G484: The Newtonian World

Newtonian Laws of Motion

Newton's First Law

- **An object will remain at rest or keep travelling at constant velocity unless it is acted on by an external force.**

Change in momentum when something bounces uses two negatives so is usually $p_1 - -p_2 = p_1 + p_2$

Example:
Find p_i (for $1 \leq i \leq 2$) when a 7 and a half tonne minibus going at 25 meters a second (p_1) and a 0.8g wasp travelling in opposite directions at the same speed (p_2)
$p_1 = mv = 7.5x10^3x25 = 1.9x10^5 kgms^{-1}$
$p_2 = mv = 0.8 \text{ x } 10^3x\text{-}25 = -2x10^{-2} \text{ } kgms^{-1}$

Newton's Second Law

Momentum is defined as mass x velocity and is vector

- *Net Force* α Rate of Change of Momentum
- *In SI units, constant of proportionality is 1 => Net Force = Rate of Change of Momentum*
- $F = \frac{\Delta p}{\Delta t}$
- **The net force acting on an object is equal to the rate of change of the linear momentum of that object. The net force and the change in momentum are in the same direction.**

In the case where mass is constant, F=ma is derived.

Example:
A 1200kg car is travelling on a road at $32ms^{-1}$. Brakes applied for 1.5 seconds. Final speed is $10ms^{-1}$. Find the magnitude of the average force applied.
$F = \frac{\Delta p}{\Delta t}$ *so ((1200x32)-(1200x10))/1.5 =* $1.8x10^4N$

Newton's Third Law

- **When two objects interact, the forces they exert on each other are equal in magnitude and opposite in direction.**

Re-arranging the general equation from Newton's second law, we get F x Δt = Δp. The LHS is equal to the impulse of the force; **Impulse = force x time = ΔMomentum**

When you have a force against time graph, the area underneath is impulse.

Measured in Ns or $kgms^{-1}$. Important when force on an object is not constant. Area under the graph of force and time is obviously impulse/ change in momentum.

Example:
A 900kg car is travelling at $12ms^{-1}$. A constant force of 1800N acts on the car for 10s. Find v_f.
ΔMomentum = Impulse = F x time = 1800 x 10 = 18000 $kgms^{-1}$. P_o = 900x12 = 10800, P_f = 18000+10800 =28800. V_f = p_f/m = 28800/900 = 32 ms^{-1}.

Collisions

The principle of the conservation of momentum states **In a closed system, the total momentum of the objects before the collision is equal to the total momentum of the objects after the collision.**
This principle is derived from Newton's Second and Third laws.

Elastic and Inelastic Collisions

In all collisions, energy and momentum are conserved. Energy may be transformed to other forms such as heat and sound. The two types of collisions are elastic and inelastic.

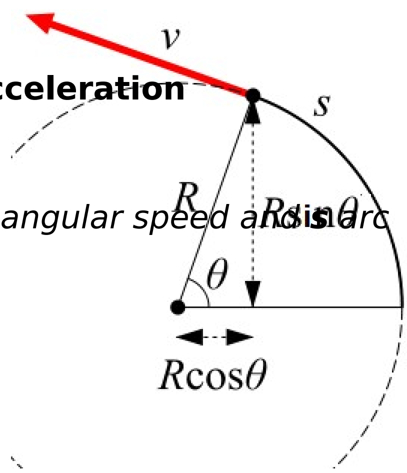
- In an **elastic collision**, the **total kinetic energy** of the system remains **constant**. Momentum and total energy are conserved.
- In an **inelastic collision**, the **total kinetic energy** is **not conserved**. Some of the kinetic energy is transformed to **heat**, **sound** etc. Momentum and total energy are conserved.

Example:
Before: 50g car at $1.2ms^{-1}$ approaching stationary 40g car
After: 50g car at $0.6ms^{-1}$ and 40g car at vms^{-1}. Find v and determine the type of collision
p_b = 50x 1.2 = $60kgms^{-1}$, p_a= 50x0.6 + 40xv = 30 + $40vkgms^{-1}$ v=3/4 ms^{-1}
$0.5(50\times10^{-3})(1.2)^2 = 0.036J$
$0.5(50\times10^{-3})(0.6)^2 + 0.5(50\times10^{-3})(0.75)^2 = 0.02J$
0.036 ≠ 0.02 ∴ collision is inelastic as momentum is conserved by kinetic energy isn't.

