UNIT G481	Module 3	1.3.4	Springs & Materials

- ***Candidates should be able to*** **:**

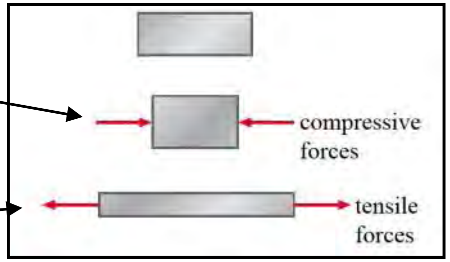
  - *Describe how deformation is caused by a force in one dimension and can be* ***tensile*** *or* ***compressive.***
  - *Describe the behaviour of springs and wires in terms of* ***force, extension, elastic limit, Hooke's Law*** *and the* ***force constant*** *(i.e. force per unit extension or compression).*
  - *Select and apply the equation* $F = kx$, *where* ***k*** *is the force constant of the spring or the wire.*
  - *Determine the* ***area under a force / extension (or compression) graph*** *to find the* ***work done by the force.***
  - *Select and use the equations for elastic potential energy* $E = \frac{1}{2} Fx$ *and* $E = \frac{1}{2} kx^2$.
  - *Define and use the terms* ***stress, strain, Young modulus*** *and* ***ultimate tensile strength (breaking stress).***
  - *Describe an experiment to determine the* ***Young modulus*** *of a metal in the form of a wire.*
  - *Define the terms* ***elastic deformation*** *and* ***plastic deformation*** *of a material.*
  - *Describe the shapes of the* ***stress / strain graphs*** *for typical* ***ductile, brittle*** *and* ***polymeric*** *materials.*

- ***TENSILE & COMPRESSIVE FORCES***

- *A* **pair of forces** *is needed to change the* **size and shape** *of a spring or wire.*

  COMPRESSIVE *forces are applied if the spring is being* **shortened or compressed.**

  TENSILE *forces are applied if the spring is being* **stretched or extended.**

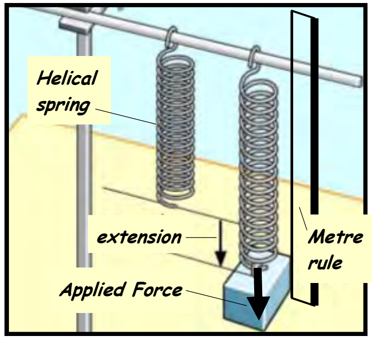


- ***STIFFNESS OF A SPRING***

- *A helical spring hangs from a rod clamped in a retort stand as shown opposite.*

  Using a mass hanger and 100 g slotted masses a force is applied to the spring and this is gradually increased.

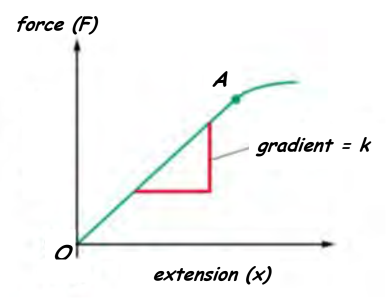
  The **EXTENSION (x)** *(i.e. the increase in length of the spring) produced for each value of the* **APPLIED FORCE (F)** *is recorded in the results table below.*



Applied Force, F/N	*Pointer Reading/m*	*Extension, x/m*
0.0		
1.0		
2.0		
3.0		
4.0		
5.0		
6.0		

- *Using the above results, a graph of* **FORCE (F)/N** *versus* **EXTENSION (x)/m** *is plotted.*

  Section **OA** *of the graph is a straight line passing through the origin, so for This section :*



Extension (x) $\propto$ Force (F)

From which : $\boxed{F = k\,x}$

(N) $\quad$ ($N\,m^{-1}$) $\quad$ (m)

$\boxed{k = F/x}$ - *is called the* **SPRING CONSTANT** *or* **STIFFNESS**.

- *can be determined from the* **GRADIENT OF THE F/x GRAPH**.
- *the* **STIFFER** *the spring is, the* **GREATER** *is the* **k-VALUE**.

- *Beyond point* **A**, *the graph is no longer a straight line. This is because the spring has been* **permanently deformed**; *it has been stretched beyond its* **ELASTIC LIMIT**.

