- ***Candidates should be able to :***

  - *Derive the **equations of motion** for constant acceleration in a straight line from a velocity-time graph.*

  - *Select and use the equations of motion for constant acceleration in a straight line :*

$\boxed{v = u + at}$ $\boxed{s = \frac{1}{2}(u + v)t}$ $\boxed{s = ut + \frac{1}{2}at^2}$ $\boxed{v^2 = u^2 + 2as}$

  - *Apply the equations for constant acceleration in a straight line, including the **motion of bodies falling in the Earth's uniform gravitational field without air resistance.***

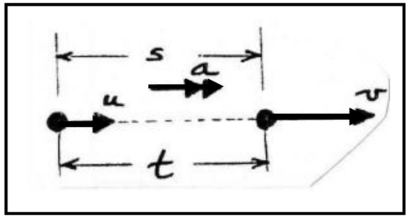
  - *Explain how experiments carried out by **Galileo** overturned **Aristotle's** ideas of motion.*

  - *Describe an experiment to determine **the acceleration of free fall (g) using a falling body**.*

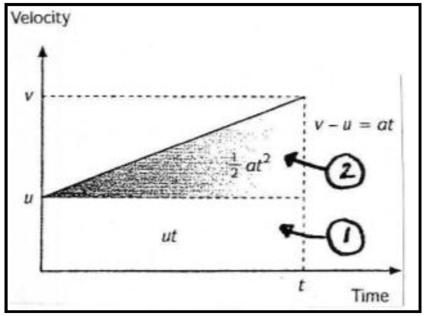
  - *Apply the equations of constant acceleration to describe and explain the motion of an object due to **a uniform velocity in one direction and a constant acceleration in a perpendicular direction.***

- ***DERIVING THE EQUATIONS OF MOTION***

*Consider an object moving with an **initial velocity (u)** which accelerates with a constant **acceleration (a)** to reach a **final velocity (v)** after a **time (t)**. The **distance moved** in this time is **(s)**.*



*This is the **velocity-time** graph for the motion.*



EQUATION 1

Acceleration = gradient of v/t graph

$$a = \frac{(v - u)}{t}$$

$$at = v - u$$

$$\boxed{v = u + at}$$

EQUATION 2

Total displacement = average velocity x time

$$\boxed{s = \frac{1}{2}(u + v)t}$$

EQUATION 3

$v = u + at$ ……… (1) and $s = \frac{(u + v)}{2}t$ ……… (2)

Substituting $(u + at)$ *for* (v) *in equation (2) gives :*

$$s = \frac{(u + u + at)\,t}{2} = \frac{2ut + at^2}{2}$$

So, $\boxed{s = ut + \tfrac{1}{2}at^2}$

EQUATION 4

$v = u + at$, so $t = \frac{(v - u)}{a}$

Substituting for (t) *in* $s = \frac{(u + v)}{2}t$ *gives :*

$$s = \frac{(u + v)}{2} \times \frac{(v - u)}{a} = \frac{(uv - uv + v^2 - u^2)}{2a}$$

$2as = v^2 - u^2$

So, $\boxed{v^2 = u^2 + 2as}$

- ***EQUATIONS OF MOTION - SUMMARY***

$\boxed{v = u + at}$ $\boxed{s = \tfrac{1}{2}(u + v)t}$

$\boxed{s = ut + \tfrac{1}{2}at^2}$ $\boxed{v^2 = u^2 + 2as}$

- *These equations can also be used* ***for objects falling without air resistance in the Earth's uniform gravitational field.*** *Then :*
  - $a = g =$ ***acceleration due to gravity*** $= 9.81\ m\ s^{-2}$.
  - g *is* ***positive*** *for objects which are* ***initially falling.***
    g *is* ***negative*** *for objects which are* ***projected upwards.***
  - $h =$ ***vertical displacement.***

- **PRACTICE QUESTIONS (1)**

1 An aircraft accelerates uniformly from ***rest*** at $3.1\ m\ s^{-2}$ to reach its take-off velocity of $100\ m\ s^{-1}$.

