UNIT G482	Module 4	2.4.4	Stationary Waves

- ***Candidates should be able to*** *:*
  - *Explain the formation of* ***stationary (standing) waves*** *using graphical methods.*
  - *Describe the* ***similarities*** *and* ***differences*** *between* ***progressive and stationary waves.***
  - *Define the terms* ***NODES*** *and* ***ANTINODES.***
  - *Describe experiments to demonstrate stationary waves using* ***microwaves, stretched strings and air columns.***
  - *Determine the standing wave patterns for* ***stretched string*** *and* ***air columns in closed and open pipes.***
  - *Use the equation :*

    Separation between adjacent nodes (or antinodes) = λ/2

  - *Define and use the terms* ***FUNDAMENTAL MODE OF VIBRATION and HARMONICS.***
  - *Determine the* ***speed of sound in air*** *from measurements on stationary waves in a pipe closed at one end.*

- *DIFFERENCE BETWEEN* ***PROGRESSIVE*** *AND* ***STATIONARY*** *WAVES*

PROGRESSIVE WAVES

These are waves which travel outwards from a source and transmit energy from one point to another.

STATIONARY (or STANDING) WAVES

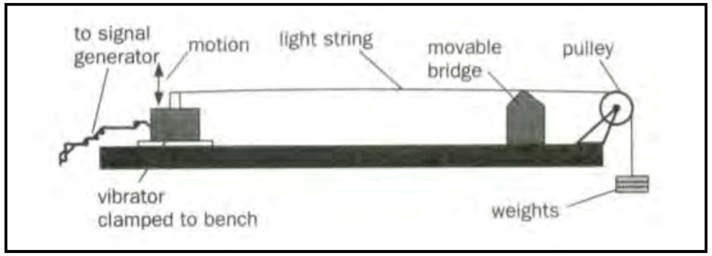
These are waves which :

- *Are formed as a result of superposition between two* ***identical*** *waves* ***(i.e. same speed, Frequency, and approximately equal equal amplitude)*** *travelling in opposite directions.*
- ***DO NOT TRANSMIT ENERGY*** *from one point to another.*
- *Stand in a fixed position and have :*

  NODES *- Points of zero displacement.*
  ANTINODES *- Points of maximum displacement.*

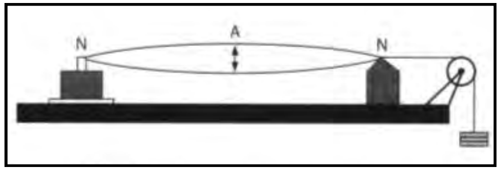
- ***OBSERVING STATIONARY WAVES***

VIBRATING STRETCHED STRINGS



The apparatus shown in the diagram above (Known as ***MELDE'S apparatus****) can be used to study the behaviour of vibrating strings.*

The vibrator sends out transverse waves along the string which are reflected from the moveable bridge. These reflected waves meet and superpose with waves coming from the vibrator. Since the two sets of waves are ***identical and travelling in opposite directions****, stationary wave formation is then possible.*



As the vibrator frequency is gradually increased from a low value, a particular frequency is reached at with the string is seen to vibrate with a large amplitude stationary wave having a ***NODE (N)*** *at each end and an* ***ANTINODE (A)*** *in the centre (as shown in the diagram above).*

This is the lowest frequency at which a stationary wave is formed and it is called the

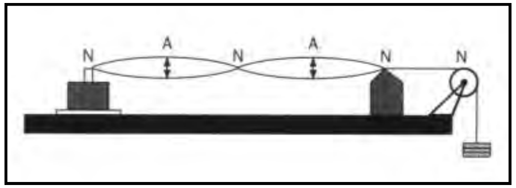
FUNDAMENTAL MODE OF VIBRATION

The frequency at which this occurs is called the ***FUNDAMENTAL FREQUENY (f_0)***

The stationary wave is formed because in the time taken for the wave to reach the end and return, the vibrator is just ready to send out a second wave and this reinforces the first wave. Thus the amplitude builds up as each new input from the vibrator is in phase with the wave in the string and the energy is added to it.

