- ***Candidates should be able to :***

  - ***State*** *and* ***use*** *each of* ***Newton's three laws of motion.***

  - ***Define linear momentum*** *as the product of mass and velocity and appreciate the* ***vector nature*** *of momentum.*

  - ***Define net force*** *on a body as equal to the* ***rate of change of its momentum.***

  - ***Select*** *and* ***apply*** *the equation* $F = \frac{\Delta p}{\Delta t}$ *to solve problems.*

  - ***Explain*** *that* $F = ma$ *is a special case of* ***Newton's second law*** *when mass (m) remains constant.*

  - ***Define IMPULSE*** *of a force.*

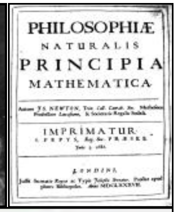
  - ***Recall*** *that the area under a* ***FORCE*** *against* ***TIME*** *graph is equal to* ***IMPULSE****.*

  - ***Recall*** *and* ***use*** *the equation :*

IMPULSE = CHANGE IN MOMENTUM

- ***NEWTON'S LAWS OF MOTION***

These three laws, published by ***Sir Isaac Newton*** *in 1687 in his* ***Philosophiae Naturalis Principia Mathematica,*** *were developed from very careful observation of the real world. They describe the effects of forces on the motion of bodies and as such, form the basis of the science of* ***KINETICS.***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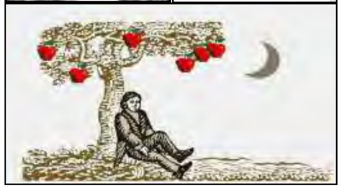


NEWTON'S FIRST LAW (NEWTON I)

All objects will continue to be ***STATIONARY (AT REST)****, or to move with* ***CONSTANT VELOCITY*** *unless they are acted on by a* ***RESULTANT FORCE.***

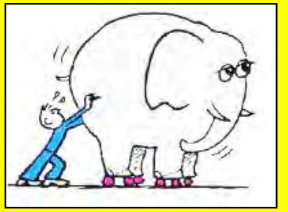
POINTS TO NOTE

- ***CONSTANT VELOCITY*** *means that there is no change in the object's* ***SPEED*** *or* ***DIRECTION OF MOTION****.*

- ***ZERO RESULTANT FORCE*** *means that the combined effect of all the forces acting on the object is zero.*

- ***FORCE*** *may then be said to be something which changes or tries to change the state of* ***REST*** *or* ***UNIFORM MOTION*** *of an object, either through* ***CONTACT*** *or* ***'FIELD' ACTION*** *(i.e. gravity, electric or magnetic fields).*

- *A **STATIONARY** object will only move when a **RESULTANT FORCE** acts on it.*



*A **MOVING** object will continue to move at constant speed in a straight line until a **RESULTANT FORCE** acts on it.*

*This **opposition to any change in speed and/or direction**, which all objects have, is called **INERTIA**.*
*The **greater the mass** of an object , the **greater is its Inertia**.*

An astronaut involved in a 'space-walk' has to be tethered to his spacecraft. In the frictionless environment of outer space, the smallest of pushes would start him moving into space and without some kind of jet propulsion unit the astronaut would be unable to change his motion.

NEWTON'S SECOND LAW (NEWTON II)

*The **RATE OF CHANGE OF MOMENTUM** of an object is directly proportional to the applied **RESULTANT FORCE** and occurs in the **DIRECTION OF THE RESULTANT FORCE**.*

POINTS TO NOTE

LINEAR MOMENTUM = MASS x VELOCITY

$$p = m \times v$$

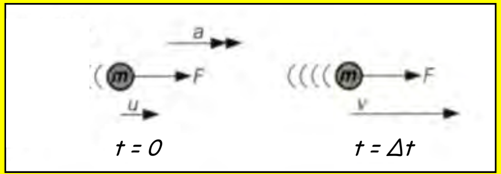
p (kg m s^{-1}), m (kg), v (m s^{-1})

*Momentum is a **vector** quantity having both **magnitude** and direction. So if momentum to the right is considered **positive**, then momentum to the left is **negative**.*

- ***FORCE** is something which can act on an object and so cause its momentum to change. The larger the force the greater is the rate at which the object's momentum changes.*

The harder the ball is struck, the greater is its rate of change of momentum and the more difficult it is for the keeper to stop.