(a) How long does it take for the aircraft to leave the ground ?

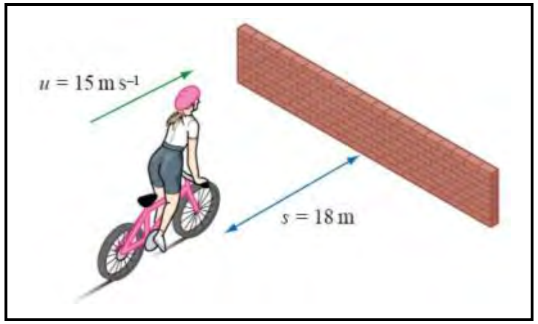
(b) How far does it travel during take-off ?

2 A motorway designer assumes that cars approaching a motorway enter a slip road with a velocity of $8\ m\ s^{-1}$ and need to reach a velocity of $25\ m\ s^{-1}$.

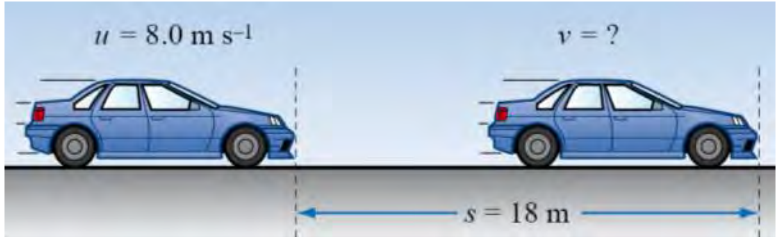
Assuming that vehicles have an acceleration of $5\ m\ s^{-2}$, calculate the ***minimum length*** for a slip road.

3 The cyclist shown in the diagram opposite is travelling at ***15 m s^{-1}***. She brakes so as to avoid colliding with the wall.

Calculate the ***deceleration*** needed in order to come to rest in ***18 m.***



4



The car shown in the diagram above is travelling along a straight road at ***8 m s^{-1}***. If it accelerates uniformly at ***1.5 m s^{-2}*** over a distance of ***18 m***, calculate its ***final velocity.***

5 A lorry accelerates from rest at a steady rate of ***1.2 m s^{-2}***.

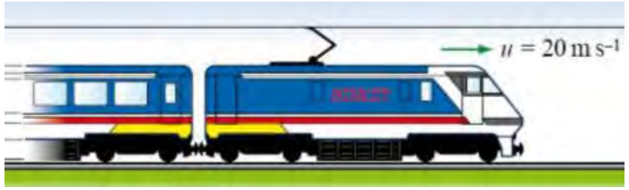
Calculate :

(a) The ***time taken*** to reach a velocity of ***15 m s^{-1}***.

(b) The ***distance travelled*** during this time.

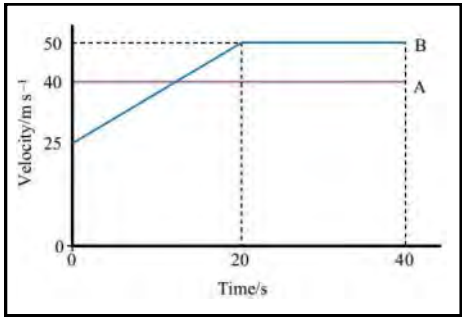
(c) The ***velocity*** of the lorry when it is ***100 m*** from the start.

6



A train travelling at ***20 m s^{-1}*** accelerates uniformly at ***0.75 m s^{-2}*** for ***25 s.*** Calculate the ***distance travelled*** by the train in this time.

7 The diagram opposite shows a velocity-time graph for two cars, ***A*** and ***B***, which are moving in the same direction over a ***40 s*** time period.



Car ***A***, travelling at a constant velocity of ***40 m s^{-1}***, overtakes car ***B*** at time, ***t = 0.***

In order to catch up with car ***A***, car ***B*** immediately accelerates uniformly for ***20 s*** to reach a constant velocity of ***50 m s^{-1}***.