Circular Motion

Angular displacement, speed, acceleration and intro to centripetal force

The **angular displacement** is the angle in radians rotated by the radius OP from some reference position. This is denoted by Φ.



d Φ/dt = **Angular Speed** = **ω**
Then $d\omega/dt = d^2\Phi/dt^2$ = **Angular Acceleration**

We know s = r Φ
- ⇨ *ds/dt = r(dΦ/dt)*
- ⇨ **v** $\boxed{= \mathbf{r\omega} =}$ 2πrT *Where **v** is speed and **ω** angular speed and s is arc length*

The position vector at a given time is:
r = rcos(ωt)**i** + rsin(ωt)**j**
- ⇨ **v** = -rωsin(ωt)**i** + rωcos(ωt)**j**
- ⇨ **a** = $-r\omega^2$cos(ωt)**i** +$-r\omega^2$sin(ωt)**j**
- ⇨ By finding l**a**l, we find that **a** = $\mathbf{r\omega^2}$

⇨ and from above v=rω, so

$$a = \frac{v^2}{r}$$

So for a particle moving in a circle of radius r with uniform angular speed ω,

- The **velocity** is **tangential** to the circle
- The **speed** around the circle is **v=rω=** 2πrT
- The **acceleration** acts towards the **centre** of the circle.
- The magnitude of the **acceleration** is **$r\omega^2$** or **v^2/rhnn**
- Since F=ma, the resultant force is acting towards the centre, the **centripedal force.** This could be **gravity**, **tension**, **friction** etc. **F = ma = mv^2/r**

<u>Example</u>:
A metal disc of diameter 12cm is spun at a rate of 480 revolutions per minute. Describe how the speed of a point on a disc depends on r. Find the maximum speed.

$v = r\boldsymbol{\omega} \therefore v \propto r$

$v = 6\times10^{-2} \times 16\pi \approx 3ms^{-1}$

Find the orbital speed and centripetal acceleration of the Earth as it orbits round the Sun at a mean distance of 1.5 x 10^{11} m.

v = 2πrT = 2 x pi x 1.5×10^{11} / 365.25x24x60x60 = 29865ms^{-1}

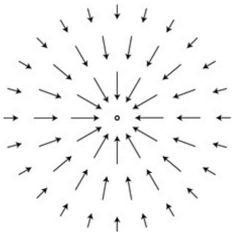
a = v^2/r = (29865^2)/(1.5×10^{11}) = 5.9×10^{-3} ms^{-2}

50g bung attached to rope and swung in horizontal circle of radius 30cm. Cross-sectional area of $4\times10^{-8}m^2$ and breaking stress 5×10^9Pa. Speed increased until snap. Find max speed.

P= f/a required centripetal force = $5\times10^9 \times (4\times10^{-8})$= 200N

F = mv^2/r => 200= 0.05(v^2)/(30×10^{-2})=> v = 34.64... ms^{-1}

Gravitational fields



The mass of an object creates a gravitational field in the space around the object. Can be mapped out using field lines. The direction of the field is the direction in which the small mass would move.

Newton's law of gravitation states **Two point masses attract each other with a force directly proportional to the product of masses and inversely proportional to the square of their separation.**

⇨ Force α product of masses

⇨ Force α Separation^{-2}

⇨ F α Mm/r^2 and using the Gravitational constant, G ($6.67\times10^{-11}Nm^2kg^{-2}$)

⇨ **F = $-GMm/r^2$**

<u>Example</u>:
A 400kg satellite orbits the Earth at a distance of 1.5×10^4km from the centre of the Earth. The mass of the Earth is 6×10^{24}kg. Find the gravitational force acting on the satellite and the acceleration and orbital speed of the satellite.

F = -6.67300 × 10^{-11}x400x6×10^{24}/$(1.5\times10^4)^2$ = 710N

F=mv^2/r so 710 = 400xv^2/1.5×10^4 sp v =1

F=ma so 710 = 400a so a=1.775 ms^{-2}
a= v^2/r sp 1.775 = $v^2/1.5x10^4$=163ms^{-1}

Gravitational Field Strength at a point in space is the gravitational force experienced per unit mass on a small object placed at that point.

