =  kg

Mass of cart 2,  $m_2$  =  kg

Speed of cart 1 before collision,  $v_1$  =   $\text{m s}^{-1}$

Speed of carts (1 + 2) after collision,  $v_2$  =   $\text{m s}^{-1}$

**CALCULATIONS**

Momentum before collision =  $m_1 v_1$  =  ×  =   $\text{kg m s}^{-1}$

Momentum after collision =  $(m_1 + m_2) v_2$  =  ×  =   $\text{kg m s}^{-1}$

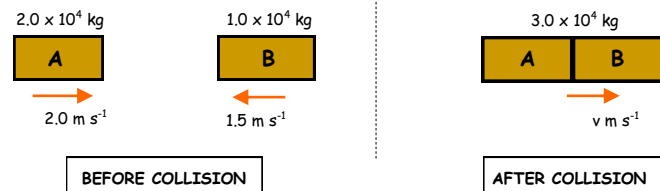
**SOME QUESTIONS**

- Is the **total momentum after the collision** equal to the **total momentum before the collision** ?
- Has the **principle of conservation of momentum** been verified ?
- Did any **external force** act on the system of colliding carts ? If so, **name** the external force.

## MOMENTUM CONSERVATION - WORKED EXAMPLES

## COLLISION

A railway truck (A) of mass  $2 \times 10^4 \text{ kg}$  travelling at  $2.0 \text{ m s}^{-1}$  collides with a second truck (B) of mass  $1.0 \times 10^4 \text{ kg}$  moving in the opposite direction at  $1.5 \text{ m s}^{-1}$ . If the trucks **couple** automatically on impact, calculate the **common velocity** with which they move after the collision.

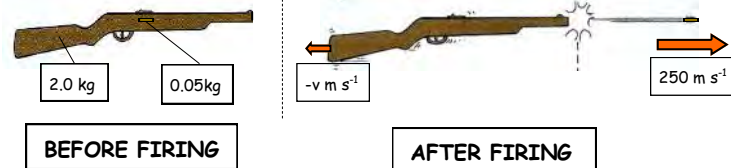


$$(2.0 \times 10^4 \times 2.0) + (1.0 \times 10^4 \times -1.5) = (3.0 \times 10^4 \times v)$$

$$v = \frac{2.5}{3.0} = \boxed{0.83 \text{ m s}^{-1}}$$

## EXPLOSION

A bullet of mass  $50 \text{ g}$  is fired from a rifle of mass  $2.0 \text{ kg}$  with a muzzle velocity of  $250 \text{ m s}^{-1}$ . What is the **initial recoil velocity** of the rifle?



$$0 = (0.05 \times 250) + (2.0 \times -v)$$

$$v = \frac{12.5}{2.0} = \boxed{6.25 \text{ m s}^{-1}}$$

- 1 A fully-laden Range Rover of total mass  $2200 \text{ kg}$  travelling at  $20 \text{ m s}^{-1}$  collides with a Mini Cooper of mass  $850 \text{ kg}$ .

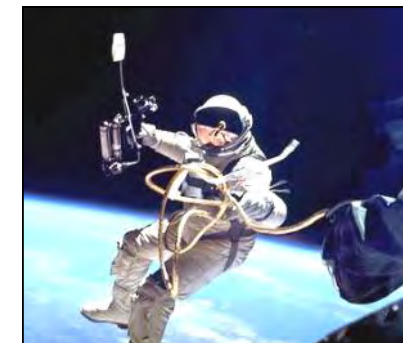
(a) Assuming that the two vehicles separate after the impact, calculate the Range Rover's velocity after impact if :

(i) The Mini is **initially stationary** and its velocity after the collision is  $10 \text{ m s}^{-1}$ .

(ii) The Mini's initial velocity is  $8 \text{ m s}^{-1}$  in the **same direction** as the Range Rover and its velocity after the collision is  $18 \text{ m s}^{-1}$ .

