- ***Candidates should be able to*** :

  - *Draw and use a **triangle of forces** to represent the equilibrium of three forces acting at a point in an object.*
  - *State that the **CENTRE OF GRAVITY** of an object is a point where the entire weight of an object appears to act.*
  - *Describe a simple experiment to determine the centre of gravity of an object.*
  - *Explain that a **COUPLE** is a pair of forces that tends to produce rotation only.*
  - *Define and apply the **TORQUE** due to a couple.*
  - *Define and apply the **MOMENT** of a force.*
  - *Explain that both the **NET FORCE** and the **NET MOMENT** on an extended object in equilibrium is **zero.***
  - *Apply the **PRINCIPLE OF MOMENTS** to solve problems, including the human forearm.*
  - *Select and use the equation for **DENSITY** - $\rho = m/V$*
  - *Select and use the equation for **PRESSURE** - $p = F/A$*

    Where F is the force normal to the area A.

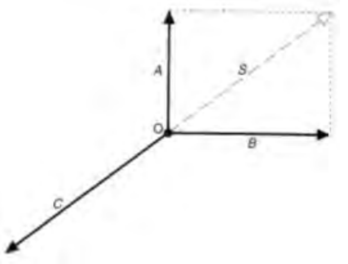
- ***TRIANGLE OF FORCES RULE***

If an object is in equilibrium under the action of three coplanar forces, and these forces are represented in magnitude and direction by vectors drawn to scale and drawn in order, they will form a closed triangle.

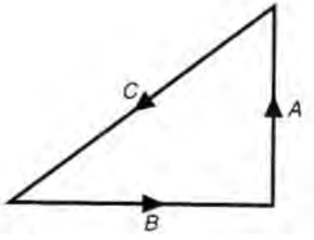
*The particle O has forces **A, B and C** acting on it.*

***A and B** can be replaced by a single force **S** which is equal and opposite to force **C**.*

*This means that the three forces A, B and C are **balanced** (i.e. object O is **in equilibrium** under the action of forces A, B and C).*



The three forces can then be represented in magnitude and direction by vectors A, B and C drawn to scale, with the head of each arrow joining with the tail of the next.



NOTE

- ***This rule can be applied to any vector quantities (e.g. velocity, Momentum etc..)***

- **EXAMPLE**

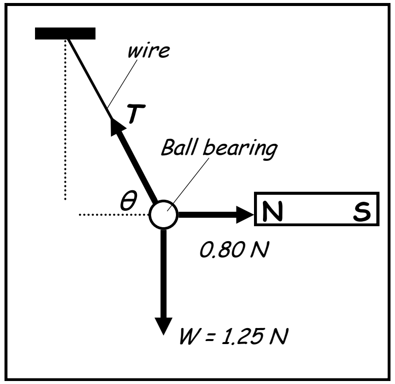
*A steel ball bearing of weight **1.25 N** is suspended using a length of piano wire attached to it.*

*The ball is then pulled into the position of equilibrium shown in the diagram by a horizontal force of **0.80 N** exerted by a bar magnet which is held close to it.*

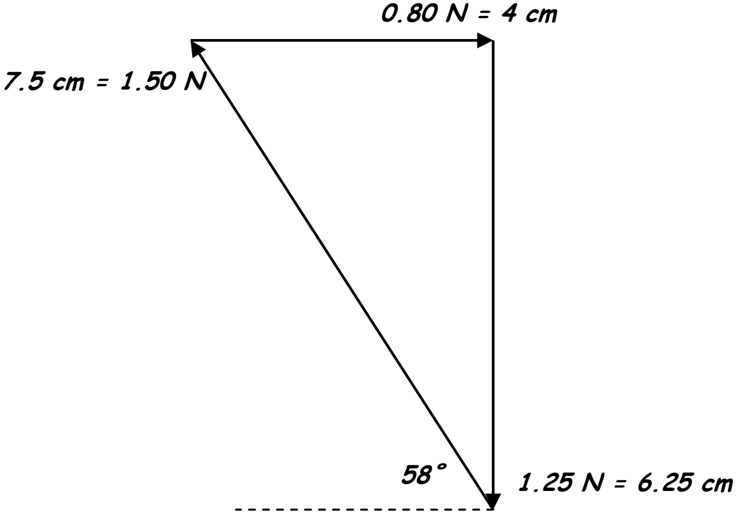
*Determine the magnitude and direction of the **TENSION (T)** in the wire :*

*(a) Using the **TRIANGLE OF FORCES** rule.*

*(b) Using **RESOLUTION OF FORCES**.*



(a) We will use a scale of **1 cm = 0.2 N** and start with the 0.80 N force.