- **g = F/m** , this is a more general form of W=mg

Applying this to the gravitational field strength of a spherical object we get

- **g = (-GMm/r²)/m**
- **g= -GM/r²**

Using these equations, it is possible to derive any other equations you need:

$$F = ma \quad a = \frac{v^2}{r} \quad v = \frac{2\pi r}{T} \quad F = \frac{GMm}{r^2}$$

Now it is also possible to derive $T^2 = (\frac{4\pi^2}{GM})r^3$
Where T is the orbital period, M the planet mass, G the constant and r the radius.

And Kepler's third law states **The square of the period of a planet is directly proportional to the cube of its distance from the sun.** i.e. $T^2 \alpha r^3$.

For communications, satellites are useful in **geostationary** orbit. **The period of the satellite is 1 day and is only possible above the equator.**
Using the above equation, the only unknown is r. Therefore it is easy to work out r to be approximately 35200km above the surface of the Earth.

<u>Example</u>:
Saturn is a giant gaseous planet with surface gravitational field strength at its surface of 10.4Nkg^{-1}. The radius of the planet is 6.04x10^7km. Determine the mass and mean density of the planet.
F = -GM/r² so 10.4 = --6.67300 × 10^{-11}x M/(6.04x10^7)² => m=5.69x10^{26} kg
p=m/v = 5.69x10^{26}/(4/3 x pi (6.04x10^7)³) = 616.5 kgm^{-3}

Jupiter has moons Europa and Callisto. Europa orbits at distance 6.7x10^5km from Jupiter's centre with period of 3.55 days and Callisto orbits at distance *d* with orbital period 16.7 days.
Using Kepler's third law, $T^2 \propto r^3$ so T^2=kr^3 so $(3.55)^2(6.7x10^5)^{-3}$=k=4.19x10^{-17} so 16.7^2=4.19x10^{-17}xr^3 => r = 1881067km

<u>Simple Harmonic Motion</u>

Recap, Displacement, Velocity and Acceleration

Displacement x – distance moved by the oscillator in a specified direction from equilibrium position.
Amplitude- A – magnitude of maximum displacement

Period- T- time for one complete oscillation
Frequency- f – number of oscillations per unit time => $f = T^{-1}$
Phase Difference- φ – Fraction of an oscillation two waves are out of synch in radians

A body executes simple harmonic motion when its acceleration is directly proportional to its displacement from its equilibrium position, and is directed towards the equilibrium position.

- ⇨ a α (–x)   the constant of proportionality=2 πf
- ⇨ **$a = -(2\pi f)^2 x$**

The quantity 2πf is the **angular frequency ω** of the oscillator.

- ⇨ Since f=1/T, **ω= 2π/T**

The displacement x of a simple harmonic oscillator is given by

$$x = a\cos(2\pi ft) \quad \text{or} \quad x = a\sin(2\pi ft)$$

The velocity is equal to the gradient of the displacement against time graph. Ditto acceleration graph. Phase difference of **π/2** between each.

- It follows that phase difference between a and x is pi. This is from a α (–x)

The maximum speed of an oscillator occurs when it travels through the equilibrium position (x=0).

- ⇨ **$V_{max} = (2\pi f)A$**

Example:
A mass is attached to the end of a spring. The mass is pulled down and released and performs simple harmonic motion. In 10s, 25 oscillations. Amplitude is 8cm. Find a_{max} and draw graph of acceleration against displacement.
Maximum acceleration occurs where the displacement is maximum. 2.5 Hz so $a=-(2\pi 2.5)^2 \times 8\times10^{-2}=19.73ms^{-1}$.
From **$a = -(2\pi f)^2 x$**, a is proportional to –x so a negative straight line.

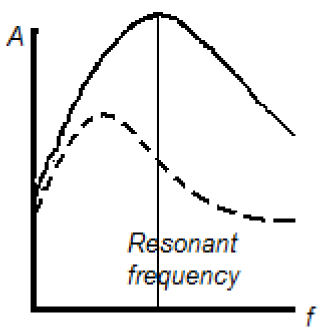
Energy of a Simple Harmonic Oscillator

An oscillator has maximum speed as it travels through the equilibrium position and is momentarily at rest when it displacement equals the amplitude.

The kinetic energy of the oscillator is zero at the extremes of the motion but the oscillator has maximum potential energy at these points.
The kinetic energy of the oscillator is maximum at the equilibrium position, but the oscillator has minimum potential energy at these points.