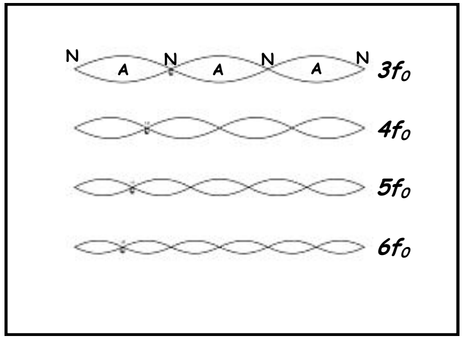
If the vibrator frequency is increased further, the single-loop stationary wave disappears.

A new stationary wave having two loops (as shown opposite) is seen when the vibrator frequency = ***2 f_0****.*

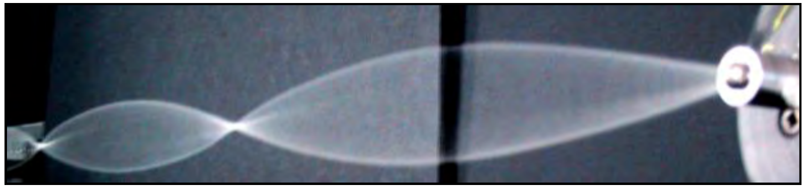


Further stationary waves having 3, 4, 5…. vibrating loops are seen when the vibrator frequency is increased to ***$3f_0$, $4f_0$, $5f_0$…..etc.*** *(as shown in the diagram opposite).*

The frequencies at which these stationary waves occur are the ***RESONANT FREQUENCIES*** *of the string under these conditions.*



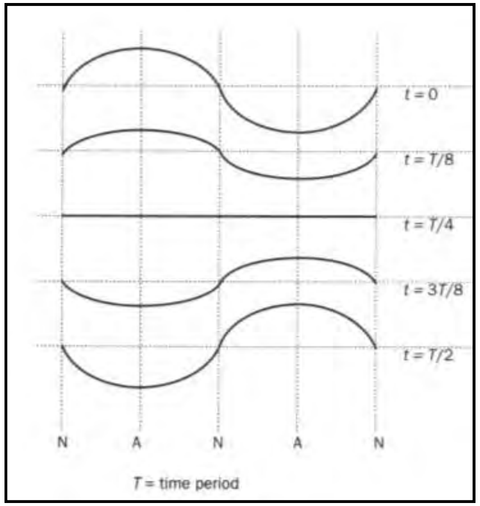
By illuminating the vibrating string with a ***stroboscope****, the actual motion of the string can be observed. If the stroboscope frequency exactly matches that of the stationary Wave being viewed, the string appears stationary. Changing the stroboscope frequency Slightly then shows the string moving in slow motion.*



*The diagram opposite shows the appearance of the string (for frequency = $2f_0$) during **½ a cycle**.*

*Each line shows the string's position after **⅛th** of a cycle.*

The stationary wave is normally represented by drawing the shape of the string in its two extreme positions.



Points To Note

- *There are points, called **NODES (N)** where the displacement of the string is **always zero**.*

  Distance between adjacent NODES = λ/2

- *There are points, called **ANTINODES (A)** where the displacement of the string is **always a maximum**.*

  Distance between adjacent ANTINODES = λ/2

- *In the region between successive nodes, all particles are moving **IN PHASE** with **differing amplitudes**.*

- *The oscillations in one loop are in **ANTIPHASE** (i.e. **180°** or **π rads** or λ/2 out of phase) with the oscillations in adjacent loops.*

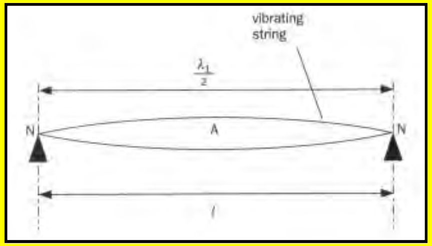
MODES OF VIBRATION OF STRETCHED STRINGS

When a stretched string (e.g. guitar string) is set into vibration, a progressive wave travels to the fixed ends where it is reflected. A stationary wave is then formed on the string as these reflected waves (of equal frequency and amplitude) become superposed.

