Physics Factsheet

January 2001 Number 09

Linear Momentum

1. What is linear momentum?

*The **linear momentum** of a body is defined as the product of its **mass** and its **velocity***

$p = mv$
p = momentum ($kgms^{-1}$) m = mass (kg) v = velocity (ms^{-1})

***Tip**: The "linear" in linear momentum is there to distinguish it from **angular** momentum. A body has linear momentum if it is moving along; it has angular momentum if it is **turning**. Angular momentum will be covered in a later Factsheet.*

Since velocity is a vector, linear momentum is a **vector quantity** – it has both magnitude and direction. Its units are $kgms^{-1}$ or, equivalently, Ns (exercise: show these units are equivalent).

Example 1. What is the momentum of a body of mass 50 grammes moving at 20 kmh^{-1} due North

***Tip:** If you would be tempted to put 50 × 20 = 1000 as your answer to this example – go back to the definition of momentum and look at the units!*

First we must change everything to the appropriate units – kg for mass and ms^{-1} for velocity

50 grammes = 0.05 kg
20 kmh^{-1} is 20 × 1000 = 20 000 metres per hour
which is $20\,000 \div 3600 = \frac{50}{9}\ ms^{-1}$
Note: it is sensible to keep this answer as a fraction to avoid rounding errors
So momentum $= 0.05 \times \frac{50}{9} = 0.278\ kgms^{-1}$ *due North (3 SF)*

***Tip:** To change kmh^{-1} to ms^{-1}, multiply by* $\frac{5}{18}$

2. Force and Momentum

Newton's Second Law states:

The rate of change of (linear) momentum of an object is proportional to the resultant force that acts upon it

If standard SI units (newtons, kilograms, metres, seconds) are used, then this becomes: Resultant force = rate of change of momentum

- If the force acting on a body is **constant**, then Newton's Second Law can be written as:
  $$\text{Force} = \frac{\text{Final momentum} - \text{initial momentum}}{\text{time}},$$
  $$\text{or } F = \frac{mv - mu}{t}$$
- If the force acting is **not constant**, then we have:
  $$\text{Average force} = \frac{\text{Final momentum} - \text{initial momentum}}{\text{time}}$$

Example 2. A force acting on a body of mass 2 kg causes its speed to increase from 2ms^{-1} to 6ms^{-1}. Find the average force acting, given that it acts for 2 seconds, if the final velocity of the body is
a) in the same direction as its initial velocity
b) in the opposite direction to its initial velocity

***Tip:** In examples where more than one direction is considered, it is a good idea to decide right at the beginning which direction you are going to take as positive. Anything going in the opposite direction will have a negative velocity, and hence a negative momentum*

Taking the direction of the initial motion as positive:

a) Initial momentum = 2 × 2 = 4$kgms^{-1}$
Final momentum = 2 × 6 = 12$kgms^{-1}$
$$\text{Average force} = \frac{\text{Final momentum} - \text{initial momentum}}{\text{time}} = \frac{12-4}{2}$$
= 4N in direction of the original motion of the body

b) Initial momentum = 4$kgms^{-1}$
Final momentum = 2 × -6 = -12$kgms^{-1}$ (since in opposite direction)
$$\text{Average force} = \frac{-12-4}{2} = -8N$$
So average force = 8N in the direction opposite to the original motion of the body

***Tip**. Common sense tells you that the answers to parts a) and b) cannot be the same. It will take a larger force to reverse the direction of the body, then increase its speed to 6ms^{-1} in the opposite direction, than it will to merely increase its speed to 6ms^{-1} in the same direction. Also, exam technique tells you that you will not be asked to do two identical calculations.*

3. Impulse

*The **impulse** of a **constant** force is given by:*
impulse (Ns) = force(N) × time for which it acts (s)

Example3. A force of 6N acts for 1 minute. Find the impulse of the force.

Impulse = 6 × 60 = 360Ns

In the section above, we met the equation

$$\text{Force} = \frac{\text{Final momentum} - \text{initial momentum}}{\text{time}}$$

for a body moving under a constant force.

Rearranging this equation, we get:

Force × time = Final momentum – initial momentum

This gives us:

Impulse = change of momentum

4. Conservation of momentum

The **principle of conservation of linear momentum** states that:

The total linear momentum of a system of interacting bodies remains constant, providing no external forces are acting.

Justifying the Principle of Conservation of Linear Momentum
Suppose two objects, X and Y, collide with each other.