Damped Motion is the result of friction and makes the x against t graph look like sin(x)/x

Resonance is when forced oscillations occur when an object is forced to vibrate at the frequency if an external source.
(A bus engine, makes parts such as the seats of the bus vibrate at the frequency of the engine)



When the **frequency** of the **engine** matches the **natural frequency** of the **seat**, the seat ***resonates*** – The **amplitude** of the **vibrations** becomes much larger.

At Resonance:
- **Natural frequency** of a **forced oscillator** is equal to the **forcing frequency**.
- The **forced oscillator** has **maximum amplitude**
- The **forced oscillator absorbs maximum energy** from the **external source**.
- The degree of **damping** affects both resonant frequency and the **amplitude** of the forced oscillator; greater degree of damping slightly **reduces resonant frequency**.

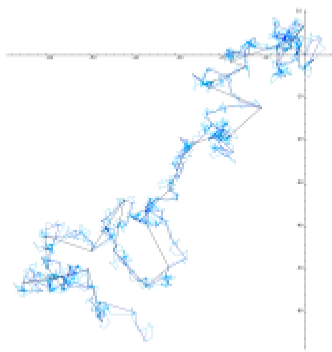
Resonance can be useful in microwaves or nuisance in cars.

<u>Example</u>:
A pendulum bob of mass 60g executes simple harmonic motion with amplitude 12cm. Period 0.52s. Find V_{max} and the maximum change in E_P.
$V_{max} = (2\pi f)A = (2\pi 0.52^{-1})(12x10^{-2}) = 1.45 ms^{1}$
$E_{k\,max} = E_{P\,max}$ $E_{K\,max} = 0.5 \times 0.06 \times 1.45^2 = 0.063J$

<u>Solids, Liquids and Gases</u>

Kinetic Model

- In **Solid** the molecules vibrate **randomly** about their **equilibrium positions**. Closely packed and exert **electrical forces** on each other.
- In **Liquid** the molecules have **translational kinetic energy** and move randomly because of collisions with other molecules. The mean separation is greater than in solids.
- In **Gas** the molecules have **random motion** and **translational kinetic energy**. Mean kinetic energy increases with temperature. Mean separation is greater than in other states and depends on the pressure. Molecules exert negligible electrical forces on each other except in collisions.



Random motion of air molecules can be observed using **Brownian Motion** (mathematical model) of the **smoke molecules** in a **smoke cell** where they move in a jerky random way because of collisions with high speed molecules of air.
A light is shone on a smoke cell full of smoke and looked under through a microscope.

A Gas in a container exerts pressure on the container walls because of molecular collisions with the container walls.
When a single molecule collides elastically with the wall, the change in momentum is **2mv**. From Newton's second law, **force= rate of change of momentum** => F= 2mv/t where t is the time between successive collisions on the wall.
Pressure on the wall = total force / area

Example:
Use the kinetic model to explain the change to the pressure exerted by gas in container whne temperature increased and volume decreased.
F = delta p over delta t *; When the temperature is increased the molecules have more energy so move at a faster velocity. Hence momentum is increased and time taken for successive collisions is less. Hence the force on the wall is greater. Hence the pressure is greater*
When volume is decreased, the time taken for successive collisions is lower so the force on the wall is greater so the pressure is greater.

Energy Changes

Internal Energy of a substance is the **sum** of the **random distribution** of **kinetic** and **potential** energies of all the atoms or molecules.

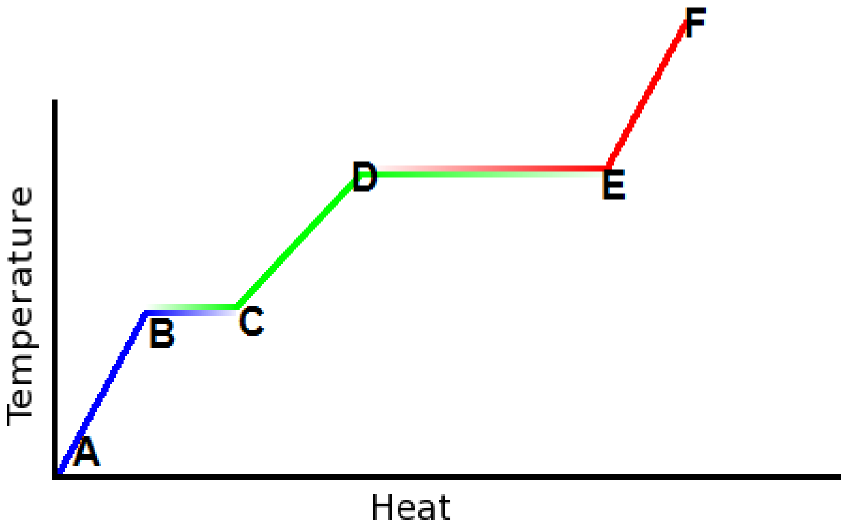
All molecules have kinetic energy (vibrations and translational motion) and potential energy (electrical attraction between the molecules).
Internal energy depends on the temperature of the substance. At 0K, all molecules stop moving and vibrating. The internal energy is potential only. As temperature is increased, internal energy of a substance increases due to molecules gaining kinetic and potential energy.