*Depending on the frequency with which it is set into vibration, the string will vibrate in a number of different **MODES**.*

FUNDAMENTAL MODE (1st HARMONIC)

*This is the simplest and lowest possible frequency of vibration, with a **NODE (N)** at the fixed ends and an **ANTINODE (A)** at the centre.*

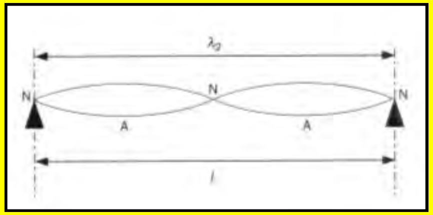


*If **(l)** is the string length and **(λ_1)** is the wavelength of the stationary wave formed, then :*

$l = \lambda_1/2$, so $\boldsymbol{\lambda_1 = 2l}$

2nd HARMONIC

This is the next possible frequency of vibration and in this case : $\boldsymbol{\lambda_2 = l}$

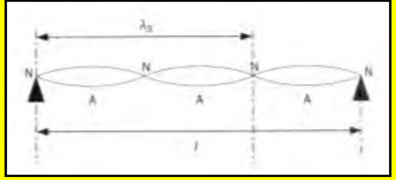


Thus $\boldsymbol{\lambda_2 = \frac{1}{2}\lambda_1}$

So, frequency of 2nd harmonic (f_1) = *2 x frequency of 1st harmonic* (f_0).

3rd HARMONIC

In this case $l = 1.5\,\lambda_3$ $\boldsymbol{\lambda_3 = 2l/3}$

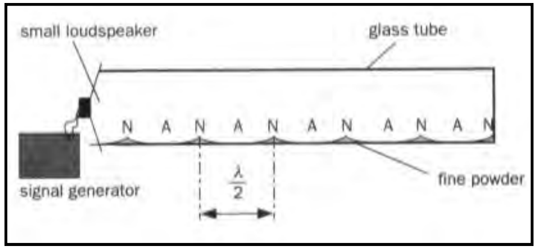


So $f_3 = 3f_0$

VIBRATIONS IN AIR COLUMNS (PIPES)

*When the air at one end of a tube is caused to vibrate, a **progressive, longitudinal** wave travels down the tube and is reflected at the opposite end (which may be closed or open to the outside air). The incident and reflected waves have the same **speed, frequency** and **amplitude** and superpose to form a **stationary, longitudinal wave.***

The apparatus shown in the diagram opposite may be used to demonstrate the formation of stationary waves in pipes.



When the frequency of the sound from the loudspeaker is steadily increased from a low value, the sound produced becomes louder at certain frequencies. These loudness peaks are caused when the air column in the tube is set into resonant vibration by the vibrating loudspeaker cone.

*At these resonant frequencies, the fine powder in the tube forms into equally-spaced heaps. This is because the air molecules vibrate longitudinally along the tube axis and the amplitude of vibration varies from a **maximum** at the **antinodes (A)** to **zero** at the **nodes (N)**. At the **antinode** positions the large amplitude vibration shifts the fine powder and so causes it to accumulate near the **node** positions, where the amplitude of vibration of the molecules is zero.*

POINTS TO NOTE

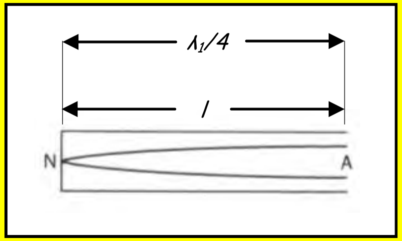
- *The stationary wave in the tube is **LONGITUDINAL.***
- *The amplitude of vibration of the air molecules is always a **maximum** at the **open end** of the tube (i.e. it is an **ANTINODE**).*
- *The amplitude of vibration of the air molecules is always **zero** at the **closed end** of the tube (i.e. it is a **NODE**).*
- *All molecules between two adjacent nodes vibrate **IN PHASE.***
- *All molecules on either side of a node vibrate **IN ANTIPHASE.***

Distance between adjacent nodes (or antinodes) = $\frac{1}{2}\lambda$

MODES OF VIBRATION OF CLOSED PIPES

FUNDAMENTAL MODE (1st HARMONIC)

*This is the simplest and lowest frequency of vibration, with an **ANTINODE (A)** at the open end and a **NODE (N)** at the closed end.*



Note that the air molecules vibrate longitudinally, but the diagram shows these vibrations Plotted along the vertical axis.