*The force vector that closes the triangle is found to be ≈ **7.5 cm** long. This makes the **TENSION** in the wire ≈ **1.50 N** at an angle of about **58° to the horizontal.***

(b) **Resolving vertically :** $T\sin\theta = 1.25$ .... (1)

Resolving horizontally : $T\cos\theta = 0.80$ .... (2)

(1) ÷ (2) : $\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{1.25}{0.80}$

$\tan\theta = 1.5625$

$\theta = \boxed{57.4°}$

$T\sin 57.4° = 1.25$

$T = \boxed{1.48\ N}$

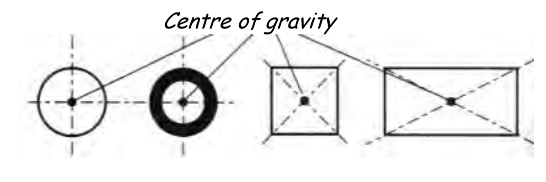
- ***CENTRE OF GRAVITY (c_g)***

- *Every particle in an object is acted on by the force of gravity and therefore has weight, but the resultant force of all these downward forces acts or appears to act at a single point which is called the **CENTRE OF GRAVITY**.*

- ***The CENTRE OF GRAVITY (c_g) of an object is the point at which the weight of the object acts or appears to act.***

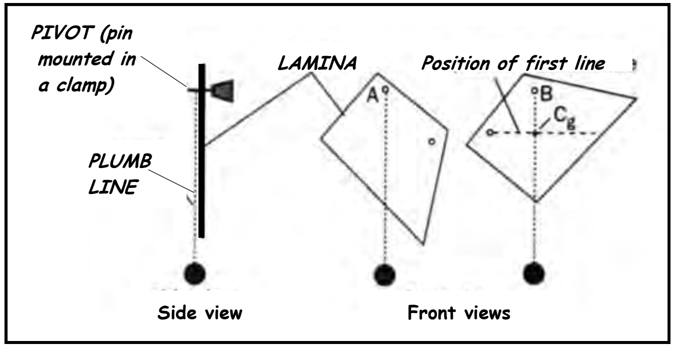
*At this level, we are only concerned with **FLAT** or **LAMINAR** objects, although the principle applies to any object.*

The diagram opposite shows the position of the centre of gravity for some regularly shaped laminas.



- *If an object is supported from a point directly above (or below) its centre of gravity, it will balance since its weight creates no moment about the support. If the object is supported at any other point, the weight creates a turning moment which unbalances the object.*

- ***FINDING THE CENTRE OF GRAVITY OF AN IRREGULAR LAMINA***



- *Suspend the lamina from a clamped pin through a hole **(A)** drilled near one edge as shown in the diagram above.*

- *Hang a plumb line from the pin and use a fine marker pen to mark the vertical position of the string onto the lamina.*

  *Since the plumb line and the lamina must hang with their centres of gravity directly below the point of suspension, the position of the **centre of gravity** (c_g) of the lamina must lie somewhere along the line marked by the pen.*

- *Hang the lamina from a second hole **(B)** and once again mark the plumb line's position onto the lamina.*

  The lamina's centre of gravity is at the point where this line intersects the previously marked line.

