- ***Candidates should be able to :***

  - *State **Kirchhoff's second law** and appreciate that it is a consequence of conservation of energy.*
  - *Apply **Kirchhoff's first and second laws** to circuits.*
  - *Select and use the equation for the **total resistance of two or more resistors in series.***
  - *Recall and use the equation for the **total resistance of two or more resistors in parallel.***
  - *Solve circuit problems involving **series** and **parallel** circuits with one or more sources of e.m.f.*
  - *Explain that all sources of e.m.f have an **internal resistance.***
  - *Explain the meaning of the term **terminal pd.***
  - *Select and use the equations :* $E = I(R + r)$ *And* $E = V + Ir$

- ***KIRCHHOFF'S LAWS***

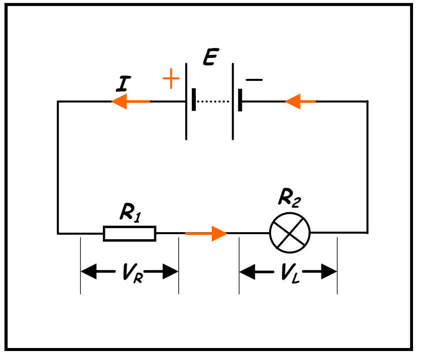
LAW 1 (K1)

The sum of the currents flowing into any point in a Circuit is equal to the sum of the currents flowing Out of the point.

i.e. $$\Sigma I_{in} = \Sigma I_{out}$$

Greek letter 'sigma', which means 'the sum of all'.

LAW 2 (K2)



Consider the circuit shown opposite.

As charge flows through the battery (of e.m.f. E), electrical energy is supplied to each coulomb. The charge then flows through the resistor of resistance (R_1) and through the filament lamp of resistance (R_2). In each of these components the electrical energy is converted to heat and heat and light energy respectively.

Energy supplied per coulomb by The battery (i.e. the e.m.f.)** = **The sum of the energies converted per coulomb in each component (i.e. the sum of the pd's)

$$E = V_R + V_L$$

And since the current (I) is the same at each point in a SERIES circuit :

$$E = IR_1 + IR_2$$

*This equation expresses **KIRCHHOFF'S SECOND LAW (K2)** which states that :*

In any closed loop in a circuit, the sum of the emf's is equal to the sum of the pd's around the loop.

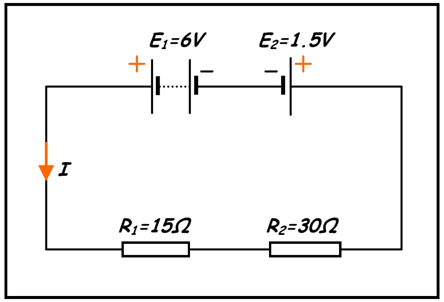
- *As we have seen from our energy analysis of a simple circuit, If a unit charge follows a closed path in a circuit :*

Total energy supplied to the charge. = Total energy dissipated by the charge.

i.e. KIRCHOFF'S SECOND LAW is a consequence Of the PRINCIPLE OF CONSERVATION OF ENERGY.

EXAMPLES OF THE USE OF KIRCHHOFF'S LAWS IN THE SOLUTION OF CIRCUIT PROBLEMS

(1) Use **Kirchhoff's Law 2** to determine the **current (I)** in the circuit shown opposite.



Net emf = sum of the pd's

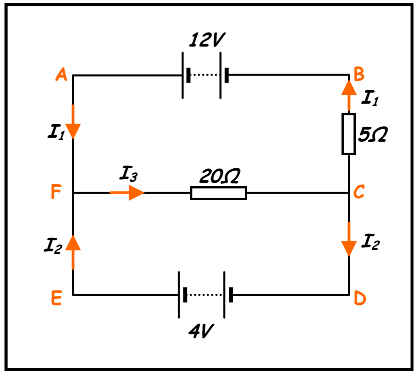
$E_1 - E_2 = IR_1 + IR_2$

$6 - 1.5 = (I \times 15) + (I \times 30)$

$4.5 = 45I$

$I = \frac{4.5}{45} = \boxed{0.1\ A}$

(2) Use **Kirchoff's Laws** to calculate the **currents I_1, I_2 and I_3** in the circuit shown opposite.