(b) If the Mini's initial velocity is  $12 \text{ m s}^{-1}$  in the **opposite direction** to that of the Range Rover and the two vehicles become **stuck together** as a result of the impact, calculate their **combined velocity** after the collision.

- 2 An astronaut who is about to attach a replacement part to the Hubble Space Telescope suddenly realises that the tether which keeps him linked to his space vehicle has broken loose. In an attempt to get back to his vehicle, he throws the replacement part into space.



The astronaut's total mass is  $175 \text{ kg}$  and that of the part is  $5.5 \text{ kg}$ . Calculate the astronaut's **velocity** if he throws the part with a velocity of  $1.5 \text{ m s}^{-1}$ .

- TYPES OF COLLISION**

- In any collision between two objects :

- TOTAL MOMENTUM IS ALWAYS CONSERVED**  
(so long as no external force acts).

$$\text{i.e. TOTAL MOMENTUM AFTER} = \text{TOTAL MOMENTUM BEFORE COLLISION}$$

- TOTAL ENERGY IS ALWAYS CONSERVED**  
(The principle of conservation of energy states that energy cannot be created or destroyed, only converted from one form into another).

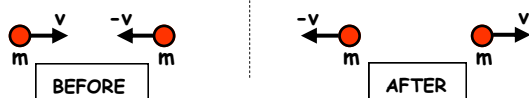
$$\text{i.e. TOTAL ENERGY AFTER} = \text{TOTAL ENERGY BEFORE COLLISION}$$

- KINETIC ENERGY 'MAY OR MAY NOT' BE CONSERVED**  
This depends on whether the collision is **ELASTIC** or **INELASTIC**.

A **PERFECTLY ELASTIC COLLISION** is one in which kinetic energy is conserved.

$$\text{i.e. TOTAL KINETIC ENERGY} = \text{TOTAL KINETIC ENERGY AFTER COLLISION BEFORE COLLISION}$$

In a **PERFECTLY ELASTIC COLLISION** :



$$\text{VELOCITY OF APPROACH OF COLLIDING OBJECTS} = \text{VELOCITY OF SEPARATION OF COLLIDING OBJECTS}$$

An **INELASTIC COLLISION** is one in which kinetic energy is not conserved.

$$\text{i.e. TOTAL KINETIC ENERGY AFTER COLLISION} < \text{TOTAL KINETIC ENERGY BEFORE COLLISION}$$

**POINTS TO NOTE**

- The collisions between gas molecules in an 'ideal' gas and those between electrons and the molecules of a superconductor are assumed to be **PERFECTLY ELASTIC**.

- The collision between two steel ball bearings in the executive toy called 'Newton's Cradle' is a close approximation to a **PERFECTLY ELASTIC** collision, but some kinetic energy is transferred to sound energy, heat energy and work done in plastic deformation. Once the toy is set in motion, the balls continue to move for some time, but eventually the initial kinetic energy is transferred to other forms and the balls come to rest.



Another close approximation to a **PERFECTLY ELASTIC** collision is that between two snooker balls.



- In practice all collisions are, to some extent, **INELASTIC** (i.e. some of the initial kinetic energy is transferred to other energy forms).

Car **CRUMPLE ZONES** are specifically designed to make a collision more **INELASTIC** and so absorb the kinetic energy in a crash.



The same thinking is used in the design of **MOTORWAY CRASH BARRIERS**.



A **PERFECTLY INELASTIC** collision is one in which **ALL** the initial kinetic energy is transferred to other energy forms.

$$\text{KINETIC ENERGY AFTER COLLISION} = 0$$

- A close approximation to a **PERFECTLY INELASTIC** collision is that which would occur between two pieces of soft dough of **equal mass** moving towards each other with **equal speed**. On collision, all the kinetic energy of each piece is transferred to sound energy, heat energy and work done in plastic deformation. So the pieces 'splat' into each other, deform into one mass of dough, heat up and come to a standstill.