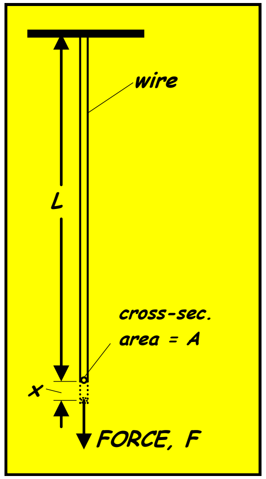
> *The* **ELASTIC LIMIT** *of a sample is that value of the stretching force beyond which the sample becomes permanently deformed (i.e. it stops behaving elastically).*

- *For section* **OA**, *the spring obeys* **HOOKE'S LAW**.

> *A material obeys* **HOOKE'S LAW** *if the* **EXTENSION** *is directly proportional to the* **APPLIED FORCE**. *This is true as long as the material's* **ELASTIC LIMIT** *is not exceeded.*

- ## *TERMS USED IN SPRINGS AND MATERIALS*

- *Consider a wire sample of* ***original length (L)*** *and* ***cross-sectional area (A)*** *subjected to a* ***force (F)*** *and suffering an* ***extension (x)***.



- *The* **STRAIN** *of a material sample is the* **EXTENSION** *produced per* **UNIT LENGTH**.

$$\text{strain} = \frac{\text{extension}}{\text{original length}} = \frac{x}{L}$$

NOTE : *Strain has* **no units** *and it is sometimes given as* **a percentage**.

- *The* **STRESS** *on a material sample is the* **FORCE** *acting per unit* **CROSS-SECTIONAL AREA** *of the sample.*

$$\text{Stress} = \frac{\text{force}}{\text{x-sec. area}} = \frac{F\ (N)}{A\ (m^2)}$$

The unit of **STRESS** *is the* **PASCAL (Pa)**.

The **STIFFNESS** *of the material being stressed is called the* **YOUNG MODULUS (E)** *of the material.*

$$\text{YOUNG MODULUS, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{x/L} = \frac{F\,L}{A\,x}$$

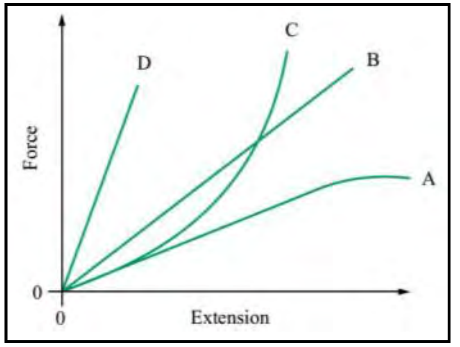
NOTE

- *The unit of E is the* **PASCAL (Pa)** *- (1 Pa = 1 N m^{-2}).*
- *E is usually a large number and so it is sometimes given in :*

  megapascal (MPa) *(i.e. Pa x 10^6)* *or* **gigapascal (GPa)** *(i.e. Pa x 10^9).*
- **E = Gradient of a stress/strain graph**

- **PRACTICE QUESTIONS (1)**

1 The diagram opposite shows the force/extension graphs for four springs labelled A to D.



Which of the four springs :

(a) Has the greatest value of **spring constant** ?

(b) Is the **least** stiff ?

(c) Does not obey **Hooke's Law** ?

2 The table opposite shows the **Young modulus** for a list of different materials. (1 Gpa = 10^9 Pa).

(a) List the metals in the table from **stiffest** to **least stiff** (by labelling them **M1, M2..**).

(b) Which **non-metal** in the table is the stiffest ? (Label it as **NM 1**).

Material	Young modulus/GPa
aluminium	70
brass	90–110
brick	7–20
concrete	40
copper	130
glass	70–80
iron (wrought)	200
lead	18
Perspex	3
polystyrene	2.7–4.2
rubber	0.01
steel	210
tin	50
wood	10 approx.

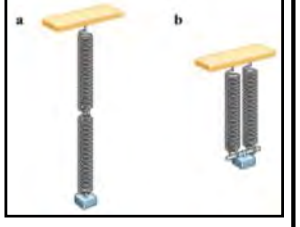
3 A metal bar of length **100 mm** and square cross section of side **20 mm** is extended by **0.030 mm** when it is subjected to a tensile force of **24 kN**.

Calculate : (a) The **stress** and the **strain** in the bar.