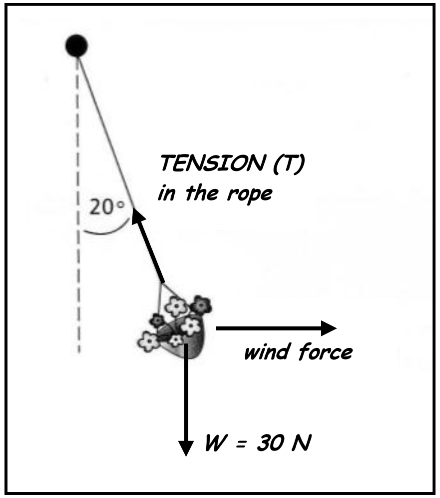
- *The lamina can be hung from a third hole and the procedure repeated in order to check the position of the c_g. You should be able to balance the lamina from this point.*

1 A point object of weight ***5.6 N*** is acted on by a horizontal force of ***3.8 N*** acting to the right and another force ***F***. The object is in equilibrium.
Apply the ***triangle of forces rule*** to draw a vector diagram to scale and use it to determine :

(a) The magnitude of ***force F***.

(b) The ***angle*** between the direction of ***F*** and the horizontal.

2 A flower basket of weight ***30 N***, is suspended by a rope attached to a fixed support. A constant wind blowing horizontally pushes the basket so that the rope makes an angle of 20° with the vertical as shown in the diagram opposite.



Determine the ***tension*** in the rope and the ***force due to the wind*** :

(a) Using the ***triangle of forces rule***, and

(b) By ***resolution*** of the forces acting on the basket.

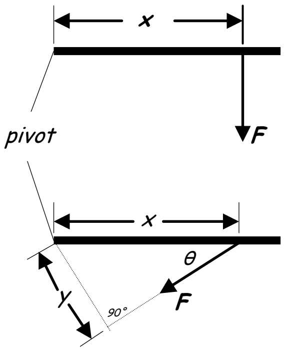
• MOMENT OF A FORCE

- *The terms **turning effect**, **moment** and **torque** all have the same meaning and they are a measure of the ability of a force to rotate a body about a given point.*

- *The **MOMENT** of a force about a point is calculated from :*

MOMENT = FORCE x PERPENDICULAR DISTANCE FROM THE LINE OF ACTION OF THE FORCE TO THE POINT

Nm **N** **m**



Moment = ☐ x ☐

Moment = ☐ x ☐

= ☐ x ☐

= ☐

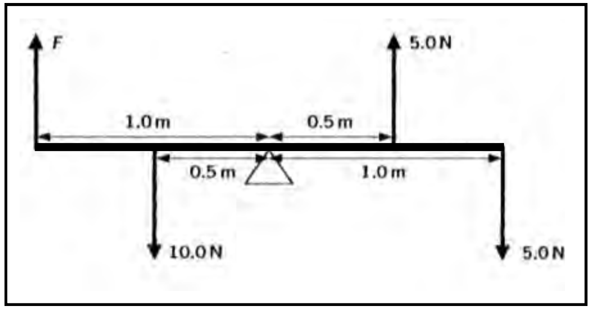
• PRINCIPLE OF MOMENTS

- *This principle can be used to :*
  - *Decide whether an object under the action of several forces will be in equilibrium or start to rotate.*
  - *Calculate an unknown force or distance for an object in equilibrium under the action of several forces.*

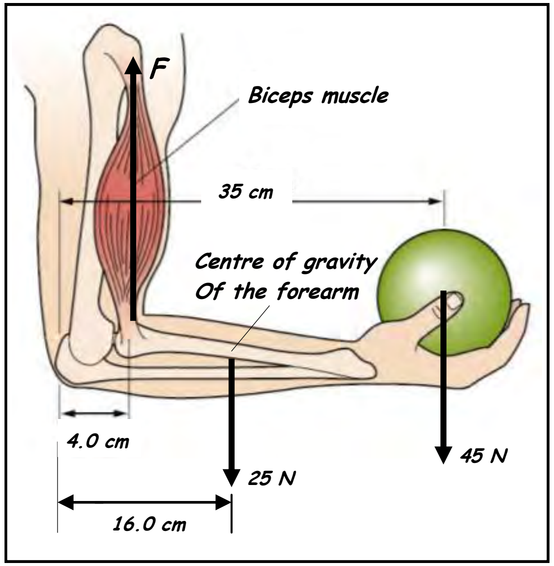
- *The **PRINCIPLE OF MOMENTS** states that if an object is in **equilibrium** (i.e. balanced) under the action of several coplanar forces, the sum of the **clockwise moments** about any point is equal to the sum of the **anticlockwise moments** about that point.*

• PRACTICE QUESTIONS (2)

1 Use the ***principle of moments*** to calculate the size of the force ***F*** needed in order to keep the rod shown in the diagram in equilibrium.