- **Mark in the currents and label the closed loops ABCDEF** as shown.
- **Applying Kirchhoff 1 to point F**

  $I_3 = I_1 + I_2$ .... (1)

- **Applying Kirchhoff 2 to loop FCDEF**

  $4 = 20\,I_3$ So $I_3 = 4/20 = \boxed{0.2\ A}$

- **Applying Kirchhoff 2 to loop ABCFA**

  $12 = 20\,I_3 + 5\,I_1 = (20 \times 0.2) + 5\,I_1 = 4 + 5\,I_1$

  $I_1 = 8/5 = \boxed{1.6\ A}$

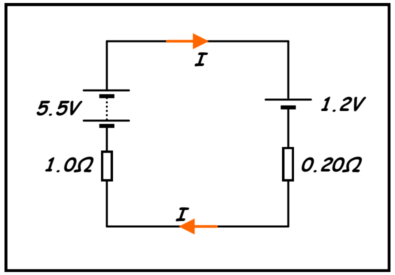
- **Substituting for I_1 & I_2 in equation (1)**

  $0.2 = 1.6 + I_2$ So $I_2 = 0.2 - 1.6 = \boxed{-1.4\ A}$

The negative sign tells us that I_2 flows in a direction opposite to that chosen.

(3) The circuit diagram opposite shows a battery charger of emf **5.5 V** and internal resistance **1.0 Ω** being used to recharge a cell of emf **1.2 V** and internal resistance **0.20 Ω**.

Use **Kirchhoff's Law 2** to determine the **charging current *I*.**



Applying Kirchhoff 2 to the loop

$5.5 - 1.2 = (I \times 1.0) + (I \times 0.20)$

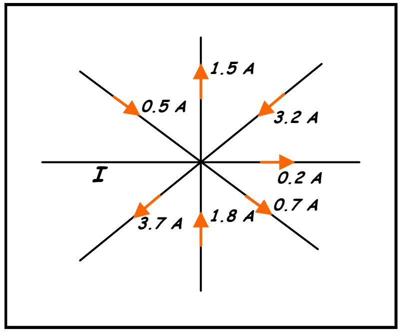
$4.3 = 1.2\,I$

$I = 4.3/1.2 = \boxed{3.58\ A}$

• PRACTICE QUESTIONS (1)

1 The diagram opposite shows a junction in a circuit.

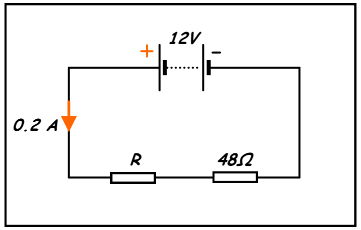
Use **Kirhhoff's First Law** ($\Sigma I_{in} = \Sigma I_{out}$) to determine the size and direction of the current I.



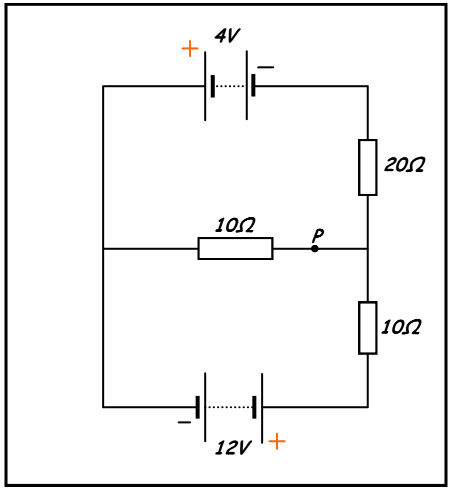
2 Use **Kirchhoff's Second Law** to Calculate :

