Edexcel Physics A-level

Topic 6: Further Mechanics

Key Points

Impulse

By combining the equations for **momentum** with **Newton's Second Law**, we can produce a definition for impulse:

$$F = ma$$

$$F = \frac{\Delta(mv)}{\Delta t}$$

$$F\Delta t = \Delta(mv)$$

Impulse can therefore be described as the **change of momentum** of an object. It is equal to the **area** under a **Force-Time graph**.

Another useful thing that these equations demonstrate is that the **force** an object experiences is equal to the **rate of change of momentum** - an idea used in vehicle safety.

Collisions

For all collisions, the **conservation of momentum** must apply. Whether or not the **kinetic energy** of the system is conserved depends on the type of collision:

- In **elastic collisions**, the kinetic energy of the system is **conserved** and so:

$$E_k \text{ final} = E_k \text{ initial}$$

- In **inelastic collisions**, the kinetic energy of the system is **not conserved** and the energy is dissipated in other forms - to work out the quantity of energy transferred to other forms you can compare the initial and final KE values:

$$\Delta E_k = E_k \text{ final} - E_k \text{ initial}$$

Radian Measures

When dealing with **circular motion**, it is often easier to make use of **angular** quantities. These make use of an angle unit known as the **radian**. The conversion between degrees and radians is:

$$2\pi \text{ radians} = 360°$$

The equation to find the **angular displacement (θ)**, which is the angle you have turned through in radians, is:

$$\theta = \frac{s}{r}$$

Where '*s*' is arc length and '*r*' is the radius of the circle.

Angular velocity (ω) is the angle an object moves through per unit time. It is given by:

$$\omega = \frac{v}{r} \qquad \omega = \frac{2\pi}{T}$$

Circular Motion

From **Newton's first law**, we know that for an object to change velocity, a **resultant force** must act. In circular motion, since the **direction** of the object is continually changing, the **velocity** must also be changing. Therefore a resultant **centripetal force** is required. This force points towards the **centre** of the object's orbit and is given by the equation:

$$F = \frac{mv^2}{r}$$

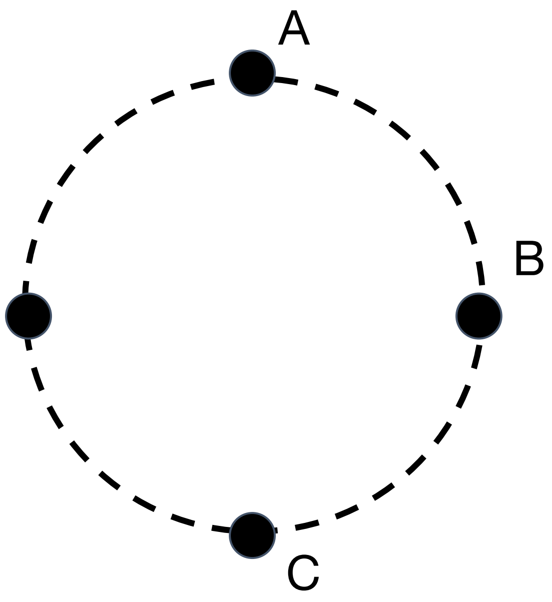
An alternative form of this equation can be given, which makes use of the object's **angular speed**.

$$F = m\omega^2 r$$

Where ω is the angular speed and is given by: $\omega = 2\pi f$

Circular Motion

It is important to consider what is contributing to the centripetal force at each point in the cycle. For a ball being spun on a string:



- At **position A**, the weight of the ball is directly contributing to the centripetal force since it is acting directly towards the centre of the circle - this means that the inwards force (tension) provided by the string is at a **minimum**
- At **position B**, the weight of the ball is acting **perpendicular** to the direction of the centripetal force, meaning it makes no contribution and the string provides the full force
- At **position C**, the weight of the ball is acting opposite to the direction of the centripetal force, meaning the inwards force of the string must overcome the weight and provide the required centripetal force - this means the tension is at a **maximum**

Kinetic Energy

You can derive an equation for the **kinetic energy** of a non-relativistic particle (one travelling below relativistic speeds) involving momentum (*p*) as shown below:

$$E_k = \tfrac{1}{2} mv^2$$

$$p = mv$$

$$v = \frac{p}{m}$$

$$E_k = \tfrac{1}{2} m \frac{p^2}{m^2}$$

$$E_k = \frac{p^2}{2m}$$

Centripetal Acceleration

The equation for **centripetal acceleration** can be derived using vector diagrams as shown below:

1. Consider an object moving at a constant speed v, in a circular path of radius r.
2. Using the diagram below, you can see that the triangles formed as part of the circular path and by the velocity vectors, are **similar** isosceles triangles. This is because they both have 2 sides of equal length (r/v) and it can be shown using circle theorems and the fact the angles in a triangle add up to 180°, that the angle between these equal sides is the same.
3. As these triangles are similar, we can write:

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}$$

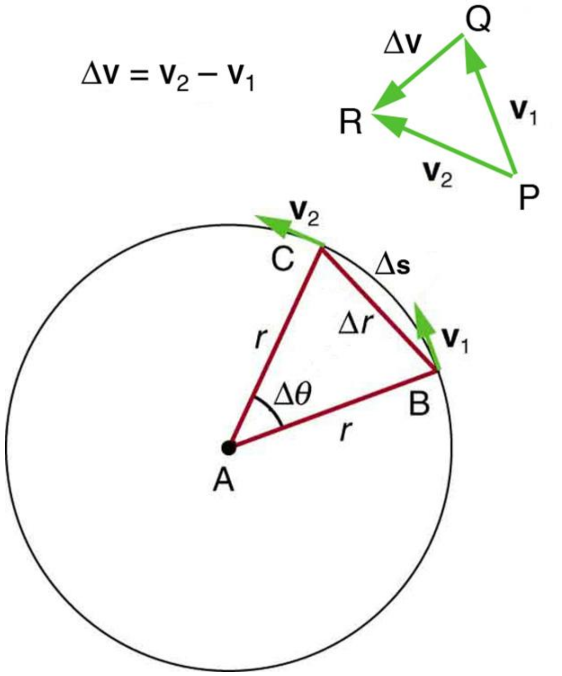
4. Rearrange to get *Δv as the subject.*

$$\Delta v = \frac{v}{r} \times \Delta s$$

5. Divide through by *Δt as a = Δv/Δt. Note that v = Δs/Δt.*

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta s}{\Delta t} \qquad a = \frac{v}{r} \times v \qquad \boxed{a = \frac{v^2}{r} = r\omega^2}$$

As $v = \omega r$



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