- *Consider a body of **mass (m)** which is acted on by a constant **resultant force (F)** and moves with constant **acceleration (a)** from a **velocity (u)** to a **velocity (v)** in a **time (Δt)**.*



Momentum change, $\Delta p = p_f - p_i = mv - mu = m(v - u)$

Rate of change of momentum, $\dfrac{\Delta p}{\Delta t} = \dfrac{m(v - u)}{\Delta t} = ma$

*From **NEWTON II** :*
resultant force (F_R) a *rate of change of momentum (= ma)*

Therefore : $F_R = kma$ *(k = constant of proportionality)*

Using the fact that :

> ***1 NEWTON (N)** is the resultant force which gives a mass of **1 KILOGRAM (kg)** an acceleration of **1 METRE PER SECOND**^{2} **(m s^{-2}).***

And substituting into $F_R = kma$
$1 = k \times 1 \times 1$ *So* $k = 1$

$$F_R = ma$$

(N) **(kg)** **(m s^{-2})**

- *The equation used to calculate the **WEIGHT (W)** of an object of **MASS (m)** in a gravitational field where there is an **ACCELERATION DUE TO GRAVITY (g)** :*

$$W = mg$$

is really just a version of $F = ma$.

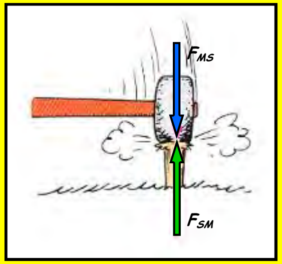
NEWTON'S THIRD LAW (NEWTON III)

> *If a body **A** exerts a force on another body **B**, then **B** exerts an **equal** force on **A** in the **opposite direction**.*
> **OR**
> ***ACTION** and **REACTION** are equal and opposite, but they act on **DIFFERENT** bodies.*

POINTS TO NOTE

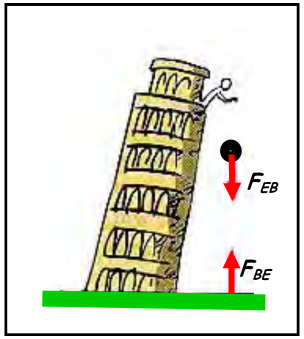
- *Forces cannot act singly, they always **ACT IN PAIRS**, but they act on **DIFFERENT** bodies.*

*Force of **mallet** on **stake**,* F_{MS} = *Force of **stake** on **mallet**,* F_{SM}



- *Consider a ball which is dropped from The leaning tower of Pisa.*

*Force of the **Earth** = Force of the **ball***
on ball, F_{EB} ***on Earth, F_{BE}***

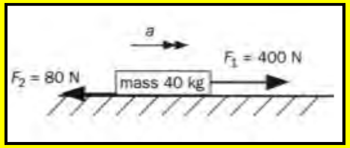


*As we know from observation, the ball accelerates towards the Earth. What may not be so obvious is that the Earth also accelerates towards the ball. Of course, because of the enormous difference in their masses, the Earth's acceleration is **infinitesimally small**.*

- ***NEWTON'S LAW PROBLEM TYPES***

1. RESULTANT FORCE

*A force of **400 N** is used to pull a box of mass **40 kg** against a constant frictional force of **80 N** as shown in the diagram opposite. Calculate the **acceleration** of the box.*



<u>From NEWTON II</u> : resultant force = mass x acceleration

$$F_1 - F_2 = ma$$

$$(400 - 80) = 40a$$

$$a = \frac{320}{40} = \underline{8\ ms^{-2}}$$

2. LIFT PROBLEM

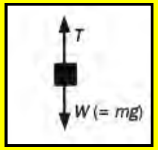
*A mass of **2 kg** is attached to a spring balance which is hung from the ceiling in a lift. Calculate the reading on the spring balance when the lift is :*

*(a) **Stationary**.*
*(b) **Accelerating upwards at 0.2 m s^{-2}**.*
*(c) **Accelerating downwards at 0.1 m s^{-2}**.*
*(d) **Moving up with a constant velocity of 0.15 m s^{-1}**.*

*Assume that **acceleration due to gravity, g = 10 m s^{-2}**.*

The two forces acting on the mass are :

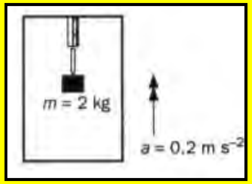
- The **weight, W (=mg)** acting downwards.
- The spring **tension, T** acting upwards.