(a) The **pd across resistor R.**

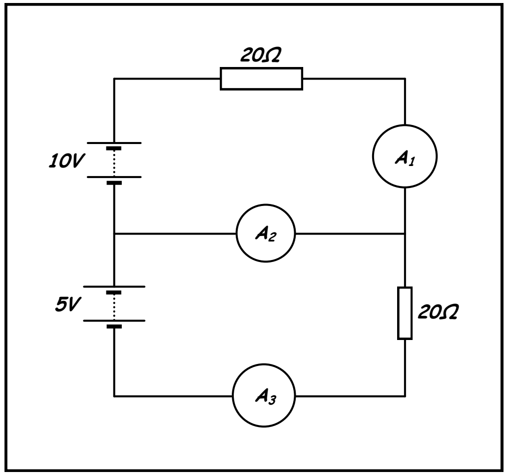
(b) The **value of R.**



3 Use **Kirchhoff's laws** to determine the size and direction of the **current at point P** in the circuit shown opposite.



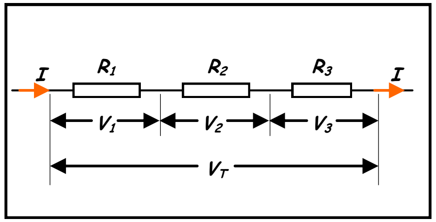
4 Use **Kirchhoff's laws** to determine the **current readings** shown on the ammeters (A_1, A_2 & A_3) in the circuit shown opposite.



• *COMBINATION OF RESISTORS*

RESISTORS IN SERIES

*Consider three resistors of resistance R_1, R_2 and R_3 connected **IN SERIES** as shown opposite.*



- ***According to Kirchhoff's Law 1*** *:* *The current (I) is the same in each resistor.*

- ***Since energy is conserved*** *:* *Total pd across the combination = The sum of the pds across each resistor*

$$V_T = V_1 + V_2 + V_3$$

(And since V = IR) $IR_T = IR_1 + IR_2 + IR_3$

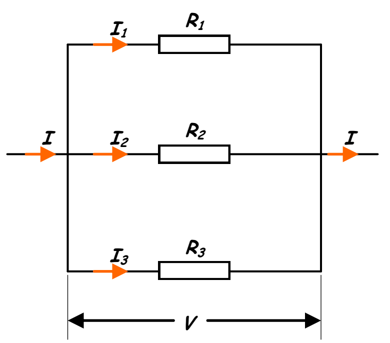
From which : $R_T = R_1 + R_2 + R_3$

*For any number of resistors connected **IN SERIES**, the **TOTAL RESISTANCE (R_T)** is given by :*

$$R_T = R_1 + R_2 + R_3$$

RESISTORS IN PARALLEL

*Consider three resistors of resistance R_1, R_2 and R_3 connected **IN PARALLEL** as shown opposite.*



- *__According to Kirchhoff's Law 1__*

  $I = I_1 + I_2 + I_3$ ....(1)

- *The pd across each resistor = V*

- *From the definition of resistance :*

  $I = V/R$

  And applying this to equation (1) :

  $$\frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

  From which : $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

*For any number of resistors connected **IN PARALLEL**, the **TOTAL RESISTANCE (R_T)** of the combination is given by :*

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- *For resistors connected __**IN PARALLEL**__ :*
  - *The **LOWEST** value resistors carry the **GREATEST** proportion of the current.*
  - *The **TOTAL RESISTANCE** of the combination is **LESS** than the **SMALLEST** resistance in the combination.*

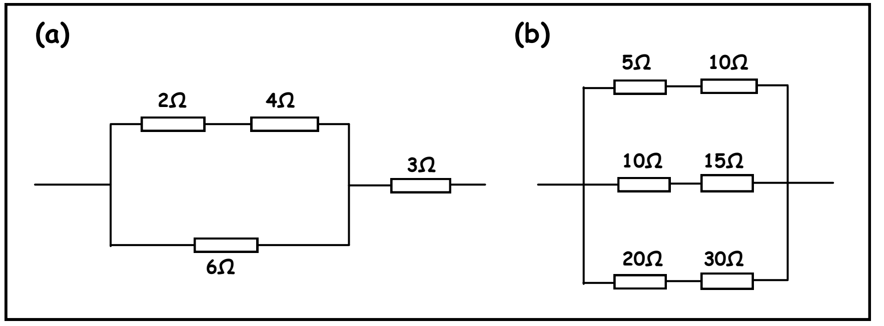
- *__SPECIAL CASE FOR RESISTORS IN PARALLEL__*

  *If **(N)** resistors having the same resistance **(R)** are connected **IN PARALLEL**, the **TOTAL RESISTANCE (R_T)** is given by :*