(b) The **Young modulus** for the material of the bar.

4 Calculate the extension of a copper wire of length **1.25 m** and radius **0.55 mm** when a tensile force of **25 N** is applied to the end of the wire. (Young modulus for copper = **1.30 x 10^{11} Pa**).

5 Springs can be combined as in (a) In **SERIES**, or as in (b) In **PARALLEL**.



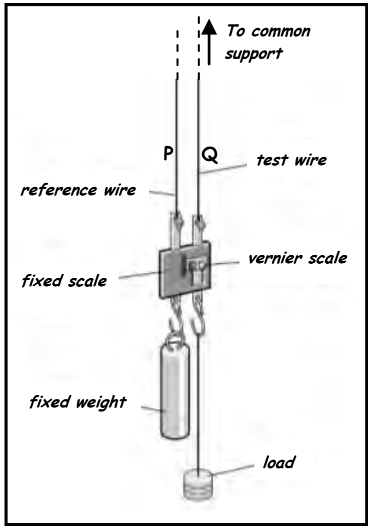
If the spring constant of a single spring is **'k'**, what is the equivalent **spring constant** for two springs : (a) In **SERIES ?**

(b) In **PARALLEL ?**

DETERMINATION OF THE YOUNG MODULUS (E)

- *Two long wires* **(P & Q)** *of the same material, length and diameter are hung from a common support.* **Q** *is the wire under test and* **P** *is the comparison wire which is used as a reference so as to avoid errors due to :*

  - Expansion occurring as a result of temperature change.
  - Sagging of the support.



- *The* **ORIGINAL LENGTH (L)** *of wire* **Q** *is measured using a metal tape measure.*

- *The* **CROSS-SECTIONAL AREA (A)** *is determined by using a micrometer screw gauge to measure the diameter of* **Q** *at several points along the length of the wire. The mean diameter and hence the mean radius (R) is calculated. Then* $A = \pi R^2$.

- *The* **EXTENSION (x)** *of wire Q when it is loaded, is accurately measured by the vernier arrangement between P and Q.*

- *The test wire Q is then incrementally loaded and the corresponding extensions are measured and noted. The results are used to plot a graph of* **FORCE (LOAD) (F)** *versus* **EXTENSION (x)** *whose* **gradient = F/x**. *Then :*

$$\text{Young modulus, } E = \frac{\text{stress}}{\text{Strain}} = \frac{F/A}{x/L} = \frac{F\,L}{x\,A} = \text{gradient of } F/x \text{ graph} \times \frac{L}{A}$$

ELASTIC AND PLASTIC DEFORMATION

- ***ELASTIC*** *behaviour is shown by a wire or spring if it returns to its original length when the applied deforming force (load) is removed.*

  - *All materials show* ***ELASTIC*** *behaviour up to the* ***ELASTC LIMIT***.
  - *When a sample (e.g. spring, wire..) is loaded beyond its* ***ELASTIC LIMIT***, *it does not regain its original dimensions when the load is removed (i.e. it suffers permanent deformation).*

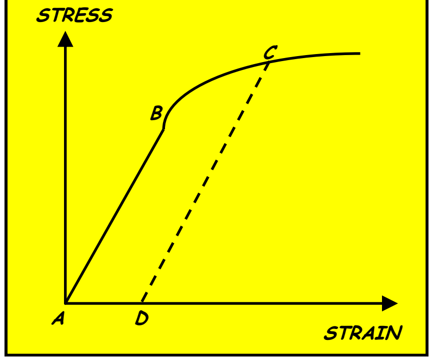
- ***PLASTIC*** *behaviour is shown by some materials when they are loaded beyond the elastic limit. The material is permanently deformed (or strained) when the load is*

- **A to B**

  There is ***ELASTIC*** *deformation - When the stress is removed, the sample goes back to its original dimensions (i.e. there is zero strain).*

- **B onwards**

  There is ***PLASTIC*** *deformation - When the stress is removed, the sample is left permanently deformed (i.e. there is a residual strain =* ***AD****).*