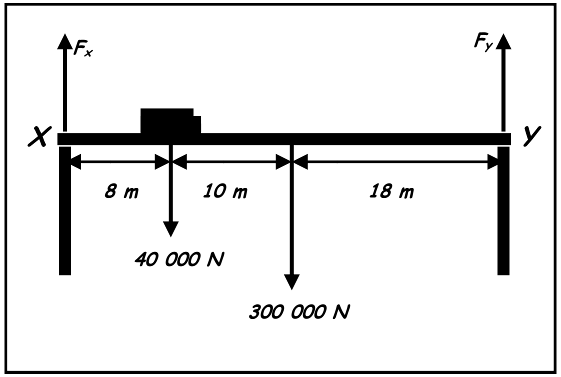
2 The internal structure of a shot putter's right arm is shown in the diagram opposite. The biceps muscle is attached to one of the bones in the forearm and provides the upward force to hold the shot.



The forearm weighs ***25 N*** and its centre of gravity is at a horizontal distance of ***16 cm*** from the elbow. The shot has a weight of ***45 N*** and it is held in the hand at a distance of ***35 cm*** from the elbow.

Assuming that the forearm is at right angles to the upper arm, calculate the size of the ***force (F)*** which the biceps muscle needs to provide.

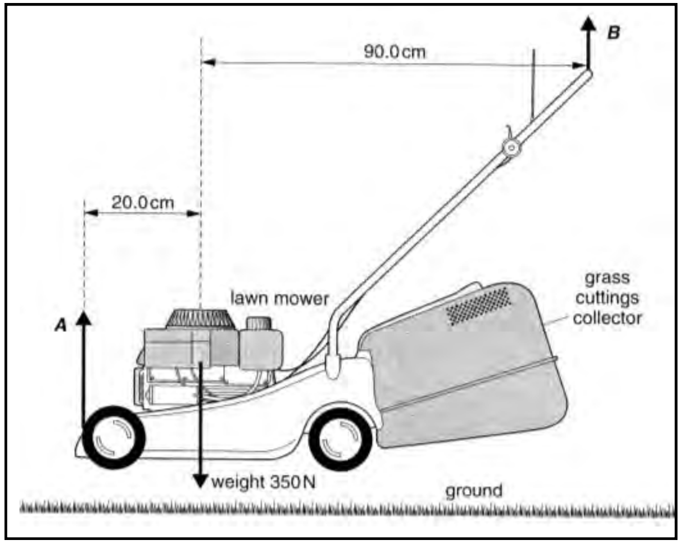
3 A lorry of weight ***40 000 N*** is stopped on a small, uniform beam bridge at a point which is ***8 m*** from a supporting pillar ***X.***



If the bridge weighs ***300 000 N***, calculate the upward reaction forces F_x and F_y on each of the pillars.

4 (a) Explain the term ***centre of gravity*** of an object. 5

(b) The diagram below shows a lawn mower which is being carried by two people.



(i) The two people apply ***forces A*** and ***B*** at each end of the lawn mower. The weight of the lawn mower is ***350 N.***

1. Explain why the weight of the lawn mower does not act in the middle of the lawn mower, that is ***55 cm*** from each end.

2. Use the principle of moments to show that the ***force B*** is ***64 N.***

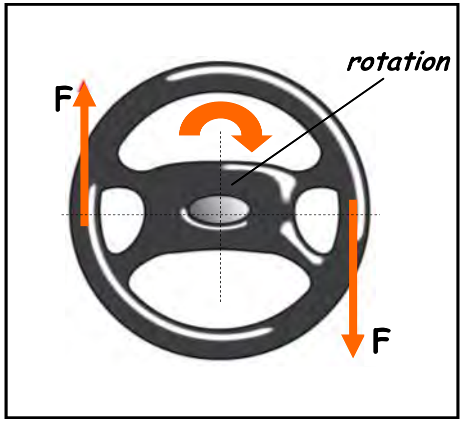
3. Determine the ***force A.***

(ii) ***State*** and ***explain*** what happens to the forces A and B if the person that applies Force B moves his hands along the handle towards the middle of the mower.

