- ***Candidates should be able to*** **:**

  - *Select and apply the equation for* ***kinetic energy*** *:*

    $$\boxed{E_k = \tfrac{1}{2}mv^2}$$

  - *Apply the definition of work done to derive the equation for the* ***change in gravitational potential energy.***

  - *Select and apply the equation for the* ***change in gravitational potential energy*** *near the Earth's surface :*

    $$\boxed{\Delta Ep = m\ g\ \Delta h}$$

  - *Analyse problems where there is an exchange between* ***gravitational potential energy*** *and* ***kinetic energy.***

  - *Apply the principle of conservation of energy to determine the speed of an object falling in the Earth's gravitational field.*

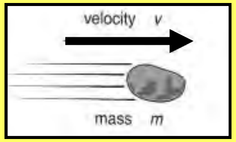
- *KINETIC ENERGY (E_k)*

- *This is the energy possessed by a* ***moving object.***

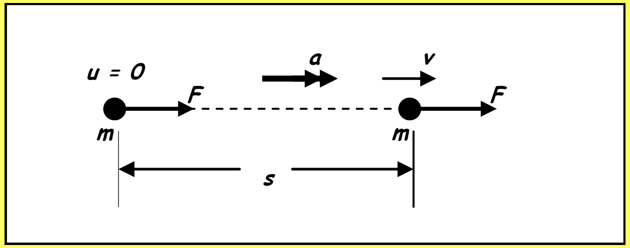
- ***KINETIC ENERGY = ½ x MASS x SPEED²***

  $$E_k = \tfrac{1}{2}\ m\ v^2$$

  (J) (kg) (m s⁻¹)



- **DERIVATION OF $E_k = \frac{1}{2}mv^2$**



Consider an object of ***mass (m)*** *acted on by a constant* ***force (F)*** *which gives it a constant* ***acceleration (a)****, and increases its velocity from* ***rest*** *to a final value* ***(v)*** *over a* ***distance (s).***

kinetic energy gained by object, E_k = *work done by force F*

E_k = *force x distance moved in the force direction*

$E_k = F \times s$

$E_k = mas$ *(since F = ma)*

But $v^2 = u^2 + 2as = 2as$ *(since u = 0)* *So* $as = \frac{1}{2}v^2$

Therefore :

$$\boxed{E_k = \tfrac{1}{2}\ m\ v^2}$$

- **PRACTICE QUESTIONS (1)**

1 A motorcycle has ***5×10^4 J*** of kinetic energy. If the brakes deliver a braking force of ***650 N***, calculate the ***shortest stopping distance*** for the motorcycle.

2 Calculate the ***increase in kinetic energy*** of a vehicle of mass ***1200 kg*** when it accelerates from ***10 m s^{-1}*** to ***25 m s^{-1}***.

3 A bullet of mass ***8.0 g*** is given ***160 J*** of kinetic energy when it is fired from a gun. Calculate the ***velocity*** of the bullet as it leaves the gun barrel.

4 Use the internet to find approximate masses and speeds of each of the following and hence estimate a value for their kinetic energy :

- A loaded family car travelling along a motorway at the speed limit.
- A male Olympic 100 m sprinter.
- A fully laden Jumbo jet aircraft at normal cruising speed.
- A tennis ball served by a Wimbledon champion.
- An electron travelling at 6.8×10^8 m s^{-1} as it exits a linear accelerator.
- The Earth moving at its orbital speed around the Sun.

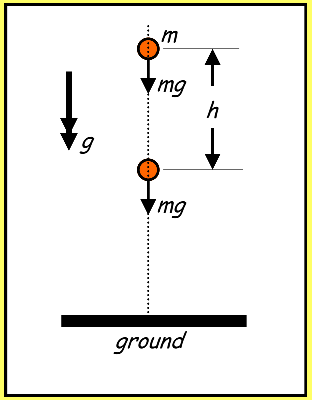
- *When an object is lifted to a higher position above the ground,* ***work is done against the force of gravity*** *and this transfers* ***gravitational potential energy*** *to the object (Strictly speaking it is the Earth-object system which gains gravitational potential energy).*

- ***GRAVITATIONAL POTENTIAL ENERGY (E_p) is the energy possessed by an object due to its position or height above the Earth.***

DERIVATION OF $E_p = m\,g\,h$

Consider an object of ***mass (m)*** *which is raised through* ***height (h)*** *above the ground.*

E_p gained by object = work done in lifting
= force x distance moved
= object weight x height Lifted
= mg x h



Therefore :

Change in gravitational potential energy (E_p) is given by :

$$\boxed{E_p = m\,g\,h}$$

- **PRACTICE QUESTIONS (2)**

1 An athlete of mass ***76 kg*** runs up a hill in the Lake District. He starts at a point ***200 m*** above sea-level and finishes at the summit which is ***950 m*** above sea-level. Calculate the athlete's increase in ***gravitational potential energy (g = 9.81 N kg^{-1}).***