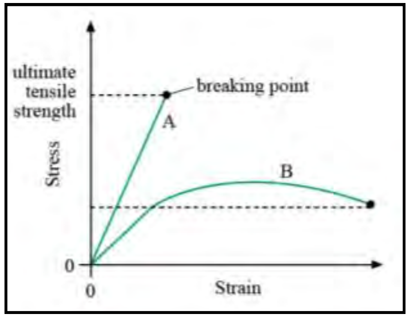


• *MATERIAL STRENGTH*

• *When we talk about the **STRENGTH** of a material, we are referring to the **STRESS** value at which the material breaks.*

> *The **ULTIMATE TENSILE STRENGTH** or **BREAKING STRESS** of a material is the stress value at which the material breaks.*

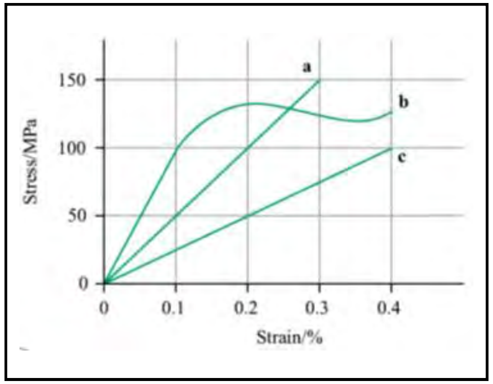
*Consider the **STRESS/STRAIN** graphs for two different materials **A & B** shown opposite.*



***Material A** has a **greater UTS value** than **material B** and this means that material A is **stronger** than material B.*

• PRACTICE QUESTIONS (2)

1 For each of the materials *a, b and c* whose ***STRESS/STRAIN*** graphs are shown opposite :

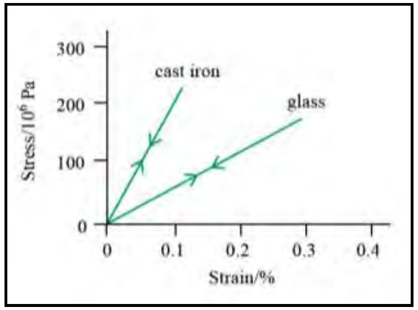


(a) Calculate the value of the ***YOUNG MODULUS (E).***

(b) State the ***ULTIMATE TENSILE STRENGTH.***

> ***BRITTLE MATERIALS** (e.g. glass, cast iron)*

• *As the stress on a **brittle** material is gradually increased, it stretches slightly, but further increase in the applied stress causes fracture.*

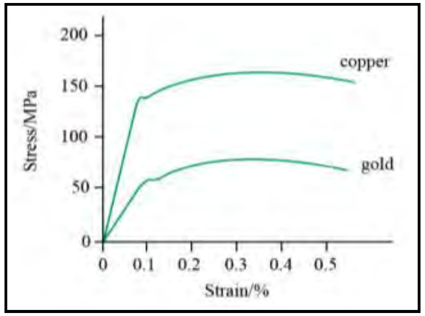


• ***Brittle** materials show **ELASTIC** behaviour up to the point of fracture (Up to that point, if the applied stress is removed the sample returns to its original length).*

• *Sudden application of a large stress will cause a **brittle** material to shatter (e.g. dropping it onto a hard floor).*

> ***DUCTILE MATERIALS** (e.g. copper, gold)*

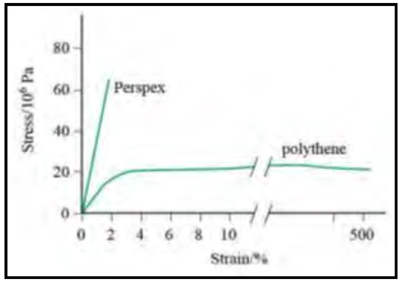
• *As stress is applied to a **ductile** material it will behave elastically up to the **ELASTIC LIMIT**, but beyond this stress value the sample stretches more and more and it does not return to its original length when the stress is removed. The material shows **PLASTIC** behaviour and it is then permanently strained.*



• ***Ductile** materials can be shaped by stretching, rolling, squashing and hammering (useful for making wires, jewellery etc.).*

POLYMERIC MATERIALS *(e.g. polythene, perspex)*

- *The behaviour of different polymers depends on their **temperature** as well as their **molecular structure.***



- *The infamous shopping carrier bag is made of **polythene** and as we all know, it is very stretchy and easy to deform. Under stress, polythene quickly undergoes **PLASTIC** deformation and stretches a lot before becoming much stiffer and snapping (behaving much like a **DUCTILE** metal).*