During the collision, X exerts a force on Y,and Y exerts a force on X.
By Newton's third law, these forces are equal and opposite.
Both forces act for the same time (however long the collision takes)
Therefore, the impulses of the forces are equal and opposite.
Since impulse equals change in momentum, the changes in momentum of X and Y are equal and opposite.
So the total change in momentum of X and Y together is zero, since their individual changes cancel out.

Using Conservation of Momentum
Conservation of momentum is used when the bodies involved are **free to move**. You would **not** use it for the collision between an object and a wall, say, because the wall is not free to move.

Calculations involving conservation of momentum involve either collisions – when you start off with two objects -or explosions/ rocket propulsion/shooting – when you start of with one object that splits into two. In either case, the following approach is helpful:

- First, decide in which direction you will take velocity to be positive.
- Draw a diagram showing the objects involved and their velocities before and after. Take all unknown velocities to be in the positive direction – if they are not, you will find out when you get a negative answer.
- Work out your total initial momentum and your total final momentum and equate them.

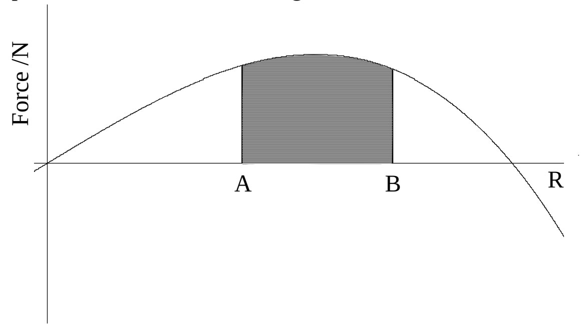
Impulse for a variable force

For a variable force, we can use:

Impulse = average force × time for which it acts
= final momentum – initial momentum,

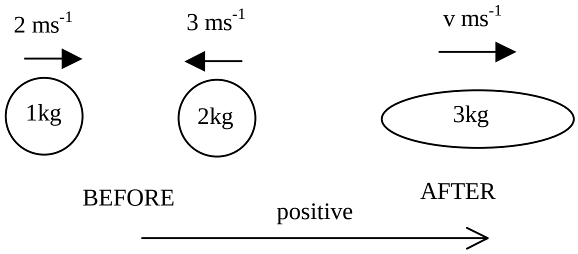
or alternatively, use
Impulse of force = area under force-time graph.

In the graph below, the shaded area is the impulse exerted between time A and time B.
Note that in region R, the force becomes negative – so the impulse it exerts will also be negative.



Example 4. A body of mass 1kg is moving with speed 2ms^{-1}. It collides head-on with a body of mass 2kg moving with speed 3ms^{-1}. The two bodies stick together, and move off with velocity v. Find v.

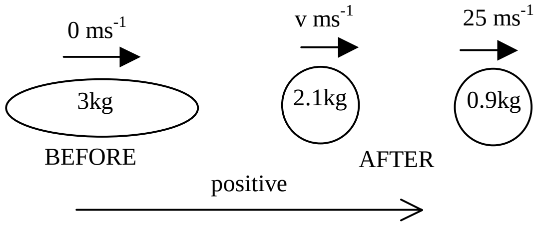
***Tip** "**Collides head on**" means they were initially moving in **opposite** directions. If one body **overtakes** the other, they were initially moving in the **same** direction.*



Total initial momentum = 1 × 2 – 2 × 3 = -5kgms^{-1}
Total final momentum = 3 × v = 3v kgms^{-1}

Principle of conservation of linear momentum: -5 = 3v ⇒ v = -1.67ms^{-1}
So v = 1.67ms^{-1} in the original direction of motion of the 2kg mass.

Example 5. An explosion causes a stationary object of mass 3kg to split into two pieces. The smaller piece, of mass 0.9kg, moves off with velocity 25ms^{-1}. Find the velocity of the larger piece.



Total initial momentum = 0
Total final momentum = 2.1v + 25 × 0.9
0 = 2.1v + 25 × 0.9
v = 10.7 ms^{-1}(3SF) *in opposite direction to smaller piece*

Typical Exam Question

(a) (i) State Newton's second law of motion. [2]
(ii) Define the term 'impulse'. [1]

(b) Water stored in a reservoir falls 100m to a turbine at the foot of a dam. The mass flow rate is 500kg s^{-1}. When the# water hits the turbine, 60% of its vertical velocity is lost. Calculate the:
(i) vertical velocity of the water just before it hits the turbine. [2]
(ii) momentum lost by the water per second as it hits the turbine. [2]
(iii) force exerted on the turbine. [1]