  $$R_T = \frac{R}{N}$$

- **PRACTICE QUESTIONS (2)**

1 Calculate the **total resistance** of the resistor combinations shown below :



2 (a) What resistor value needs to be connected **in parallel** with a **20 Ω** resistor to give a combined **total resistance** of **10 Ω** ?

(b) You are provided with several **100 Ω** resistors. How could you combine the **__minimum number__** of these resistors to give a **total resistance** of **250 Ω** ?

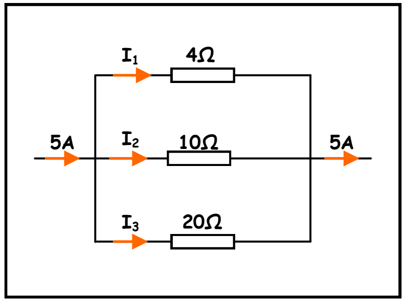
3 You are provided with a **400 Ω** resistor and **two 200 Ω** resistors. Calculate the **total resistances** which may be obtained by connecting **__some__** or **__all__** of these resistors in various combinations.

4 For the resistor network shown calculate :

(a) The **total resistance** of the combination.

(b) The **pd (V)** across each resistor.

(c) The value of the **currents** **(I_1, I_2 & I_3)** through each of the resistors.



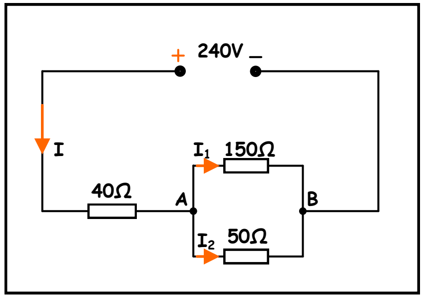
5 For the circuit shown opposite, calculate :

(a) The **resistance (R_{AB})** of the parallel combination.

(b) The **total resistance (R_T)** of the circuit.

(c) The currents **I**, **I_1 and I_2**.

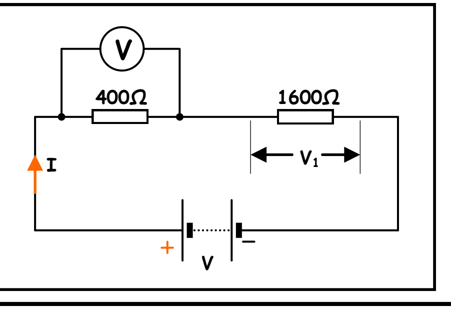
(d) The pd across the **40 Ω** resistor.



6 In the circuit shown opposite the voltmeter gives a reading of **8 V**. Calculate :

(a) The **pd's V** and **V_1**.

(b) The **pd across the 1600 Ω** resistor when the voltmeter Is disconnected.

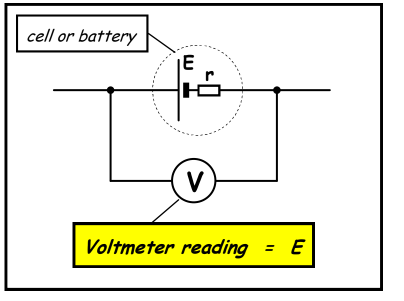


- *EMF (E), TERMINAL PD (V) AND INTERNAL RESISTANCE (r)*

- *All sources of emf have some* ***INTERNAL RESISTANCE (r)***, *since they are made from materials (e.g. metal wires, electrodes, chemical electrolytes) which have some electrical resistance.*