(a) **<u>Lift stationary</u>**

$$F_R = ma$$
$$T - W = m \times 0 = 0$$
$$T = W = mg = 2 \times 10 = \underline{20\ N}$$

(b) **<u>Lift accelerating upwards</u>**



Since the lift is accelerating **upwards**, the resultant force must be **upwards**.

$$T - W = ma$$
$$T - mg = ma$$
$$T = m(g + a)$$
$$T = 2(10 + 0.2) = \underline{20.4\ N}$$

*This result accounts for the sensation felt when a lift starts to ascend or comes to a halt when descending. In each case the acceleration is upwards and we experience a momentary **increase** in the contact force between our feet and the floor.*

(c) **Lift accelerating downwards**

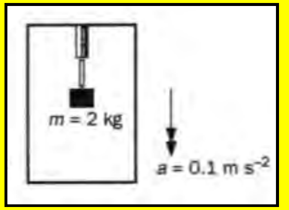
Since the lift is accelerating downwards, the resultant force is downwards.

$F_R = ma$

$W - T = mg - T = ma$

$T = m(g - a)$

$T = 2(10 - 0.1) =$ **19.9 N**



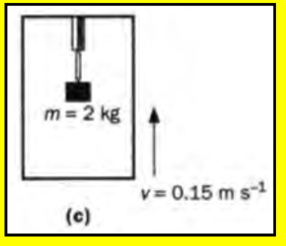
Passengers in a lift which is starting to descend or comes to a halt when it is ascending, experience a momentary decrease in the contact force between their feet and the floor.

(d) **Lift moving with constant velocity**

$F_R = ma$

$T - W = m \times 0 = 0$

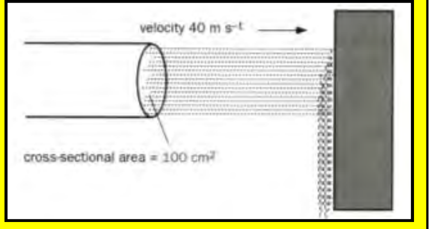
$T = W = mg = 2 \times 10 =$ **20 N**



Passengers in a lift which is either ***stationary*** *or moving with* ***constant velocity*** *experience a* ***steady*** *contact force with the floor equal to their weight.*

3. HOSEPIPE PROBLEM

A hosepipe of cross-sectional area ***0.01 m*** ^{2} *ejects a jet of water horizontally at a speed of* ***40 m s*** $^{-1}$*. The water strikes a wall perpendicularly and runs down the wall without rebounding, as shown in the diagram opposite.*



Calculate the ***force exerted on the wall.***
(density of water = ***1 x 10*** ^{3} ***kg m*** $^{-3}$*)*

From **NEWTON II** :

force exerted by the wall on the water = rate of change of momentum of the water

= mass of water striking wall per second × velocity change of water

= (volume of water/s) × (water density) × (velocity change of water)

$= (40 \times 0.01) \times (1 \times 10^3) \times (40 - 0)$

$=$ **1.6×10^3 N**

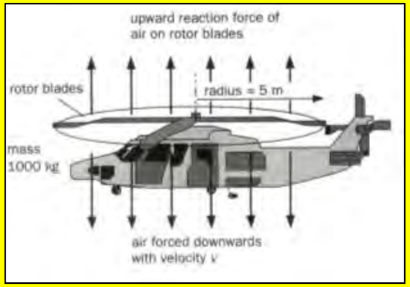
From **NEWTON III**, there must be an equal and opposite force exerted by the water on the wall.

Therefore, force exerted on the wall = **1.6×10^3 N**

4. HELICOPTER PROBLEM

*Rotating helicopter blades force air downwards and from **NEWTON III** there is an equal and oppositely directed force exerted by the air on the blades. A helicopter can hover in mid-air when this upward force is equal to its weight. The same principle applies to all forms of hovering flight, from the natural wonders of the bumble bee and humming bird to the technological brilliance of the harrier, vertical take-off aircraft.*

*A hovering helicopter of mass **2000 kg** is shown in the diagram opposite. If the length of each rotor blade is **5 m**, calculate the velocity of the air forced downwards by the rotor in order that the helicopter hovers in mid-air.*



*density of air = **1.3 kg m** $^{-3}$.*
*Acceleration due to gravity, g = **10 m s** $^{-2}$.*

From **NEWTON II** :

upward force of air on blades = rate of change of momentum of air forced down by blades = helicopter weight

helicopter weight = (mass of air forced down per sec) x (velocity change of air being forced down)