(OCR AS Physics - Module 2821 - June 2007)

- *COUPLES*

- *A COUPLE :*

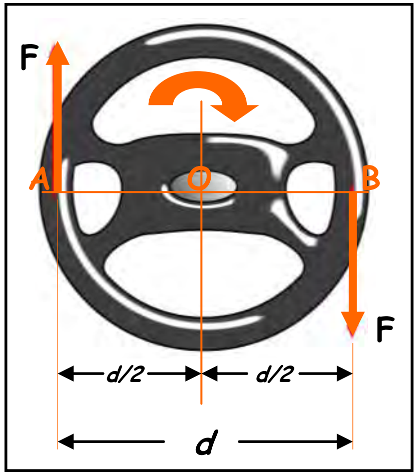


- *Consists of two **equal**, **oppositely directed-parallel**, **coplanar** forces whose **lines of action do not intersect**.*
- *Always **tends to produce rotation**.*
- *Cannot produce a **resultant force** (Because the Forces are equal and opposite) and so **cannot Produce linear motion**.*

TORQUE DUE TO A COUPLE

The diagram shows a steering wheel which is free to rotate about an axis through point O.

If two equal, opposite, parallel forces (F) act as shown, the clockwise TORQUE (T) due to the couple is given by :



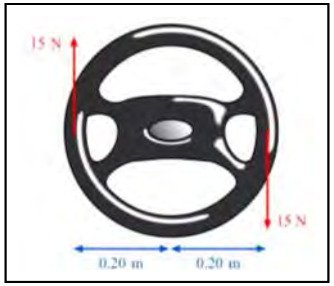
$$T = (F \times d/2) + (F \times d/2)$$

$$T = F \times (d/2 + d/2)$$

$$T = F \times d$$

Torque = Force x Perpendicular distance between the forces

(Nm) (N) (m)



*Calculate the **TORQUE** for the steering wheel shown on the left.*

• CONDITIONS FOR EQUILIBRIUM

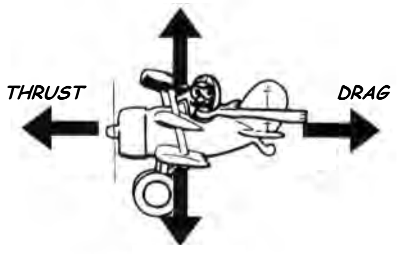
- *A body which is in **equilibrium** is not suffering any changes in its motion and it is therefore either :*
  - ***STATIONARY*** *or*
  - ***MOVING WITH CONSTANT VELOCITY*** *(i.e. **not accelerating**)*

- *For an object to be in equilibrium under the action of forces, the following conditions must be satisfied :*

> • **THE RESULTANT OR NET FORCE ON AN OBJECT MUST BE ZERO AND SO THE LINEAR ACCELERATION WILL BE ZERO.**

- *The test for equilibrium is to resolve all the forces acting on the object into vertical and horizontal components.*

The object is in equilibrium if the resultant force is zero vertically and horizontally.



The plane is in equilibrium when it is flying at constant velocity at a fixed altitude.

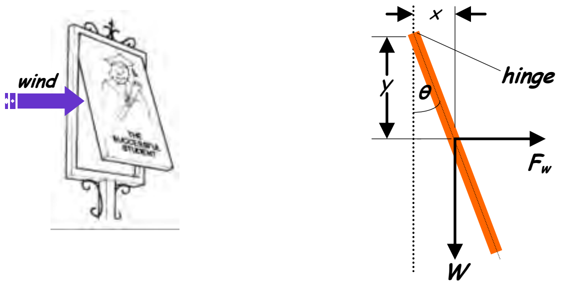
DRAG = THRUST

So horizontal resultant force = 0

LIFT = WEIGHT

So vertical resultant force = 0

> • **THE RESULTANT OR NET TORQUE ACTING ON THE OBJECT MUST BE ZERO AND SO THE ANGULAR ACCELERATION IS ZERO.**
>
> *(This condition is commonly called the **PRINCIPLE OF MOMENTS**)*