- ***Perspex** is a very tough, hard plastic which, when stressed, stretches elastically until it snaps (**BRITTLE** material behaviour). When it is warmed however, it ceases to be brittle and can be moulded into any shape. It is used for contact lenses, dentures, artificial eyes and a huge variety of other industrial uses.*

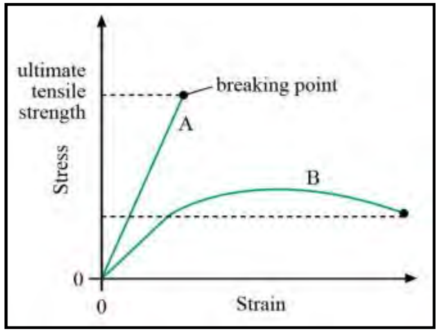
MATERIAL BEHAVIOUR SUMMARY

- *All materials show **ELASTIC** behaviour up to the **ELASTIC LIMIT**.*
- ***BRITTLE** materials break at the **ELASTIC LIMIT** and shatter when subjected to a large , sudden stress.*
- ***DUCTILE** materials exhibit **PLASTIC** behaviour (i.e. suffer permanent deformation) when they **are stretched beyond the ELASTIC LIMIT.***

- ***ULTIMATE TENSILE STRENGTH (UTS)***

- *The value of the stress at which a material breaks is called the **ULTIMATE TENSILE STRESS (UTS)** or the **BREAKING STRESS**.*

  *In the stress/strain graph shown opposite, material A is stronger than material **B**, since its UTS is greater.*



- ***ELASTIC POTENTIAL (STRAIN) ENERGY***

- *This is the **energy stored** when a solid is **stretched** or **compressed**.*

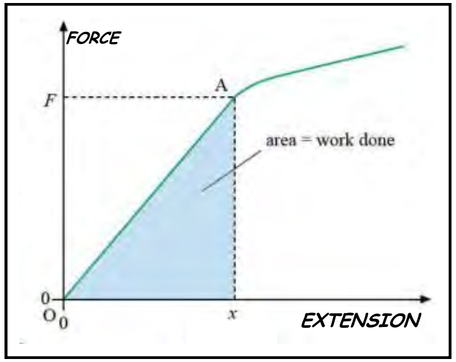
- *If the solid has been strained **ELASTICALLY**, the stored energy can be recovered.*

  *If the solid has been strained **PLASTICALLY**, some of the work done has gone into moving atoms past each other and the energy is non-recoverable.*

- *Consider a wire subjected to a gradually increasing force.*

  Up to the elastic limit :

  Extension $\propto$ ***applied force***



Initial value of force = 0
Final value of force = F
Average value of force = $\frac{1}{2}$ F

Elastic Potential Energy stored in the wire = Work done by the average force
= average force x extension
= $\frac{1}{2}$F x x

Elastic Potential energy stored in the wire = $\frac{1}{2}$ Fx
(J) (N) (m)

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- *Elastic Potential Energy stored in the wire = Area Enclosed by the F/x Graph*

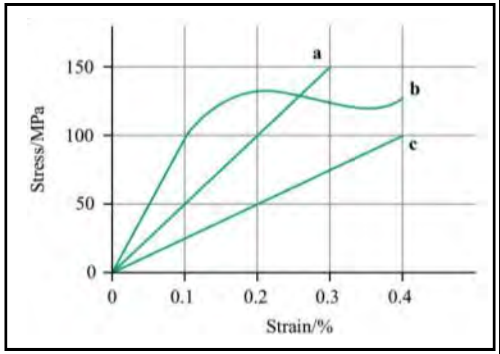
  $= \frac{1}{2} \times base \times height$

  $= \frac{1}{2}\, x \times F$

  $= \frac{1}{2} Fx$

- **PRACTICE QUESTIONS (3)**

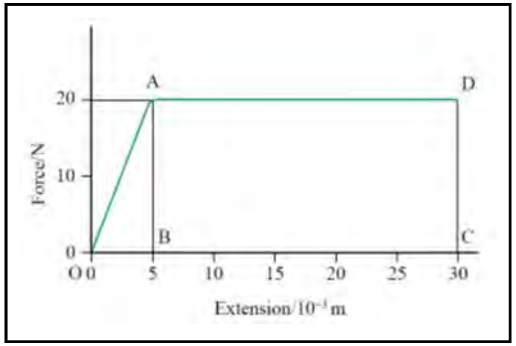
1 For each of the materials, ***a, b*** and ***c*** shown in the stress/strain graph opposite, deduce the values of the ***YOUNG MODULUS*** and the ***ULTIMATE TENSILE STRESS.***



2 The diagram opposite shows a simplified **FORCE - EXTENSION** graph for a metal.

Use the graph to calculate :

(a) The **STRAIN ENERGY** stored when the metal is stretched to its **ELASTIC LIMIT**.