Helicopter weight = (volume of air forced down per sec) x (density of air) x (velocity change of air being forced down)

$$mg = \pi R^2 v \times \rho \times v$$

$$v^2 = \frac{mg}{\pi R^2 \rho} = \frac{2000 \times 10}{\pi \times 5^2 \times 1.3} = 195.9$$

$$v = \sqrt{195.9} = \underline{14 \text{ m s}^{-1}}$$

1 A rocket has a total mass of **3.0 x 10^{6} kg** of which **1.2 x 10^{5} kg** is its initial fuel load at take-off. If its engines can provide a thrust of **3.0 x 10^{7} N,** calculate :

(a) The **initial acceleration** at take-off.

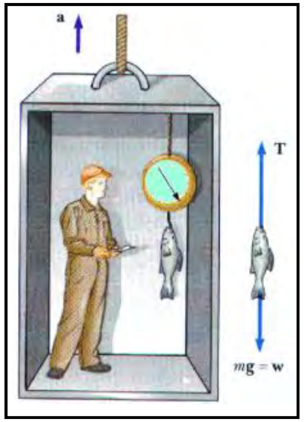
(b) The **acceleration** when **1.0 x 10^{5} kg** of the fuel has been used up.

2 A submarine of mass **5.0 x 10^{6} kg** is moving with a velocity of **8.5 m s^{-1}** while fully submerged. The power is suddenly shut off, and the submarine takes **5.5 minutes** to come to rest. Calculate :

(a) The **average deceleration.**

(b) The **average decelerating force.**

3 A fisherman is weighing his fish inside the lift of a tall building. If he hooks a sea bass of mass **4.0 kg** onto the spring balance, calculate the indicated **weight** when the lift is :



(a) **Stationary.**

(b) **Accelerating upwards at 2.5 m s^{-2}.**

(c) **Accelerating downwards at 3.0 m s^{-2}.**

(d) **Moving upwards at a constant velocity of 4.0 m s^{-1}.**

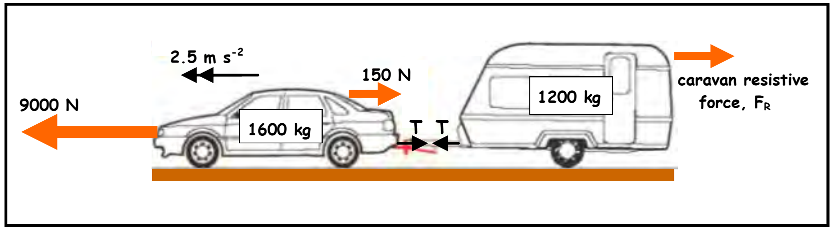
4 In a severe storm, the wind is blowing with a velocity of **30 m s^{-1}** perpendicular to a barn wall of area **80 m^2.** Assuming that the air Moves parallel to the wall after striking it, calculate :

(a) The **force** acting on the wall.

(b) The **pressure** exerted on the wall. (Density of air = **1.3 kg m^{-3}**)

5 A large hoverfly of mass **1.2 g** suspends itself in mid-air by using its wings to push air downwards. If the total area swept out by the beating wings is **1.5 cm^2**, calculate the **velocity** of the air which is being pushed downwards.
(Density of air = **1.3 kg m^{-3}**)

6



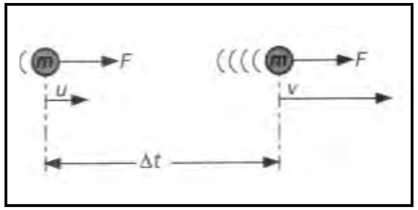
A car of mass **1600 kg** is towing a caravan of mass **1200 kg** along a straight, level road. The car engine provides a motive force of **9000 N** and makes the car accelerate with a constant acceleration of **2.5 m s^{-2}**. If the total resistive force acting on the car due to Road friction and air resistance is 150 N, calculate :

(a) The total **resistive force (F_R)** acting on the caravan.

(b) The **tension (T)** in the rigid tow-bar linking the car and caravan.