If l is the length of the tube and λ_1 is the wavelength of the stationary wave formed, then : $l = \lambda_1/4$,

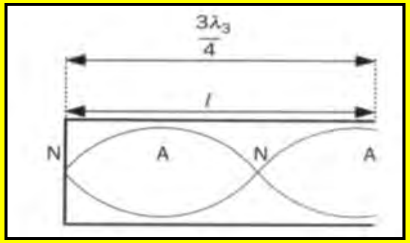
So $\lambda_1 = 4l$

This is the longest wavelength possible for this length of closed pipe, so the note produced has the lowest possible frequency.

3rd HARMONIC

This is the next possible frequency of vibration and in this case :

$$l = \frac{3\lambda_3}{4}$$



So $\lambda_3 = \frac{4l}{3}$

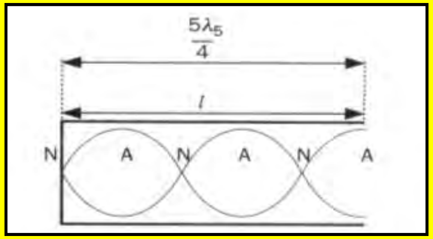
*This wavelength is **one third** of the wavelength of the **1st harmonic,** so the note produced has a frequency **three** times higher than the **fundamental frequency**.*

*Thus a **CLOSED PIPE** can only produce **ODD** harmonics.*
***i.e.** f_0, $3f_0$, $5f_0$,……….**etc**.*

5th HARMONIC

In this case : $l = \frac{5\lambda_5}{4}$

So $\lambda_5 = \frac{4l}{5}$



The frequency of this mode of vibration is ***5 x the frequency of the fundamental.***

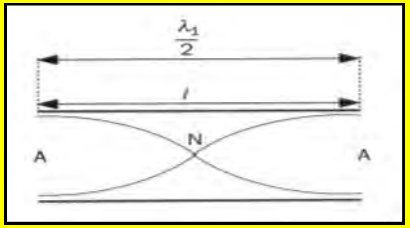
The 7th, 9th etc harmonics are formed with increasing numbers of nodes And antinodes and the notes produced have frequencies of 7 and 9 x that Of the fundamental.

MODES OF VIBRATION IN OPEN PIPES

- *The term* ***OPEN PIPE*** *means a pipe which is open to the atmosphere at both ends.*

FUNDAMENTAL MODE (1st HARMONIC)

This is the simplest and lowest frequency mode of vibration, with an ***ANTINODE (A)*** *at the open ends and a* ***NODE (N)*** *at the centre.*



In this case : $l = \frac{\lambda_1}{2}$

So $\lambda_1 = 2l$

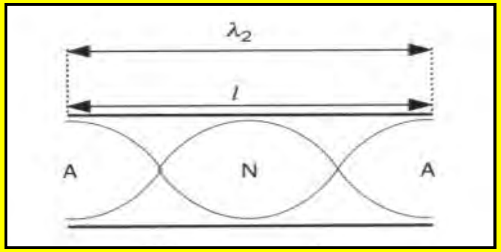
This wavelength is HALF of that produced by a closed pipe sounding in its Fundamental mode, so :

Fundamental frequency of an ***OPEN*** *pipe = 2 x the Fundamental frequency of a* ***CLOSED*** *pipe*

2nd HARMONIC

This is the next possible Frequency of vibration And in this case :

$\lambda_2 = l$



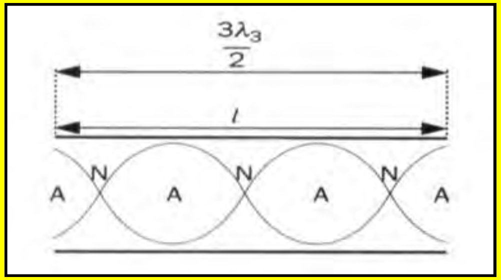
This is ½ the wavelength of the fundamental mode and so the frequency of this note is ***2 x fundamental frequency.***

3rd HARMONIC

In this case :

$L = \frac{3\lambda_3}{2}$

So $\lambda_3 = \frac{2l}{3}$



This is 1/3 the wavelength of the fundamental mode and so the frequency of this note is ***3 x fundamental frequency.***