*A rectangular pub sign hinged along its upper edge is held in equilibrium at an angle (θ) to the vertical by the wind. Assuming that the friction at the hinge is negligible, equilibrium is possible so long as the **RESULTANT TORQUE = 0**.*

Therefore :

Clockwise torque due to the sign's weight	=	Anticlockwise torque due to the wind force

$$W \times x = F_w \times y$$

• DENSITY (ρ)

- *The **DENSITY (ρ)** of a substance is the mass per unit volume of the substance.*

  i.e.

$$\text{density } (\rho) = \frac{\text{mass } (m)}{\text{volume } (V)}$$

Units: density $(Kg\ m^{-3})$, mass (kg), volume (m^3)

UNIT G481	Module 2	1.2.3	Equilibrium

- *The **density** of an element depends on :*
    - *The **mass of each of its atoms**, and*
    - *The **way the atoms are packed together**.*

*Despite its lowly 76th place in the Periodic Table **OSMIUM's** atoms very efficiently packed together and this makes it the densest known element **(ρ = 2.25 x 10^4 kg m^{-3}).** This density is nothing compared to the density of nuclei at around **4.0 x 10^{17} kg m^{-3}!***

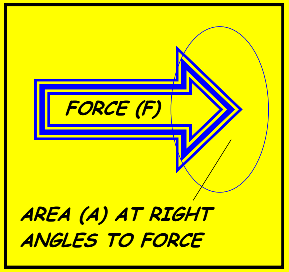
- ***PRESSURE (p)***

- ***<u>PRESSURE</u>** is the 'normal' force per unit area acting on a surface.*

PRESSURE = NORMAL FORCE / CROSS-SECTIONAL AREA

$$p = \frac{F}{A}$$

F (N), A (m^2), p (N m^{-2})



- *The unit of pressure is the **Newton per metre^2** which is called the **pascal (Pa).***
-

1 Pa = 1 N m^{-2}

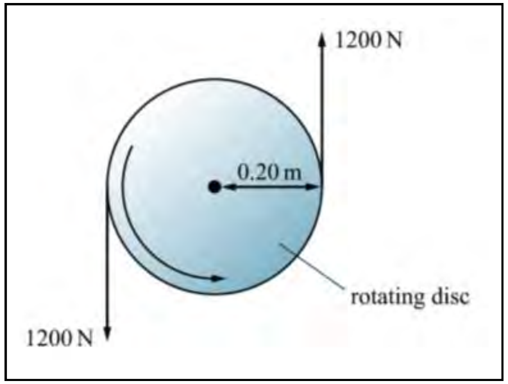
- **HOMEWORK QUESTIONS**

1 (a) What is a ***COUPLE ?***

(b) What does a couple ***tend to produce*** and what can it ***not produce ?***

(c) Define the ***TORQUE*** of a couple.

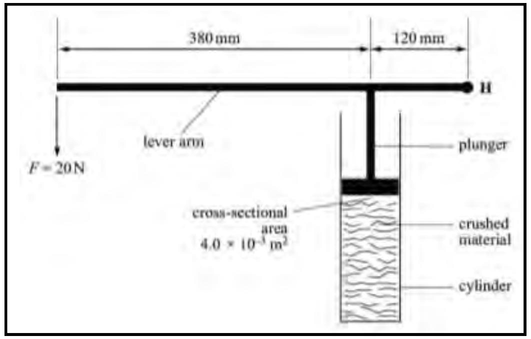
(d) Calculate the ***TORQUE*** produced by the forces shown in the diagram opposite.



(OCR AS Physics (part question) - Module 2821 - June 2006)

2 (a) (i) Define ***PRESSURE.***
(ii) Define ***MOMENT*** of a force.

(b) The diagram shows a device used for compressing materials. A vertical force of ***20 N*** is applied at one end of a lever system.



The lever is pivoted about a hinge H. The plunger compresses the material in the cylinder.

(i) Two forces acting on the lever arm are its weight and the force F. ***State two*** other forces acting on the lever arm, including the direction in which they act.

(ii) By taking moments about H, show that the force acting on the plunger is ***83 N.*** The weight of the lever arm may be neglected.