MOMENTUM CHANGE DUE TO A CONSTANT FORCE

*Consider a body of **mass (m)** which is acted on by a **constant force (F)** for a **time (Δt)** and so changes its velocity from an initial value **(u)** to a final value **(v)** as shown in the diagram opposite.*



From __NEWTON II__ :

Resultant force = rate of change of momentum

$$F = \frac{mv - mu}{\Delta t}$$

Therefore :

$$\boldsymbol{F\Delta t = mv - mu}$$

(kg) — m; (N) — F; (s) — Δt; (m s^{-1}) — v, u

IMPULSE = RESULTANT FORCE x TIME = MOMENTUM CHANGE

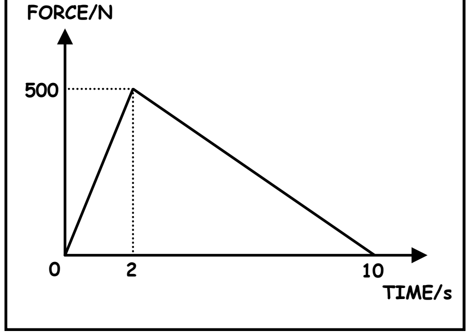
(N s or kg m s^{-1}) (N) (s) (kg m s^{-1})

MOMENTUM CHANGE DUE TO A VARYING FORCE

The analysis for momentum change produced by a constant force is also applicable to that due to a varying force because the product ***FΔt*** *can be thought of as being equal to the* ***AREA ENCLOSED*** *by a graph of* ***FORCE*** *against* ***TIME****.*

EXAMPLE

A body is acted upon by a force which varies with time as shown in the **FORCE-TIME** graph shown opposite.



Use the graph to calculate the **impulse** given to the body in a time of **10 s.**

What is the **momentum gained by the body ?**

Impulse = change of momentum = Area enclosed by the F/t Graph

= ½ x 10 x 500

= 2500 N s (or kg m s^{-1})

IMPLICATIONS OF IMPULSE AND CHANGE OF MOMENTUM

The ***impulse*** *needed to decelerate a moving object and bring it to rest, or to accelerate a stationary object from rest can be provided either by a* ***small force*** *for a* ***long time*** *or a* ***large force*** *for a* ***short time.***

LARGE Δt means SMALL F for a given momentum change

- *Crickets try to slow down a ball* ***gradually*** *when catching it and so exert a* ***small force*** *for a* ***long time.*** *Stopping it too rapidly needs a large force and that hurts!*

- *Cars have* ***crumple zones*** *designed to collapse progressively on impact. This increases the time taken for the car to come to rest in a crash, thereby reducing the force exerted on the car and its occupants.*

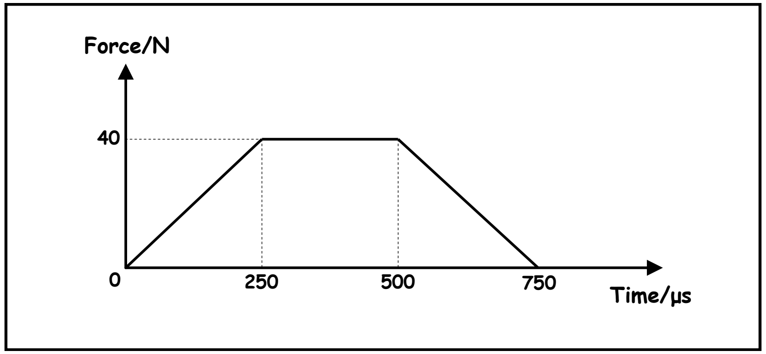
- *Airmen whose parachutes have failed to open have sometimes survived falling thousands of metres by landing in deep snow, on steep hillsides. The time taken for them to come to rest is increased and this means that the force exerted on them is small enough to cause only slight injury.*

- *In any sport where balls are struck (golf, tennis, football, etc.), the player trying for maximum speed always tries to* ***follow-through*** *so as to* ***increase the contact time*** *between the club, racquet, foot, etc.*

  This maximises the impulse given to the ball, producing a greater change in momentum.

- **PRACTICE QUESTION (2)**

1



A force (F) acts on a body which is initially stationary. The graph shows how F varies with time (t).

(a) sketch a **velocity/time** graph for the **750 μs** period and **explain** its shape.

(b) **Explain** what the **area enclosed by the FORCE/TIME graph** Represents.

(c) Use the graph to calculate the **momentum gained** by the body.

- **HOMEWORK QUESTIONS**

1 In a motorway accident, a small car of mass **800 kg** which was moving at **5 m s^{-1}** was struck from behind by a larger vehicle. The impact which lasted **0.8 s** made the speed of the car increase to **9.5 m s^{-1}.**

Calculate : (a) The **change of momentum** of the car which results from the collision.