- *If a voltmeter is connected across the terminals of an electrical supply (e.g. a cell or battery), it indicates what is called the* ***TERMINAL PD (V)*** *of the cell or battery.*

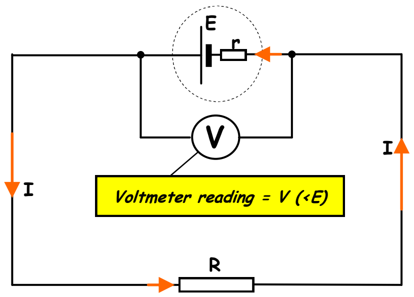
If the cell or battery is not part of an external circuit and the voltmeter is 'ideal' (i.e. it has ***infinite resistance****), then* ***zero current*** *is drawn and :*



voltmeter reading = cell emf (E)

The emf (E) of a cell or battery can be defined as its TERMINAL PD when it is NOT supplying a current.

If an external circuit is connected to the cell or battery (or the voltmeter is not perfect and draws some current), the reading on the voltmeter drops to a value ***less than E***.



This is because when there is a current through the cell, some of its energy is converted into heat by the cell's internal resistance.

The decrease in voltage is called the ***'LOST VOLTS'*** of the cell and it is proportional to the current.

The reading (V) which is < E indicated by the voltmeter is the ***TERMINAL PD*** of the cell and also the ***pd across the resistor R***.

Applying Kirchhoff's Law 2 to the circuit :

Emf of the cell = terminal pd + pd across the internal resistance
(= pd across R)

$$E = V + Ir$$

$$E = IR + Ir = I(R + r)$$

$$I = \frac{E}{(R + r)}$$

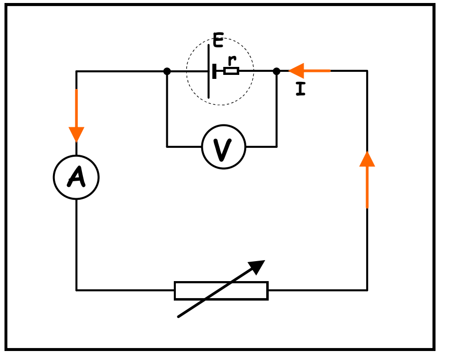
MEASURING EMF (E) AND INTERNAL RESISTANCE (r)

- *A good estimate of the emf (E) of a cell, battery or power supply may be obtained by simply measuring the terminal pd with a* ***DIGITAL VOLTMETER*** *(These have a high resistance and will therefore only draw a very small current).*

Value of E given by the digital voltmeter = V

PROCEDURE

- *The circuit shown opposite is used to obtain a more accurate determination of the* ***emf (E)*** *of a cell as well as its* ***internal resistance (r)****.*



Corresponding values of the ***current (I)*** *in the cell and* ***pd (V)*** *across the cell are obtained by adjusting the variable resistor. The results are recorded in the table shown below.*

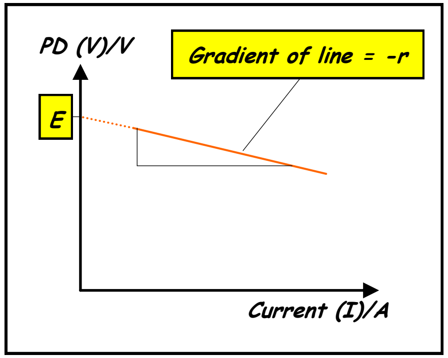
RESULTS

PD (V)/V										
Current (I)/A										

GRAPH

- *When a graph of pd (V) is plotted, a best fit straight line should be obtained as shown opposite.*

__NOTE__ : In practice the graph is curved because (r) changes as the current (I) increases.



ANALYSIS OF GRAPH

- *The equation which links **EMF (E), TERMINAL PD (V), CURRENT(I)** and **INTERNAL RESISTANCE (r)** is :*

$$E = V + Ir$$

Rearranging : $V = -Ir + E$

Comparing with the equation

For a straight line : $y = mx + c$

It can be seen that :