Calculate :

(a) The ***distance travelled*** by car A during the first ***20 s.***

(b) Car ***B's acceleration*** during the first ***20 s.***

(c) The ***distance travelled*** by car ***B*** during the first ***20 s.***

(d) The ***additional time*** taken for car ***B*** to catch up with car ***A.***

(e) The ***total distance*** each car will have travelled.

8 A coin is dropped down a mine shaft and falls through a height of ***55 m*** before hitting the bottom of the shaft. Assuming ***negligible air resistance*** and taking the ***acceleration due to gravity (g)*** as ***9.81 m s⁻²***, calculate :

(a) The ***time taken*** for the coin to hit the bottom.

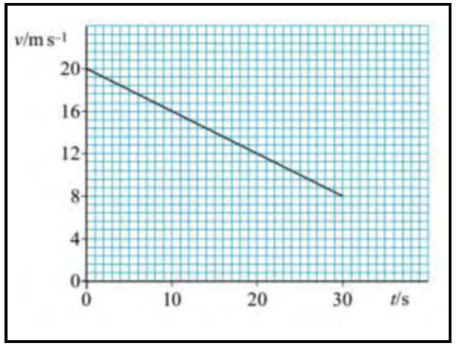
(b) The ***velocity*** of the coin on impact.

9 A stone is projected vertically upwards from the ground with an initial velocity of ***30 m s⁻¹***. Assuming ***air resistance is negligible*** and taking the ***acceleration due to gravity (g)*** as ***9.81 m s⁻²***, Calculate :

(a) The ***maximum height*** reached by the stone.

(b) The ***total time taken*** to return to the ground.

10 The diagram opposite shows a velocity-time graph for a vehicle travelling along a straight road for a time of ***30 s.***



(a) ***Describe*** the motion of the vehicle.

(b) Use the graph to determine the ***acceleration*** of the vehicle over the ***30 s period.***

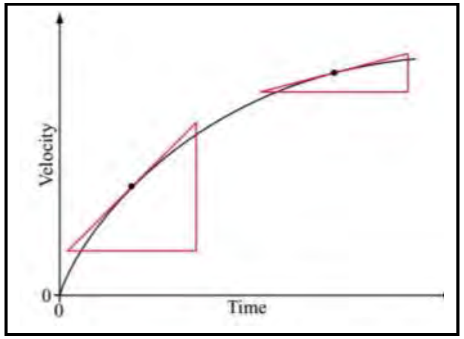
(c) Use the graph to determine the ***displacement*** of the vehicle over the ***30 s period.***

(d) Check your answer to part (c) by calculating the ***displacement*** using a suitable equation of motion.

- *NOTE ON NON-UNIFORM ACCELERATION*

- *The **EQUATIONS OF MOTION** only apply to objects moving with **CONSTANT or UNIFORM ACCELERATION**.*

- *The v/t graph opposite shows the motion of an object which is moving with **non-uniform acceleration.***

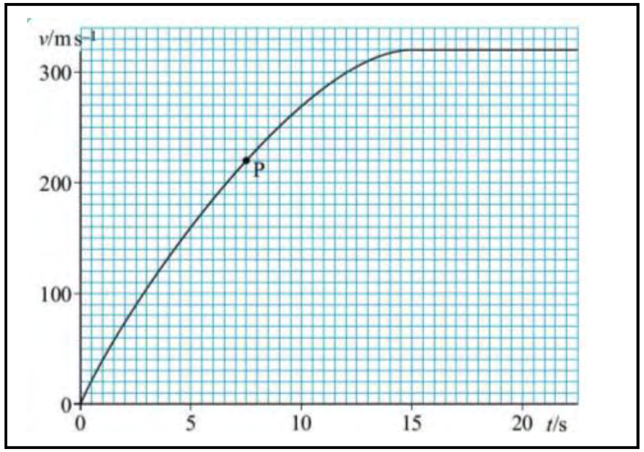


*The **acceleration at any time** is given by the **gradient of the v/t graph** at that time.*

To find the acceleration at Any given time :

- ***At the time in question, mark a point on the v/t graph.***
- ***Draw a tangent to the curve at that point.***
- ***Construct a large right-angled triangle and use it to calculate the gradient.***

Use the procedure outlined above to calculate the acceleration at Point P in the v/t graph shown below.