(b) The **impact force (impulse)** on the car.

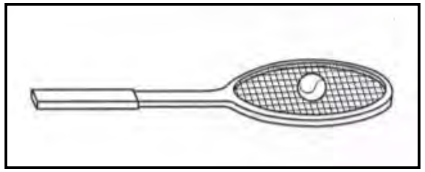
2 A neutron of mass **1.67 x 10^{-27} kg** moving with a velocity of **10^{4} m s^{-1}** collides perpendicularly with a rigid surface and rebounds with the same speed in the opposite direction. If the collision time is **25 ns,** calculate :

(a) The **momentum change** experienced by the proton.

(b) The **force** experienced by the proton.

(c) The **momentum change** and the **force** experienced by the proton if it collides with the same surface at an angle of **75° to the surface.** (**Hint** : You will need to calculate the proton velocity perpendicular to the surface before and after impact).

3 (a) The diagram opposite shows a ball of mass **0.050 kg** resting on the strings of a tennis racquet held horizontally.



(i) **Draw and label** the two forces acting on the ball.

(ii) Each of these forces has a corresponding equal and opposite force according to **Newton's third law of motion. Describe** these equal and opposite forces and **state the objects on which they act.**

(iii) Calculate the **difference in magnitude between the two forces on the ball** when the racquet is accelerated upwards at **2.0 m s^{-2}.**

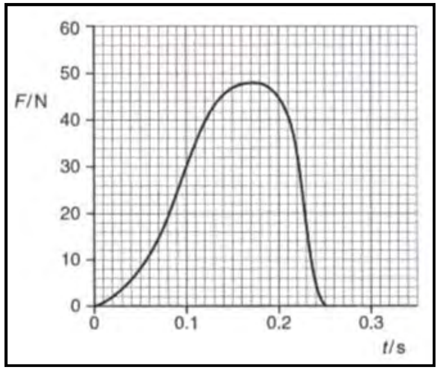
(b) The ball is dropped **from rest** at a point **0.80 m** above the racquet head. The racquet is fixed rigidly. Assume that the ball make an elastic collision with the strings and that any effects of air resistance are negligible. Calculate :

(i) The **speed** of the ball just before impact.
(ii) The **momentum** of the ball just before impact.
(iii) The **change in momentum** of the ball during the impact.
(iv) The **average force** during the impact for a contact time of **0.050 s.**

(OCR A2 Physics - Module 2824 - January 2003)

4 This question is about kicking a football.

(a) The graph shows how the **force (F)** applied to the ball varies with **time (t)** whilst it is being kicked horizontally. The ball is initially at rest.



(i) Use the graph to find :

1. The **maximum force** applied to the ball.
2. The **time** the boot is in contact with the ball.

(ii) The mean force multiplied by the time of contact is called the **impulse** delivered to the ball. Use the graph to estimate the **impulse** delivered to the ball.

(b) The mass of the ball is 0.50 kg. Use your answers to (a) to calculate :

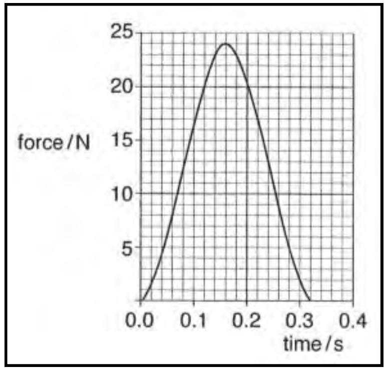
(i) The maximum acceleration of the ball.
(ii) The final speed of the ball.
(iii) The kinetic energy of the ball after the kick.

(c) The ball hits a wall with a speed of **14 m s^{-1}.** It rebounds from the wall along its initial path with a speed of **8 m s^{-1}.** The impact lasts for **0.18 s.** Calculate the **mean force** exerted by the ball on the wall.

(OCR A2 Physics - Module 2824 - June 2005)

5 A cricketer throws a cricket ball of mass **0.16 kg.**

(a) The graph shows how the **force** on the ball from the cricketer's hand varies with **time.** The ball starts from rest and is thrown horizontally.



(i) Estimate the **area under the graph.**

(ii) The area under the graph represents a change in a physical quantity for the ball. **State** the name of this quantity.

(iii) Calculate the **speed** of the ball, mass **0.16 kg,** when it is released.

(iv) Calculate the **maximum horizontal acceleration** of the ball.

(Part question - OCR A2 Physics - Module 2824 - January 2006)