(c) The cross-sectional area of the plunger is ***4.0 x 10^{-3} m^2.*** Calculate the ***pressure*** exerted by the plunger on the material in the cylinder.

(OCR AS Physics - Module 2821 - January 2006)

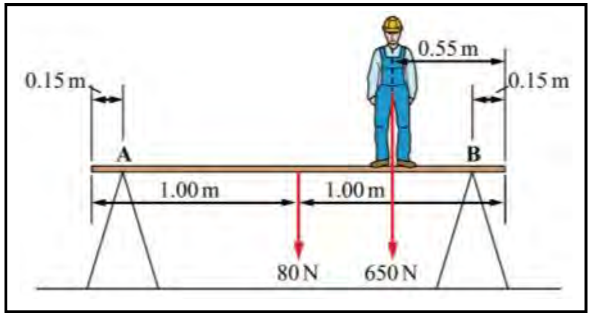
UNIT G481	Module 2	1.2.3	Equilibrium

3 A rectangular tombstone has dimensions ***1.50 m x 0.75 m x 0.10 m*** and it is made from a stone of density ***4500 kg m^{-3}.*** Calculate :

(a) The ***weight*** of the tombstone.

(b) The ***minimum force*** needed to lift one end of the tombstone if it is lying flat on its largest face.

4 (a) State the ***two conditions*** necessary for a system to be in ***equilibrium.***



(b) The diagram above shows a painter's plank resting on two supports ***A and B.*** The plank is uniform, has a weight of ***80 N*** and a length of ***2.00 m.*** A painter of weight ***650 N*** stands ***0.55 m*** from one end.

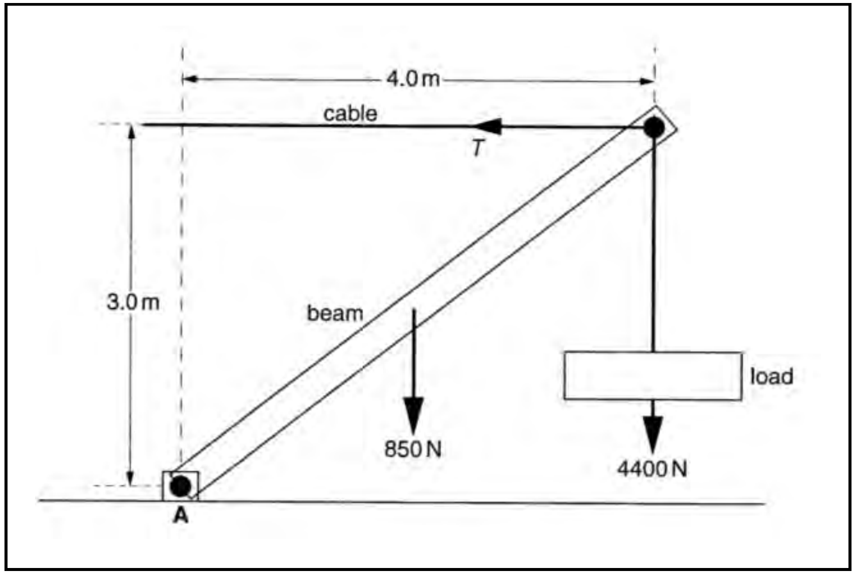
(i) Show that the force acting on the plank at the support ***B*** is approximately ***540 N*** by taking moments of all the forces about the support at ***A.***

(ii) Calculate the ***force*** acting on the plank at support ***A.***

(iii) ***Describe and explain*** what happens to the forces on the plank at ***A*** and ***B*** if the painter moves towards the support at ***A.***

(OCR AS Physics - Module 2821 - May 2008)

5 The diagram below shows a system for supporting a load.



The load of weight ***4400 N*** is hanging from a uniform beam that is supported by a horizontal cable. The beam has a weight of ***850 N*** and is hinged at ***A.***

(a) Take moments about ***A*** and show that the ***tension T*** in the cable is ***6400 N*** (to 2 significant figures).

(b) ***State and explain*** what force, in addition to those shown, must act on the beam to keep it in equilibrium. You are not expected to calculate this force.

Draw this force on the diagram above and label it ***F.***

(OCR AS Physics - Module 2821 - May 2008)