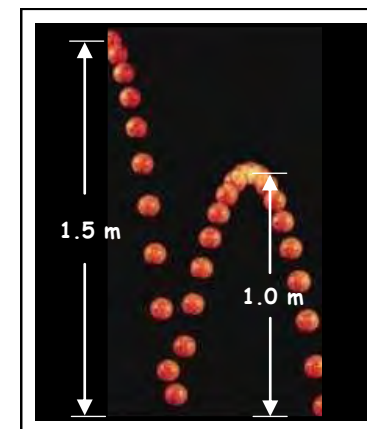
- In a school experiment, a cart of mass **2.5 kg** moving with a velocity of **0.5 m s<sup>-1</sup>** collides with a second, **stationary** cart of mass **3.0 kg**. The velcro fastenings attached to the cart ends causes them to stick Together on impact and they move off with a common velocity of **0.227 m s<sup>-1</sup>**.

- Show that **momentum is conserved** in this collision.
- Calculate the **kinetic energy before** and **after** the collision.
- Explain** whether the collision is **elastic** or **inelastic**.

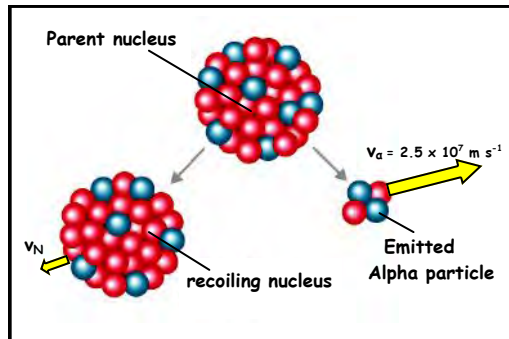
- A tennis ball of mass **0.06 kg** is dropped from a height of **1.5 m** above a hard surface and it rebounds to a height of **1.0 m** above the surface.

- Calculate the **kinetic energy lost** on impact with the surface.

- Explain** whether this collision is **elastic** or **inelastic**.



- 3 A stationary radioactive nucleus of mass  $3.98 \times 10^{-25} \text{ kg}$  undergoes radioactive decay in which it emits an alpha particle of mass  $6.68 \times 10^{-27} \text{ kg}$ .

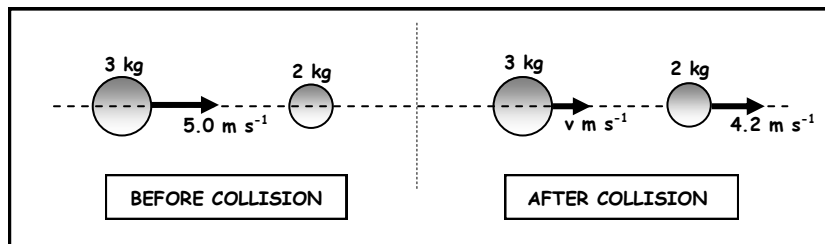


- (a) If the speed of the alpha particle is  $2.5 \times 10^7 \text{ m s}^{-1}$ , what is the speed of the recoiling nucleus?

(HINT : The mass of the recoiling nucleus = mass of the parent nucleus - mass of the alpha particle).