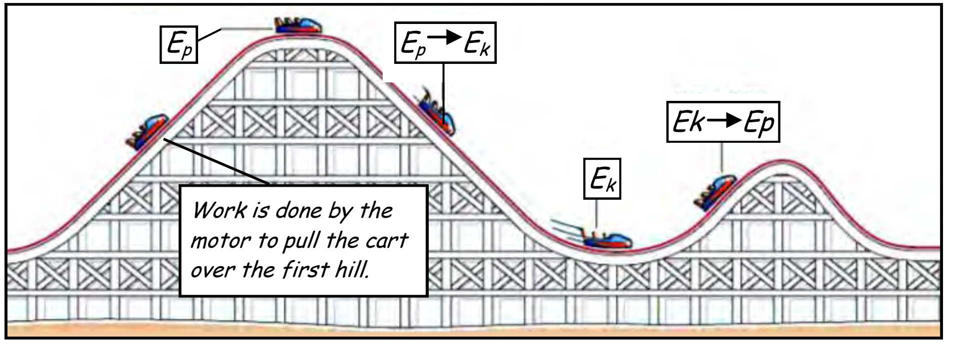
2 A catapult has ***25 J*** of elastic energy. If all its elastic energy is used to project a marble of mass ***5.0 g*** vertically upwards, what is the ***maximum height*** reached by the marble?
(Take ***g = 9.81 N kg-1***).

3 A ball of mass ***0.25 kg*** drops from a height of ***12 m*** and rebounds to a height of ***8.5 m.*** Assuming negligible air resistance, calculate the ***energy lost on impact*** with the ground ***(g = 9.81 N kg^{-1}).***

- ***E_p-E_k TRANSFORMATIONS ON A ROLLER COASTER***

- *A roller coaster provides us with an excellent example of transformations of gravitational potential energy to kinetic energy and vice versa.*

  A motor pulls the cart over the top of the first hill. It then runs down the other side, accelerating as it goes. The second hill is lower than the first and the cart is moving just fast enough to make it over the top and once again accelerate down the second slope. The work done on the cart by the motor is transferred to gravitational potential energy which is then transformed to kinetic energy as it speeds down the slope.

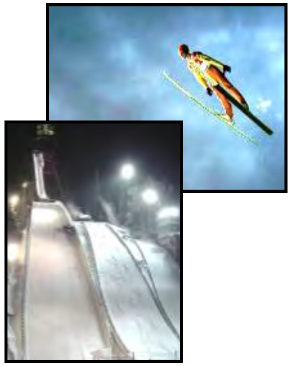


- *The diagram above shows the cart at various positions on the roller coaster as well as the energy transformations which occur at each point.*
  - *At the top of the first hill, the cart is momentarily stationary and only has **E_p**.*
  - *As it accelerates down the slope it loses **E_p** and gains **E_k** (i.e. **E_p** is being transformed into **E_k**).*
  - *At the bottom of the slope, the cart's initial **E_p** has been transformed into **E_k**.*
  - *As it runs up the second hill, work is done against gravity and so the cart slows down. It loses **E_k** and gains **E_p** (i.e. **E_k** is being transformed into **E_p**).*

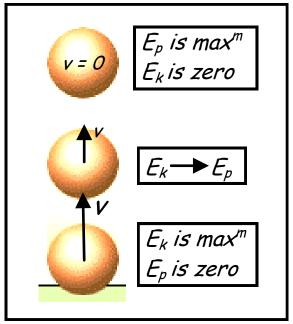
As the cart moves along, some of its kinetic energy is used to do work against friction and air resistance. Thus, some of the cart's kinetic energy is transformed into heat and sound energy and it is therefore unavailable for transformation into gravitational potential energy. For this reason, the cart cannot return to its original height and so the second hill must be lower than the first and the third must be lower than the second and so on.

- *There are many other examples of Ep – Ek transformation.*

  - *In the Winter Olympics **Ski Jump** competitors slide down a long snow-covered slope from a great height. E_p is transformed into E_k and this is used by the jumper to achieve the maximum possible jump distance.*

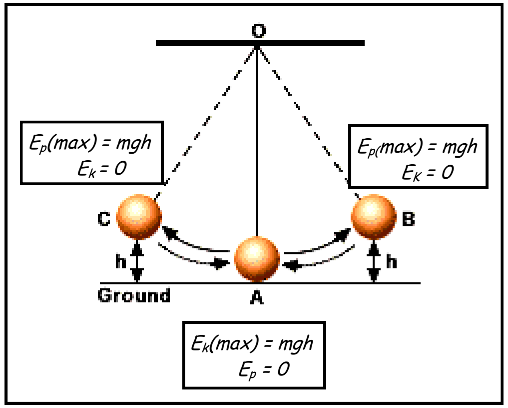


  - *If an object is thrown upwards, its initial E_k is transformed into E_p as it rises and slows down. Eventually, when it reaches the maximum height, all the E_k is transformed into E_p (assuming zero air resistance).*



- *In the case of an oscillating **simple pendulum**, there is a continuous interchange of E_p and E_k as the bob moves from C→A→B→A→C.*