Acceleration = ——————— = ______ m s⁻²

- ***HOW GALILEO'S EXPERIMENTS OVERTURNED ARISTOTLE'S IDEAS OF MOTION***

- *The Greek philosopher **ARISTOTLE** thought that a force must act all the time in order to keep an object moving. Our own experience seems to support this idea in that a car, for example, will slow down and eventually stop when the engine is switched off. But, does this happen because the driving force has been switched off ? The reality is that the slowing down and stopping occurs because there is an opposing **FRICTIONAL FORCE**.*

- *Experiments carried out by **GALILEO** (about 1600 years after Aristotle) showed that **Constant force is not needed to maintain motion,** but **force is needed to :***

  - ***Start and stop motion,***
  - ***Change the speed of an object,***
  - ***Change the direction of an object.***

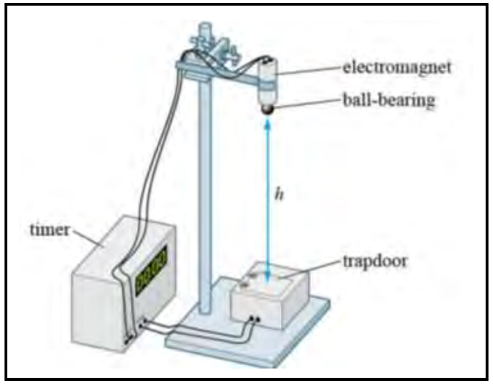
- ***<u>GALILEO'S EXPERIMENTS</u>***

  - ***Galileo** simultaneously dropped two objects of different weight from the top of the **leaning Tower of Pisa** and found that they hit the ground at the same time. He concluded that **any two objects will fall at the same rate, regardless of their relative weights.***

  - *Using a 'dripping water' clock which counted the volume of water drips as a measure of time, **Galileo** was able to measure the time taken by a ball to travel equal distances down a slope from rest. His results showed that the ball **accelerates as it rolls down the slope and that the greater the slope, the greater is the acceleration.** From this, he concluded that **an object falling vertically will accelerate.***

- ***DETERMINATION OF THE ACCELERATION OF FREE FALL (g)***

- *A steel ball-bearing is held by an electromagnet. When the current to the magnet is switched off, the ball is released and the timer is started. The ball strikes and opens a trapdoor which then stops the timer. The **time taken (t)** for the ball to fall through a given **height (h)** is recorded. The timing is repeated and an **average t-value** is calculated.*



The procedure is repeated for several different h-values and the results are recorded in the table below :

h/m	t_1/s	t_2/s	t_3/s	t_{av}/s	t^2/s^2

$h = ut + \frac{1}{2}gt^2$

And since $u = 0,\ ut = 0$

So $h = \frac{1}{2}g\ t^2$

Compare with $y = m\ x + c$

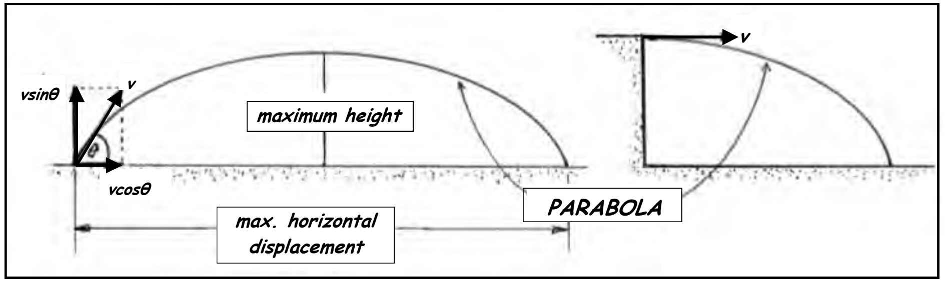
*Therefore, plotting a graph of **(h)** against **(t^2)** gives a best-fit, straight line through the origin, whose **gradient** = $\frac{1}{2}$ **g***