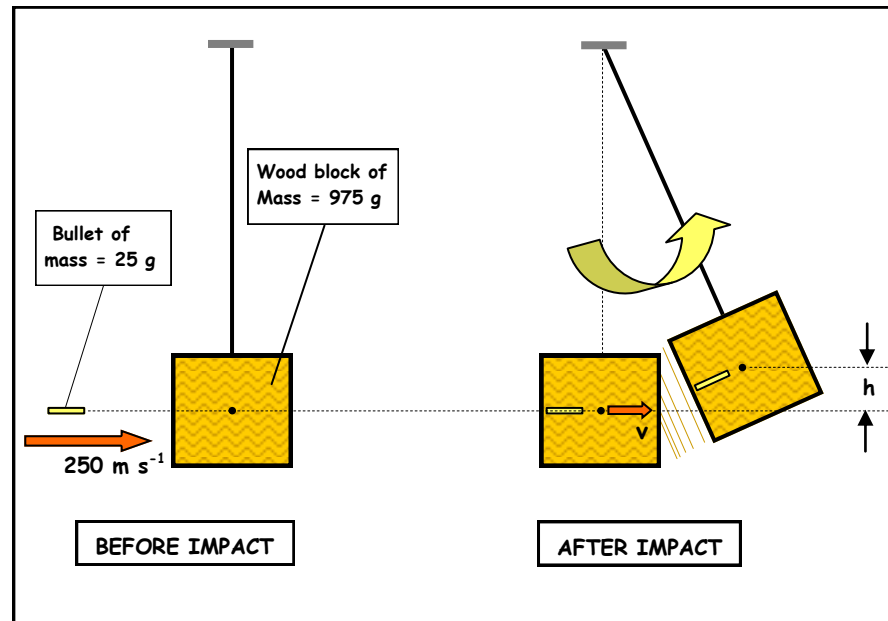
- (b) If all the energy released in this decay is the sum of the kinetic energies of the alpha particle and the recoil nucleus, calculate the **alpha particle's kinetic energy as a fraction of the total energy released**.

4



A **3 kg** sphere moving with a velocity of  $5.0 \text{ m s}^{-1}$  collides head-on with a stationary sphere of mass **2 kg**. If the **2 kg** sphere moves off with a velocity of  $4.2 \text{ m s}^{-1}$ , calculate :

- (a) The **velocity (v)** of the **3 kg** sphere after the collision.
- (b) The **kinetic energy lost** as a result of the collision.



A bullet of mass **25 g**, travelling horizontally at a speed of  $250 \text{ m s}^{-1}$  strikes and embeds itself in a stationary wooden block which is suspended so that it can swing freely.

- (a) Calculate :
- The **common speed** of the block and embedded bullet immediately after impact.
  - The **maximum vertical height** through which the block rises as a result of the impact.
- (b) How much of **the bullet's initial kinetic energy** is lost as a result of the impact and into what forms is this 'lost' kinetic energy transferred?
- (c) **Explain** whether this collision is **elastic** or **inelastic**.

This question is about the interactions between three **identical, perfectly elastic, solid** cubes. 7

• **HOMEWORK QUESTIONS**

- 1 (a) (i) Define the **momentum** of a body.
- (ii) A body, initially at rest, explodes into two unequal fragments of mass  $m_1$  and  $m_2$ . Mass  $m_1$  has a velocity  $v_1$  and mass  $m_2$  has a velocity  $v_2$ . Using the principle of conservation of momentum, **derive an expression for  $v_1/v_2$** .
- (b) An isolated nucleus of mass  $4.0 \times 10^{-25}$  kg is initially at rest. It decays, emitting an alpha particle of mass  $6.7 \times 10^{-27}$  kg with a kinetic energy of  $1.2 \times 10^{-14}$  J.

(i) Show that the speed of the alpha particle is about  $2 \times 10^6$  m s<sup>-1</sup>.

(ii) Calculate the **momentum of the alpha particle**.

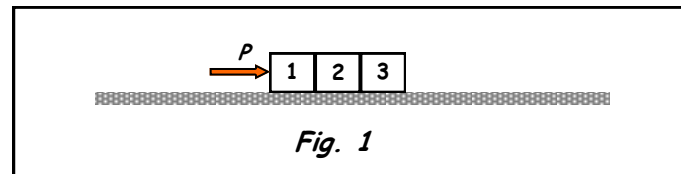
(iii) Hence find the **speed of the recoiling nucleus**.

- (c) Before the decay described in (b) The nucleus is at **point P** as shown below.