Thus, an ***OPEN PIPE*** *can produce* ***ALL*** *harmonics* ***(odd and even)*** *:*

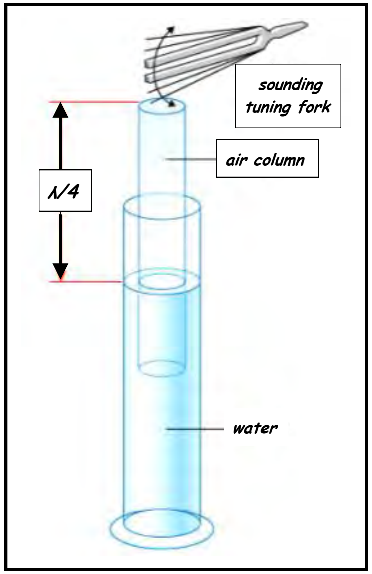
i.e. f_0, $2f_0$, $3f_0$, $4f_0$, ……….. ***etc.***

SUMMARY wavelengths and positions of NODES and ANTINODES for strings & pipes

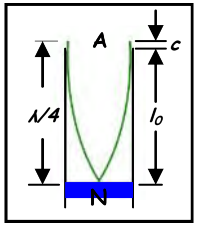
Harmonic	String	Closed pipe	Open pipe
1st	$\lambda_1 = 2l$	$\lambda_1 = 4l$	$\lambda_1 = 2l$
2nd	$\lambda_2 = l$	Impossible	$\lambda_2 = l$
3rd	$\lambda_3 = 2l/3$	$\lambda_3 = 4l/3$	$\lambda_3 = 2l/3$
4th	$\lambda_4 = l/2$	Impossible	$\lambda_4 = l/2$

DETERMINATION OF THE SPEED OF SOUND IN AIR

*In the **RESONANCE TUBE** shown in the diagram opposite, an air column can be set into vibration by holding a sounding tuning fork over the open end.*

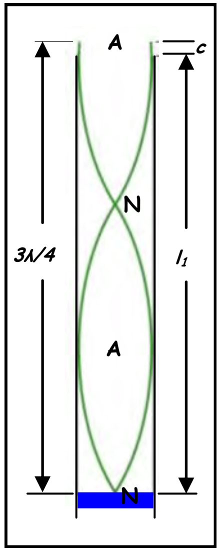


*A given length of air column has a particular natural frequency of vibration and if the tuning fork frequency matches this, the air column is set into **RESONANT** vibration and the tuning fork sounds **much louder**.*
*With the tuning fork sounding over the open end, the inner tube is slowly raised, increasing the air column length until a **LOUD** sound is heard.*



*In this first resonance position the air column is vibrating in its **fundamental mode** with a **node (N)** at the closed end and an **antinode (A)** at a distance **(c)** above the open end. 'c' is called the '**END CORRECTION**'. Then :*

$\lambda/4 = l_0 + c$ .... (1)



*By further increasing the length of the air column, a **second resonance position** is obtained.*

*The air column is now vibrating in its first harmonic with two **nodes (N)** and two **antinodes (A)** as shown opposite. In this case :*

$3\lambda/4 = l_1 + c$ .... (2)