(b) The **TOTAL WORK** which needs to be done to break the metal.

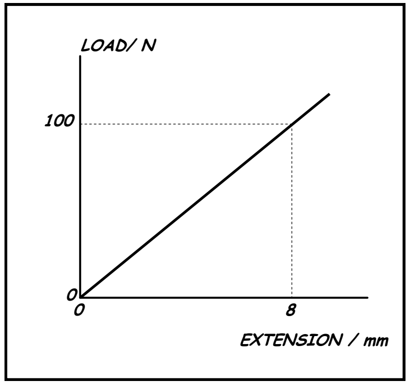
3 A spring has a force constant of **3.8×10^3 N m^{-1}**. Calculate the **ELASTIC POTENTIAL ENERGY** stored in such a spring when it is stretched by **8.5 mm.**

4 An elastic string of cross-sectional area **3 mm^2** and length **2.5 m,** stretches by **2.0 cm** when a force of **4.0 N** is applied to it.

Calculate : (a) The **YOUNG MODULUS** of the string.

(b) The **ELASTIC POTENTIAL ENERGY** stored in the string when it is stretched by **2.0 cm.**

5 The graph opposite shows the variation of extension with applied load for a wire of length **4.0 m** and radius **0.25 mm.**



Calculate :

(a) The **STRESS** in the wire when the load is **100 N.**

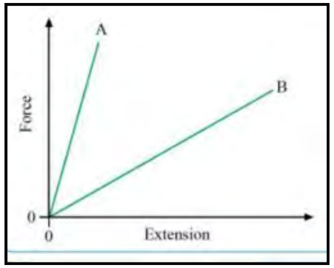
(b) The **ELASTIC POTENTIAL ENERGY** stored in the wire when the load is **100 N.**

6 The diagram shows the force/extension graph for two pieces of polymer. **State with an explanation** which polymer :

(a) Has the ***greater stiffness.***

(b) Requires the ***greater force*** to break it.

(c) Requires the ***greater amount of work*** to be done in order to break it.



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- **HOMEWORK QUESTIONS**

1 (a) (i) Define ***STRESS.*** (ii) Define ***STRAIN.***

(b) Describe an experiment to determine the ***YOUNG MODULUS*** of a metal in the form of a wire. Your description should include :

- A labelled diagram of the apparatus.
- The measurements to be taken.
- An explanation of how the equipment is used to obtain the measurements.
- An explanation of how the measurements would be used to determine the Young modulus.

(OCR AS Physics - Module 2821 - Jan 2005)

2 (a) Define the ***YOUNG MODULUS.***

(b) The wire used in a piano string is made from steel. The original length of wire used was ***0.75 m.*** Fixing one end and applying a force to the other stretches the wire. The extension produced is ***4.2 mm.***

(i) Calculate the ***STRAIN*** produced in the wire.

(ii) The Young modulus of the steel is 2.0×10^{11} Pa and the cross-sectional area of the wire is 4.5×10^{-7} m^2. Calculate the ***FORCE*** required to produce the strain in the wire calculated in (i).

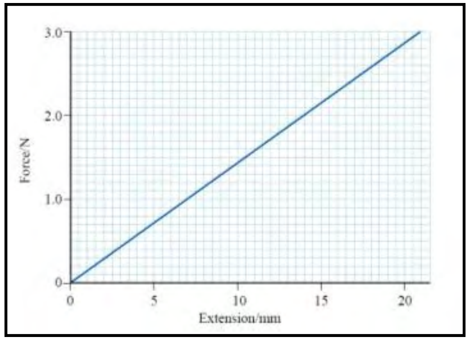
(c) A different material is used for one of the strings in the piano. It has the same length, cross-sectional area and force applied. Calculate the ***EXTENSION*** produced in this wire if the Young modulus of this material is ***half*** that of steel.