• P

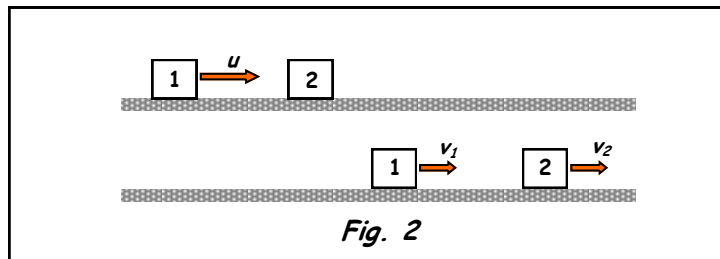
- (i) Place a small cross at a possible position, **to full scale**, of the alpha particle  $8.0 \times 10^{-9}$  s after emission.
- (ii) Indicate with an arrow, starting at P, the **direction of movement of the recoiling nucleus**.
- (iii) Estimate how far the recoiling nucleus has moved in  $8.0 \times 10^{-9}$  s.

*(OCR A2 Physics - Module 2824 - June 2004)*



- (i) Show that the acceleration of cube 3 is  $P/3m$ .
- (ii) Write down an expression for the **resultant force  $F_3$**  on cube 3.
- (iii) Write down expressions in terms of  $P$  for :
1. The **acceleration,  $a_2$**  of cube 2.
  2. The **resultant force,  $F_2$**  on cube 2.
- (iv) Hence write down an expression for the magnitude of the **force** applied by :
1. Cube 3 on cube 2,  $F_{32}$ .
  2. Cube 1 on cube 2,  $F_{12}$ .

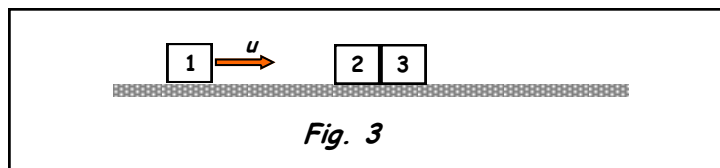
(b) Cube 3 is removed. Cube 1 is moved to the left and then projected towards the stationary cube 2 with speed  $u$ . Suppose that after collision cube 1 moves with speed  $v_1$  and cube 2 moves with speed  $v_2$ . See Fig. 2.



(i) Write down equations for the **conservation of momentum** and of **energy** in terms of  $m$ ,  $u$ ,  $v_1$ , and  $v_2$ .

(ii) Put the values  $v_1 = 0$  and  $v_2 = u$  into your equations in (i) and show that they are solutions.

(c) Cube 3 is replaced in its original position close to cube 2. Cube 1 is moved to the left and projected towards cube 2 with speed  $u$ . See Fig. 3.



(i) After the collision, cubes 1 and 2 remain **at rest** and cube 3 moves with speed  $u$ . **Explain** these observations.

(ii) Cubes 2 and 3 are now glued together. **Describe without calculation or explanation** what happens in this situation when cube 1 is projected towards cube 2 as shown in Fig. 3.

3 In a motorway collision, a lorry of mass **3500 kg** moving at a speed of **24 m s<sup>-1</sup>** rammed the rear end of a car of mass **1000 kg** which was travelling in the **same direction** as the lorry with a speed of **14 m s<sup>-1</sup>**. After the impact, the car shot forward at a speed of **20 m s<sup>-1</sup>**. 8

(a) Use the principle of conservation of momentum to calculate the **speed of the lorry after the collision**.

(b) (i) Calculate the **total kinetic energy** before and after the collision.

(ii) Explain whether the collision is *elastic* or *inelastic*.

4 A man of mass **95 kg** jumps off a landing stage into a rowing boat of mass **250 kg** which is untied and lying stationary in the water. If the man's velocity as he lands in the boat is **4.0 m s<sup>-1</sup>** at an angle of **30° to the vertical**, calculate the **speed** with which the man and boat move off together.

5 (a) Define the **momentum** of a particle. State the principles of conservation of **linear momentum** and of the conservation of **energy** as applied to head-on collisions between particles. **Explain** the conditions under which linear momentum and kinetic energy are conserved.

(b) Use one or both of the principles in (a) to explain why in **elastic** collisions a small particle **bounces back** from a massive particle (Fig. 1), whereas a large massive particle incident on a small particle **does not bounce back** (Fig. 2).