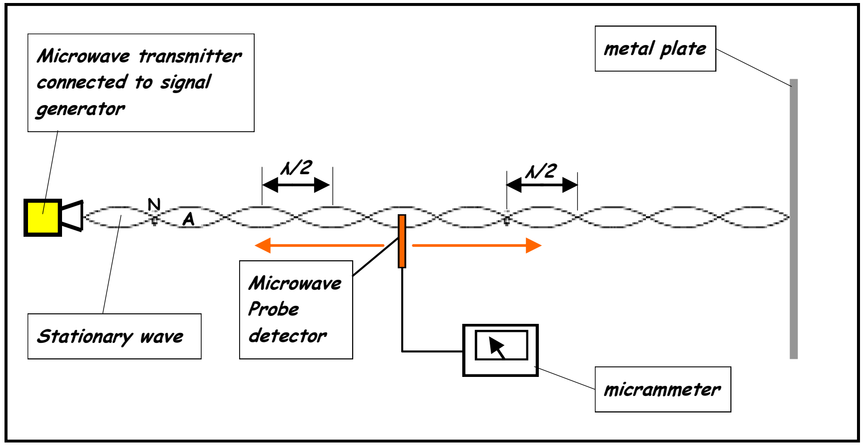
(2) - (1) → $\lambda/2 = l_1 - l_0$

So $\lambda = 2(l_1 - l_0)$

Then speed of sound, $v = f\lambda$

$$v = 2f(l_1 - l_0)$$

STATIONARY WAVES WITH MICROWAVES



*A microwave transmitter connected to a signal generator is used to direct microwaves at a metal plate as shown in the diagram above. The microwaves are reflected from the metal plate and superposition between the incident and reflected waves can produce a **stationary wave.***

*A microwave probe detector moved between the transmitter and the plate indicates alternating points of **maximum (ANTINODE, A)** and **minimum (NODE, N)** microwave intensity (i.e. high and low readings are obtained on the microammeter).*

*The microwave **wavelength (λ)** can be determined by measuring the distance moved by the probe detector as it goes through a number of nodes (indicated as minimum readings on the microammeter). Moving the detector a distance, **D** through say, **10** nodes, means that :* $5\lambda = D$ *and so* $\lambda = D/5$.

*The microwave **frequency (f)** is given by the signal generator and so the **speed (c)** of the microwaves can be calculated from* $c = f\lambda$.

NOTE *If a loudspeaker is connected to the signal generator, a **sound** stationary wave pattern can be produced. A microphone connected to an oscilloscope is then used to detect the positions of the nodes and antinodes.*

1 Calculate the **wavelength** of the stationary wave in a guitar string of length **0.84 m** when it is caused to vibrate :

(a) In its **FUNDAMENTAL** mode.

(b) In its **SECOND HARMONIC** mode.

In each case, draw a diagram to show the appearance of the string.

(c) For the **FUNDAMENTAL** mode of vibration, calculate the **speed** of the wave along the wire, if the note produced has a frequency of **256 Hz.**

2 An organ pipe of length **1.4 m** is **open** at one end and **closed** at the other. Given that the speed of sound in air is **340 m s^{-1},** calculate the **wavelengths** and **frequencies** of :

(a) The **FUNDAMENTAL** mode of vibration.

(b) The **THIRD HARMONIC** mode of vibration.

Sketch the stationary wave pattern formed in each case.

3 Microwaves are directed at a flat metal plate. When a detector is moved along the line between the transmitter and the plate, it is observed to register zero microwave intensity at points **18 mm** apart.

(a) **Explain** why the detector registers zero intensity at these points.

(b) Calculate the **wavelength** of the microwaves.

- **HOMEWORK QUESTIONS**

1 (a) A **tranverse** wave pulse passes along a slinky coil. **State** how any single coil in the slinky will move as the pulse passes it.

(b) **Fig. 1** shows a large measuring cylinder. The air column in the cylinder can be made to produce a note by blowing horizontally across the top of the cylinder.

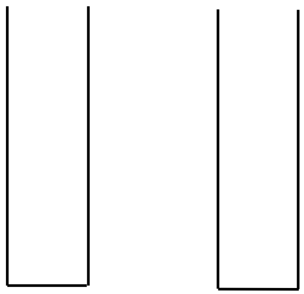


Fig.1 Fig.2

(i) **State** the direction in which the particles in the air column move when the note is produced.

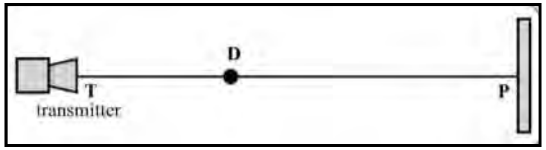
(ii) The air column in Fig. 1 is producing its **lowest frequency** note (the **fundamental**). **Label** on Fig. 1 the positions in the cylinder of a **node**, with the letter **N**, and an **antinode**, with the letter **A.**

(iii) The length of the air column is **0.40 m** and the speed of sound in air is **340 m s^{-1}**. Calculate the **frequency** of the **lowest (fundamental)** note.

(iv) **Label** on **Fig. 2** the positions of **nodes (N)** and **antinodes (A)** when a note of higher frequency is being produced.