(d) (i) Define ***DENSITY.***

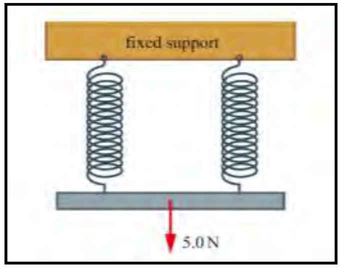
(ii) ***State and explain*** what happens to the density of the material of a wire when it is stretched. Assume that when the wire stretches, ***the cross-sectional area remains constant.***

(OCR AS Physics - Module 2821 - June 2004)

3 Part of a force against extension graph for a spring is shown opposite. The spring obeys ***HOOKE'S LAW*** for forces up to ***5.0 N.***



(a) Calculate the ***EXTENSION*** produced by a force of ***5.0 N.***



(b) The diagram above shows a second identical spring that has been put in parallel with the first spring. A force of ***5.0 N*** is applied to this combination of springs.

For this arrangement, calculate :

(i) The ***EXTENSION*** of each spring.

(ii) The ***ELASTIC POTENTIAL ENERGY*** stored in the springs.

(c) The Young modulus of the wire used in the springs is ***2.0×10^{11} Pa.*** Each spring is made from a straight wire of length ***0.40 m*** and cross-sectional area ***2.0×10^{-7} m^2.*** Calculate the ***EXTENSION*** produced when a force of ***5.0 N*** is applied to this straight wire.

(d) Describe and explain, ***without further calculations***, the ***difference in the elastic potential energies*** in the straight wire and in the spring when a force of 5.0 N is applied to each.

(OCR AS Physics - Module 2821 - June 2006)

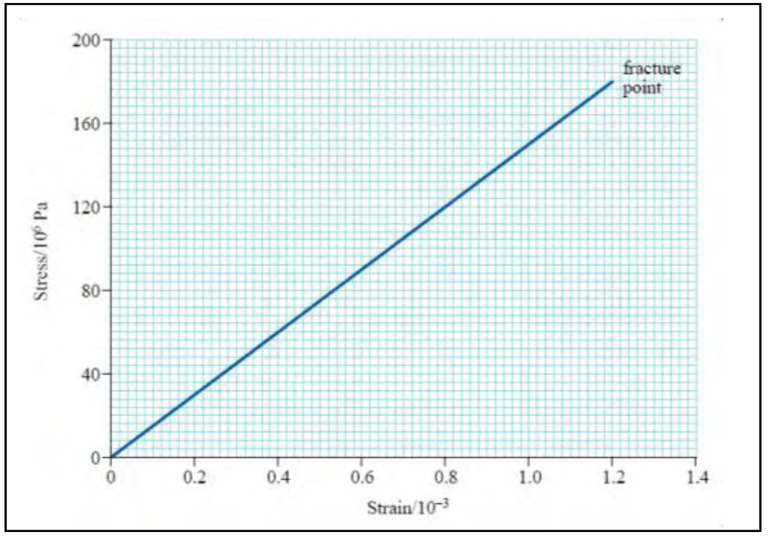
4 Use the words ***ELASTIC, PLASTIC, BRITTLE*** and ***DUCTILE*** to deduce what the following observations tell you about the materials described.

(a) If you tap a ***cast iron*** bath gently with a hammer, the hammer bounces off. If you hit it hard, the bath shatters.

(b) ***Aluminium*** drinks cans are made by forcing a sheet of aluminium into a mould at high pressure.

(c) ***'SILLY PUTTY'*** can be stretched to many times its original length if it is pulled gently and slowly. If it is pulled hard and rapidly, it snaps.

5



The diagram above shows a stress against strain graph up to the point of fracture For a rod of cast iron.

(a) The rod of cast iron has a cross-sectional area of ***1.5×10^{-4} m²***. Calculate :

(i) The ***FORCE*** applied to the rod ***at the point of fracture.***

(ii) The ***YOUNG MODULUS*** of cast iron.

(b) Using the graph or otherwise, describe the ***stress against strain behaviour*** of the cast iron up top and including the point of fracture.

(OCR AS Physics - Module 2821 - Jan 2006)