Melting
- Solid state to liquid state

Boiling
- Liquid state to gaseous state

During these two steps when temperature is **increased**
- Mean Separation increases
- Electrical potential energy increases
- Mean kinetic energy of molecules remains the same (bonds are broken but molecules do not move any faster)
- Internal energy increases

Evaporation
- Liquid state to gas state
- Fast moving molecules escape from the surface leaving the slower molecules, cooling the substance.
- Occurs at ALL temperatures of a liquid
- Rate increased by blowing over surface and heating.



A to B, C to D, E to F
- In solid state.
- As temp increases, **internal energy** increases **mainly** due to more **vibrations**

B to C, D to E
- Melting/ Boiling
- Energy used to break molecular bonds
- Mean separation increases

- **Internal energy** increases **mainly** due to increased **electrical potential energy**. (Temp is constant)

Electrical potential energy of a gas is zero (which is maximum because electrical potential energy is negative for lower temperatures).

When a hot object is in contact with a cooler object, there is a net heat transfer from the hot object to cool object. Eventually, both objectsd reach the same termperature and are in **thermal equilibrium**.

The difference between degrees and Kelvin is plus/minus **273**.

Thermal Properties of materials

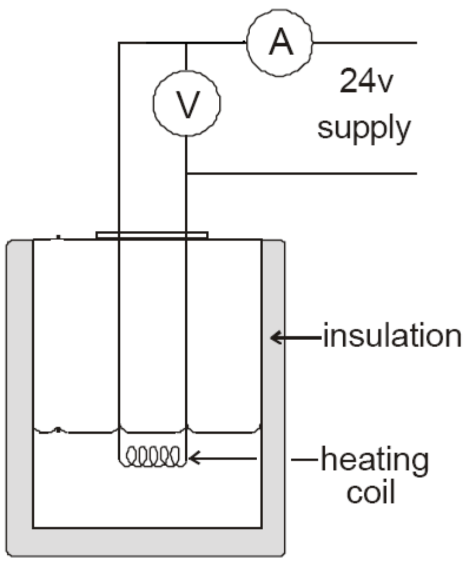
The **specific heat capacity of a substance is the energy required per unit mass of the substance to raise its temperature by 1 K**.

- ⇨ **c = E/mΔt**

<u>Example</u>:
A beaker contains 80g of water at a temperature 20^{o}c. The glass bulb of 24W filament lamp is immersed in the water. The temperature doubles after 5 minutes. The specific heat capacity of water is 4200Jkg^{-1o}C^{-1}. Assuming no loss of heat, find the efficiency of the lamp as a heater.
*E=mc*Δt= 0.08x4200x20= 6720J of energy
24W = 24Js^{-1}= 7200J of energy
672/720 x 100 = 93% efficiency

An experiment to determine the heat capacity of a solid or liquid.

An electrical heater heats the substance. It's temperature is measured using a thermometer. Ammeter and voltmeter connected and insulation to prevent heat loss.



- Measure m and $Ø_0$ of the substance.
- Start stopwatch, turn on, and measure I and V.
- After a while, stop stopwatch and turn off.
- Measure $Ø_i$ and t from stopwatch
  c= VItm(Øf-Øi)

There is no temperature change when a substance changes state. Total **kinetic energy** of the molecules remains **constant** but their **electrical potential energy increases** as molecular bonds are broken. Term used for energy supplied to change the state is latent heat. (latent means hidden)

Latent heat of fusion is the energy supplied to MELT a solid substance to liquid

Latent heat of vaporisation is energy supplied to BOIL a liquid substance to gas

Ideal gases

One mole of any substance has 6.02×10^{23} particles.
One mole is the amount of substance that has the same number of particles as there are atoms in 12g of carbon 12 isotope.