(OCR AS Physics - Module 2823 - June 2002)

The diagram shows an arrangement where microwaves leave a transmitter **(T)** and move in a direction **TP** which is perpendicular to a metal plate **(P).**



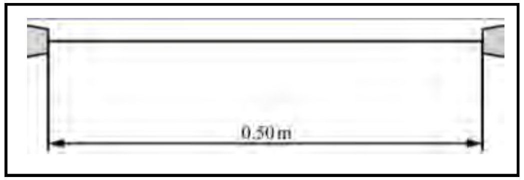
(a) When a microwave detector **(D)** is slowly moved from **(T)** towards **(P),** the pattern of the signal strength received by **(D)** is **high, low, high, low**........ Etc. Explain :

- Why these **maxima and minima** of intensity occur.
- How you would measure the **wavelength** of the microwaves.
- How you would determine their **frequency.**

(b) Describe how you could test whether the microwaves leaving the transmitter are **plane polarised.**

(OCR AS Physics - Module 2823 - June 2004)

3 The diagram shows a stretched wire held horizontally between supports **0.50 m** apart. When the wire is plucked at its centre, a standing wave is formed and the wire vibrates in its **fundamental mode (lowest frequency).**



(a) **Explain** how the standing wave is formed.

(b) Draw the fundamental mode of vibration of the wire. Label the position of any **nodes with the letter N** and any **antinodes with the letter A.**

(c) What is the **wavelength** of the standing wave ?

(OCR AS Physics - Module 2823 - January 2006)

4 (a) **Figure 1** shows a string stretched between two points **A** and **B**.

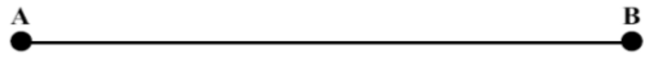


Figure 1

State how you would set up a standing wave on the string.

(b) The standing wave vibrates in its **fundamental mode** (i.e. the **lowest frequency** at which a standing wave can be formed). Draw this standing wave.

(c) **Figure 2** shows the appearance of another standing wave formed on the same string.

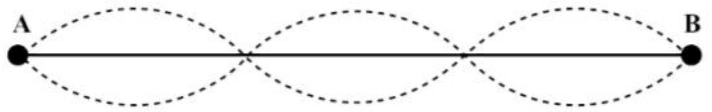


Figure 2

The distance between **A** and **B** is **1.8 m.** Use Figure 2 to calculate :

(i) The distance between neighbouring **NODES.**

(ii) The **wavelength** of the standing wave.

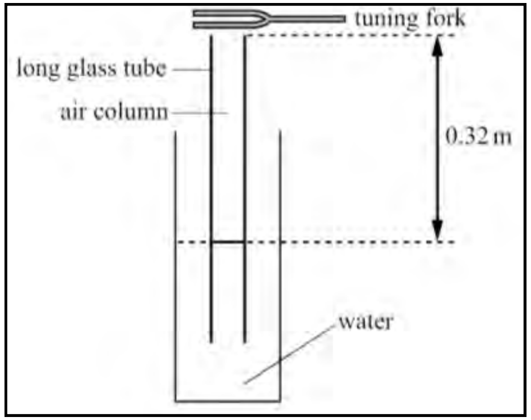
(OCR AS PHYSICS - Module 2823 - June 2003)

5 (a) In standing waves there are **NODES** and **ANTINODES.**

Explain what is meant by :

(i) A **NODE.** (ii) An **ANTINODE.**

(b) The diagram shows a long glass tube within which standing waves can be set up. A vibrating tuning fork Is placed above the glass tube and the length of the air column is adjusted, by raising or lowering the tube in the water until a sound is heard.



(i) The standing wave formed in the air column is the **fundamental (the lowest frequency).** Make a copy of the diagram and show on it the position of a **NODE** - Label as **N,** and an **ANTINODE** - Label as **A.**

(ii) When the **fundamental** wave is heard, the length of the air column is **0.32 m.** Determine the **wavelength** of the standing wave formed.

(iii) The speed of sound in air is **330 m s^{-1}.** Calculate the **frequency** of the tuning fork.

(OCR AS Physics - Module 2823 - June 2005)