*From which, **acceleration of free fall, g** = 2 x **gradient of h/t^2 graph***

= 2 x

= $\boxed{\qquad\qquad m\ s^{-2}}$

- ***MOTION DUE TO CONSTANT VELOCITY IN ONE DIRECTION AND A CONSTANT ACCELERATION IN A PERPENDICULAR DIRECTION***



*The motion of objects projected at an angle or horizontally from some height above the ground is called **PROJECTILE MOTION**.*

*All **PROJECTLIES** have the following in common :*

- *They are given some initial velocity (by kicking, firing etc.)*

- *Throughout their flight, the only force acting (neglecting air resistance) is their **WEIGHT** due to gravity, which exerts a constant force acting vertically downwards .*

  This gives the projectile :

  CONSTANT DOWNWARD ACCELERATION = g (9.81 m s^{-2})

- *Neglecting air resistance, the **HORIZONTAL COMPONENT OF VELOCITY REMAINS CONSTANT THROUGHOUT THE MOTION** and the path followed is a **PARABOLA**.*

SUMMARY FOR SOLVING PROJECTILE PROBLEMS

Resolve the initial velocity into :

HORIZONTAL and VERTICAL COMPONENTS.

*Assume **NEGLIGIBLE AIR RESISTANCE***

For the Horizontal Direction :

The HORIZONTAL COMPONENT of velocity remains constant throughout the flight.

horizontal displacement = horizontal velocity x flight time

For the Vertical Direction :

The acceleration is constant, equal to 'g'.

The equations of motion apply.

$$v = u + gt \qquad h = \tfrac{1}{2}(u + v)t$$

$$h = ut + \tfrac{1}{2}gt^2 \qquad v^2 = u^2 + 2gh$$

At the maximum height, vertical velocity = 0.

When the projectile has returned to the level of its launch point :

- ***Vertical displacement = 0***
- ***Final vertical velocity is equal, but oppositely Directed to the initial vertical velocity.***

- **Practice Questions (2)**

1 A helicopter is flying in a straight line at a speed of ***20 m s^{-1}*** and at a constant height of ***180 m.*** A small object is released from the helicopter and falls to the ground. Assuming air resistance is negligible, calculate :

(a) The ***time taken*** for the object to reach the ground.

(b) The ***vertical component of velocity*** of the object when it hits the ground.

(c) The ***horizontal component of velocity*** of the object when it hits the ground.

(d) The ***horizontal displacement*** of the object in the time taken to reach the ground.

2 During a European Champions League match, a free kick was taken by Steven Gerrard and the ball was projected with a velocity of ***20 m s^{-1}*** at an angle of ***35°*** to the pitch. Assuming that air resistance is negligible, calculate :

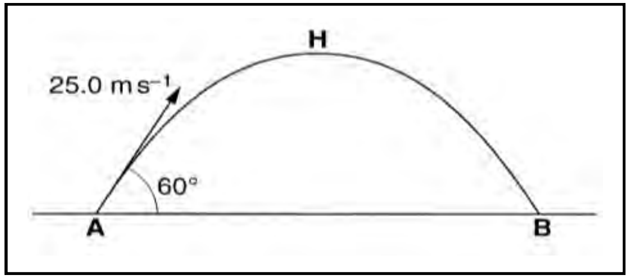
(a) The ***initial vertical and horizontal components of velocity***.

(b) The ***time taken*** for the ball to reach its maximum height.

(c) The ***maximum height*** reached by the ball.

(d) The ***horizontal displacement*** of the ball in the time taken to return to the ground.

3



The diagram above shows the path of a ball that is thrown from ***point A*** to ***point B.*** The ball reaches its maximum height at ***point H.*** The ball is thrown with an initial velocity of ***25.0 m s^{-1}*** at ***60°*** to horizontal. Assume that there is no air resistance.

(a) (i) Show that the ***vertical component of the initial velocity*** is ***21.7 m s^{-1}***.