<u>Example</u>:
Mr of He is 4 g mol^{-1}. Determine the mass of a helium atom.
One mole is 6.02×10^{23} atoms. So $0.004/(6.02 \times 10^{23})$ kg = 6.64×10^{-27}kg

Boyle's Law states:
The pressure exerted by a fixed amount of gas is inversely proportional to its volume, provided its temperature is constant.

- ⇨ $p \propto 1V$ so $pV=constant$

For a gas at constant temperature, the product of pressure and volume remains constant. Doubling the pressure will halve the volume. i.e. $p1V1=p2V2$

The above applies to ideal gases (H, He, O are ideal gases at s.t.p but as temp lowers or pressure very high, some gases depart from their ideal behaviour because electrical forces between atoms aren't negligible)

Experiments show

$p \propto T$ *(for constant volume)* and $V \propto T$ *(for constant pressure)*

Combining the above with Boyle's law we have

$pVT=constant$ so $p1V1T1=p2V2T2=$(*constant)*

The constant depends on the number of moles (n) and the molar gas constant (R) so:

$pV=nRT$

This is the ideal gas equation.

<u>Example</u>:
Oxygen consists of O_2 molecules. The molar mass of oxygen is 32g mol^{-1}. For 160g of oxygen, find number of moles, molecules and the volume at 20°C and 1.01×10^5 Pa.
160/32=5 mols, 5 x $6.02 \times 10^{23} = 3.01 \times 10^{24}$, $pV = nRT \Rightarrow 1.01 \times 10^5 \times V = 5 \times 8.31 \times (20+173)$
$V= 0.12m^3$

Translational kinetic energy

Increasing the **temperature** of the gas makes the atoms travel faster and their means **translational kinetic energy increases**. The mean **kinetic energy** of the atoms is **related** to the **thermodynamic temperature** of the gas.

Mean translational kinetic energy of atoms $\propto$ ***T***

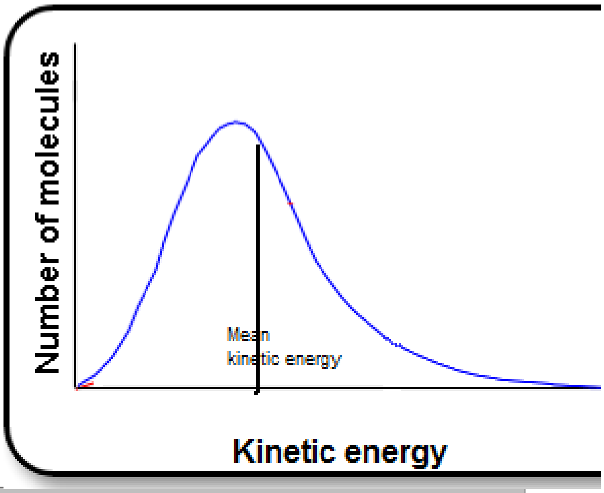
The internal energy of the gas is almost entirely equal to the total kinetic energy of the atoms, so

Internal energy ∝ T

For individual atoms, the mean translational kinetic energy E is given by

E= 32kT

Where k is the Boltzmann constant. The mean kinetic energy is equal to $0.5mv^2$. At a given temperature, the gas atoms have a random motion and travel at different speeds. However, the mean kinetic energy of the atoms remains constant at a given temperature.



<u>Example</u>:
The outer surface of a star had H and He atoms. Temp is 8000K. Mr(H)=1 Mr(He)=4. Do hydrogen and helium have the same mean kinetic energy? Calculate mean speed of H atoms. Determine ratio of mean speed of hydrogen atoms over mean speed of helium atoms.

32kT= E=Ek=12mv2, *so T ∝ E_k. So yes. V = root(3kT/m)* $10^{-3}/(6.02x10^{23})=1.66x10^{-27}$*kg each hydrogen atom. V= root((3x1.38x*10^{-23}*x8000)/(1.66x*10^{-27}*)) = 1.4 x*10^4 *ms*$^{-1}$*. V* ∝m *so root(4) = 2, =2.*