- ***Intercept on the y-axis = E =*** ______ V
- ***Gradient of the V/I graph = r =*** ______ Ω

SOME EFFECTS OF INTERNAL RESISTANCE

- ***Low voltage sources** from which **large currents** are drawn (e.g. car batteries) should have **LOW internal resistance**, otherwise their terminal PD (V = E - Ir) would be **very low.***

- ***High voltage sources** (e.g. 5 kV supplies which are sometimes used in schools) have a **HIGH internal resistance** so as **to limit the current supplied** should they be short-circuited accidentally.*

- *The headlamps on a car will dim if the vehicle is started while they are switched on. This is because the starter motor draws a **large current** and this causes the **battery terminal PD (V = E - Ir)** to drop sharply.*

- *In order to get an **efficient transfer of energy** from a source of emf to an external component of resistance **(R),** the internal resistance **(r)** of the source **<< R.***

PRACTICE QUESTIONS (3)

1 A battery of emf **4.5 V** and internal resistance **1.5 Ω** is connected to a **10 Ω** fixed resistor. Draw a circuit diagram of the arrangement and calculate the **current** in the circuit.

2 A battery of emf **15 V** and internal resistance **1.2 Ω** is connected to an **8 Ω** resistor. Calculate :

(a) The **total resistance** of the circuit.

(b) The **current** through the battery.

(c) The **"lost volts"**

(d) The battery's **terminal pd.**

3 A battery of emf **(E)** and internal resistance **(r)** was connected in series with a variable resistor of resistance **(R)** and an ammeter. If the ammeter reading was **2.0 A** when R was set to **4.0 Ω** and it dropped to **1.5 A** when R was set to **6.0 Ω**, calculate the values of **E** and **r**.

4 The pd across the terminals of a battery is found to be **3.0 V** when it is measured using a **very high resistance** voltmeter. The battery is then connected to a **10 Ω** resistor and its terminal pd drops to **2.8 V**. Calculate the **internal resistance** of the battery.

5 A **high resistance** voltmeter gives a reading of **1.5 V** when connected to a dry cell on **"open circuit"**. When the cell is connected to a lamp of resistance **R,** there is a current of **0.30 A** and the voltmeter reading drops to **1.2 V**. Calculate :

(a) The **emf** of the cell.

(b) The **internal resistance** of the cell.

(c) The value of the **resistance R.**

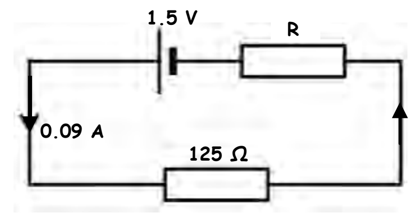
6 A car battery has an emf of **12 V** and an internal resistance of **0. 05 Ω**. The current drawn by the starter motor is **96 A**.

(a) Calculate the **terminal pd** of the battery when the car is being started.

(b) If the headlamps are rated at **12 V, 36 W,** what is their **resistance ?**

(c) Calculate the value of their **power output** when the starter motor is in operation.

1 (a) On which conservation laws are **Kirchhoff's first** and **second laws** based ?



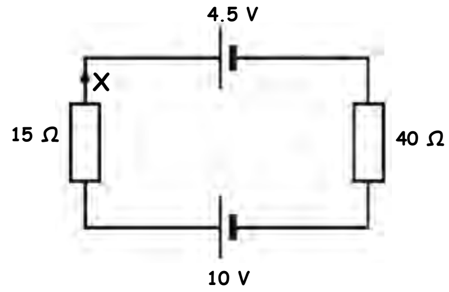
(b) For the circuit shown opposite, calculate :

(i) The **pd across the 125 Ω** resistor.