  *At **C and B** where the bob momentarily comes to rest, the bob has **zero Ek** and **maximum E_p** and at **A** where the bob is moving at its **maximum velocity**, **E_p is zero** and **E_k = mgh has its max value.***

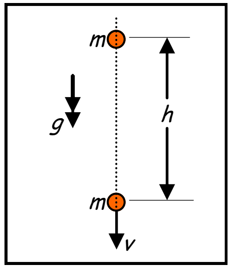


- *As we have seen in all the examples considered, when an object falls its E_p **decreases** and its E_k **increases**. Assuming no energy is lost in the process :*

$$E_p \text{ lost} = E_k \text{ gained}$$

*This relation can be used to solve a variety of problems, such as the **velocity** attained by an object when it falls from a given height.*

*Consider an object of **mass (m)** which Falls from a **height (h)** above the ground.*



$$E_k \text{ gained} = E_p \text{ lost}$$

$$\tfrac{1}{2}mv^2 = mgh$$

From which : $v = \sqrt{(2gh)}$

• PRACTICE QUESTIONS (3)

1 A high diver reaches the highest point in his jump at which his centre of gravity is ***11.4 m*** above the water surface. Assuming that all the diver's gravitational potential energy is transformed into kinetic energy during the dive, calculate the ***velocity*** with which he enters the water (Take $g = 9.81\ m\ s^{-2}$).

2 An object of mass ***0.75 kg*** is projected vertically upwards with a velocity of $12\ m\ s^{-1}$. If it reaches a height of ***6.75 m,*** calculate the energy loss caused by air resistance (Take $g = 9.81\ m\ s^{-2}$).

3 A steel ball bearing of mass ***0.05 kg*** at a height of ***2.0 m*** above a steel table is released from rest and it is found to rebound to a height of ***1.8 m.*** Calculate :

(a) The ***gravitational potential energy*** lost during the fall.

(b) The ***kinetic energy*** and ***velocity*** of the ball bearing just before impact.

(c) The ***gravitational potential energy*** gained by the ball bearing when it rebounds to a height of ***1.8 m.***

(d) The ball bearing's ***rebound velocity.***

(Take $g = 9.81\ m\ s^{-2}$).

• HOMEWORK QUESTIONS

1 Describe the energy changes that occur in each of the following :

(a) A cyclist freewheels from rest down a hill and then uses the brakes to stop at the bottom.

(b) The bob on a simple pendulum is displaced from equilibrium with the thread taut and then released. The bob swings across to maximum displacement on the other side of the equilibrium position.

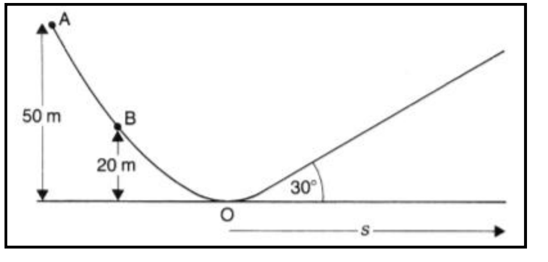
2 A rock falls from the top of a ***75 m*** high cliff and strikes the ground at the bottom with a velocity of $35\ m\ s^{-1}$.

(a) What ***percentage of the rock's initial gravitational potential energy*** is transformed into kinetic energy as a result of the fall ?

(b) Explain what happens to the rest of the rock's initial energy.

3 The diagram opposite shows the vertical section through a ski track.

A skier of mass ***76 kg*** starts from rest at ***A.***

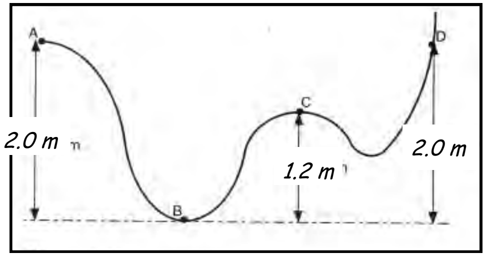


Assuming ***friction to be negligible,*** calculate :

(a) The skier's ***velocity at point B.***

(b) The ***maximum horizontal distance (s)*** from point O that the skier reaches.

4 A toy car of mass ***0.50 kg*** is released from point ***A*** on a frictionless track.



Calculate the car's :

(a) ***Kinetic energy*** at point ***B.***

(b) ***Velocity*** at point ***B.***

(c) ***Gravitational potential energy*** at point ***C.***

(d) ***Kinetic energy*** at point ***C.***

(e) ***Velocity*** at point ***C.*** (Take $g = 9.81\ m\ s^{-2}$).

5 A roller coaster cart of total mass ***1500 kg*** moving with an initial velocity of $2\ m\ s^{-1}$ descends through a height of ***70 m*** to reach a velocity of $36\ m\ s^{-1}$ after travelling a distance of ***120 m*** along the track.

(a) Calculate : (i) Its ***loss of gravitational potential energy.***

(ii) Its ***gain of kinetic energy.***

(b) Show that the ***average frictional force*** acting on the cart during the descent was ***500 N.***