(ii) Calculate the ***time taken*** for the ball to reach ***point H.***

(iii) Calculate the ***displacement*** from ***A to B.***

(b) For the path of the ball shown in the diagram, draw sketch graphs, with labelled axes but without numerical values, to show the variation of :

(i) The ***vertical component of the ball's velocity*** against ***time.***

(ii) The ***distance travelled along its path*** against ***time.***

(OCR Physics AS - Module 2821 - May 2008)

UNIT G481	Module 1	1.1.4	Linear Motion

- **HOMEWORK QUESTIONS**

1 When the brakes are applied in a car which is moving at ***40 m s⁻¹***, the velocity is reduced to ***25 m s⁻¹*** over a distance of ***140 m.*** If the deceleration remains constant, what ***further distance*** will the car travel before coming to rest ?

Sketch a ***velocity-time*** graph for the whole motion showing numerical values on the axes.

2 Calculate the deceleration of a bullet initially travelling at ***400 m s⁻¹***, if it is brought ***to rest*** after travelling ***10 cm*** through a wooden block.

3 A hawk is hovering above a field at a height of ***50 m.*** It sees a Mouse directly below it and dives vertically with an acceleration of ***12 m s⁻².*** Calculate :

(a) The hawk's ***velocity*** at the instant it reaches the mouse.

(b) The ***time taken*** to reach the mouse.

4 A train accelerates steadily from ***4.0 m s⁻¹*** to ***24.0 m s⁻¹*** in a time of ***180 s.*** Calculate :

(a) The train's ***acceleration.***

(b) The ***average velocity*** of the train.

(c) The ***distance travelled*** by the train during the acceleration.

5 Traffic police investigators use the length of skid marks left on the road by a decelerating vehicle in order to determine whether or not the speed limit has been exceeded. 8

If the skid marks at the scene of an accident are ***52m*** long and other tests on the road surface show that the skidding car was decelerating at ***6.5 m s⁻²***, was the car breaking the speed limit of ***30 m s⁻¹*** ?

6 A sandbag is dropped from a height of ***180 m***, from a helicopter that is moving vertically upwards with a velocity of ***6 m s⁻¹.*** If air resistance is neglected, calculate :

(a) The ***initial velocity*** of the sandbag.

(b) The ***final velocity*** of the sandbag.

(c) The ***time taken*** for the sandbag to reach the ground.

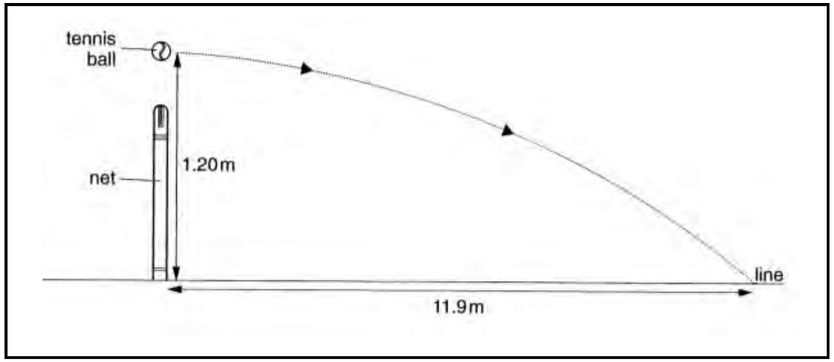
7 An aid parcel is released from a plane flying horizontally at ***65 m s⁻¹***, at a height of ***800 m*** above the ground.

(a) Calculate the ***horizontal*** and ***vertical*** components of the parcel's ***initial velocity.***

(b) How long does it take for the parcel to reach the ground ?

(c) At what ***horizontal distance*** from the target should the plane be when the parcel is released ?

8



The diagram above shows the path of a tennis ball after passing over the net.

As it passes over the net, it is travelling at a height of ***1.20 m.*** The ball strikes the ground on a line which is ***11.9 m*** from the net.

(a) Assuming air resistance to be negligible,

(i) Show that the ***time taken*** for the ball to reach the line After passing over the net is ***0.495 s.***

(ii) At the instant the ball strikes the line, calculate :

1. The ***horizontal component of its velocity.***

2. The ***vertical component of its velocity.***

(OCR Physics AS - Module 2821 - June 2007)