(ii) The **pd across resistor R.**

(iii) The **resistance of resistor R.**

2 (a) State **Kirchhoff's Second Law.**



(b) Apply Kirchhoff's Second Law to the circuit shown opposite to determine the **current at point X.**

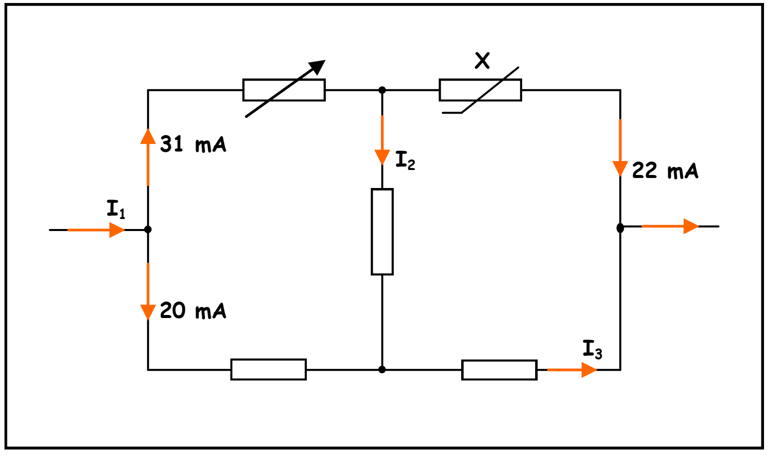
3 You are given three resistors of resistance **4 Ω, 6 Ω and 8 Ω.** Using all the resistors **draw** the combination to give :

(a) The **largest** resistance. (b) The **smallest** resistance.

In each case calculate the **total resistance** of the combination.

4 (a) State **Kirchhoff's First Law.**

(b) The diagram below shows part of an electrical circuit.



(i) Name the component marked **X.**

(ii) Determine the magnitude of the currents I_1, I_2 and I_3.

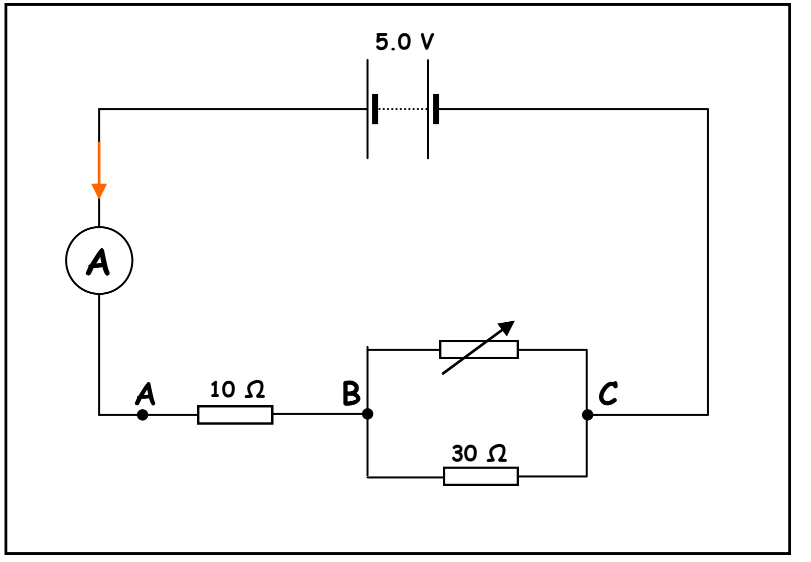
(OCR AS Physics - Module 2822 - January 2004)

5 The results of an experiment to determine the **emf (E)** and **internal resistance (r)** of a power supply are given in the table below.

V/V	1.43	1.33	1.18	1.10	0.98
I/A	0.10	0.30	0.60	0.75	1.00

Plot a suitable graph and use it to find **E** and **r**.

6 The diagram below shows a circuit diagram including three resistors.



(a) The variable resistor is set on its maximum resistance of **20 Ω.**

Calculate the resistance between points :

(i) **B and C.** (ii) **A and C.**

(b) In the circuit shown in the diagram above, the battery has negligible internal resistance and an emf of **5.0 V.** The vaiable resistor is now set on its lowest resistance of **0 Ω.**

Calculate the **reading on the ammeter.**

(OCR AS Physics - Module 2822 - January 2006)