(a) (i) rate of change of momentum of a body is proportional ✓ to resultant force acting on it, & is in the direction of that force ✓
(ii) impulse acting on a body = change in momentum of the body ✓
(or: impulse of a force = magnitude of force × time it acts for)
(b) (i) Using: loss in p.e = gain in k.e each second ✓
$500 \times 100 \times g = \frac{1}{2} \times 500 \times v^2$
$44.3\ ms^{-1} = v$ ✓
(ii) $500 \times v$ ✓
$= 22140\ kgms^{-1}$ *per second* ✓
(iii) impulse = 22140 Ns
time = 1 second
$\Rightarrow F = 22140\ Ns$ ✓ *(f.t.)*

5. Elastic and Inelastic Collisions

In most collisions, some **kinetic energy** is lost – it is transferred to heat and sound, or is used to damage or change the shape of the bodies involved.

- *A (perfectly) **elastic collision** is one in which **kinetic energy is conserved***
- *An **inelastic collision** is one in which **some kinetic energy is lost***
- *A **totally inelastic collision** is one in which the bodies **stick together after impact***

In practice, perfectly elastic collisions are very rare – almost all collisions make some noise. But hard objects – like pool balls – will be closer to having elastic collisions than soft objects, because when soft objects collide, energy is used in deforming the objects.

***Exam Hint:** Questions may ask you to check whether a collision is elastic – this means you have to calculate the total kinetic energy before and total kinetic energy after, and compare them.*

Typical Exam Question

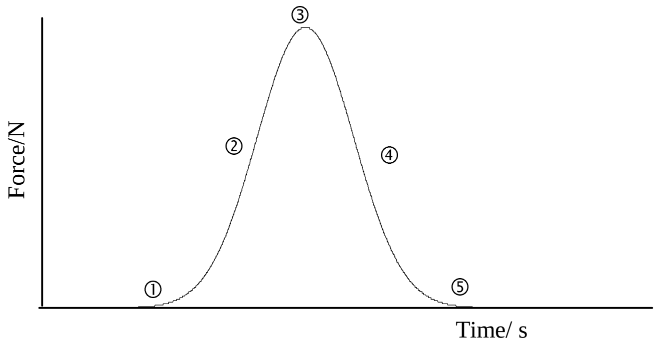
A trolley of mass 2kg moving at a speed of 3ms^{-1} catches up and collides with another trolley of mass 1kg moving at a speed of 1ms^{-1} which is travelling in the same direction. Following the collision, the first trolley has velocity $^5/_3$ ms^{-1}
Show that the collision is perfectly elastic [6]

$2 \times 3 + 1 \times 1 = 2 \times {^5/_3} + 1 \times v$ ✓ *(conserving momentum)*
$7 = {^{10}/_3} + v \Rightarrow {^{11}/_3} = v$ ✓ *(Velocity of 2nd trolley)*
KE before $\frac{1}{2} \times 2 \times 3^2 + \frac{1}{2} \times 1 \times 1^2$ ✓ $= 9\frac{1}{2}$ *J* ✓

KE after $= \frac{1}{2} \times 2 \times \left(\frac{5}{3}\right)^2 + \frac{1}{2} \times 1 \times \left(\frac{11}{3}\right)^2$ ✓ $= \frac{25}{9} \times \frac{121}{118} = 9\frac{1}{2} J$ ✓

Force-time Graphs During Collisions

The diagram below shows a possible graph of force against time during a collision (for example, a ball hitting a wall). The graph shows how the horizontal force on the ball varies with time. Before and after the collision, there is no horizontal force on the ball. The total change of momentum of the ball in its collision with the wall would be given by the area under the graph.



① ball first hits wall
② ball increasingly deforms ("squashes") due to force exerted by wall
③ ball reverses direction
④ ball starts to return to normal shape
⑤ ball loses contact with wall

The exact shape of the graph would depend on the material the ball was made of. If the ball was very hard, then it would not deform significantly during the collision, and the "spike" on the graph would be much narrower, showing that the collision is much shorter in duration. A very "squashy" ball would deform readily, leading to a flatter graph and a longer-lasting collision.

Typical Exam Question

(a) An pellet of mass 0.30kg is fired with velocity 20ms^{-1}, at a stationary ball of mass 0.10kg. The collision is perfectly elastic.
(i) State the change in total kinetic energy on impact [1]
(ii) Write an equation for the momentum of the pellet and the ball before and after the elastic collision. Use the symbols v_{pellet} and v_{ball} to represent the velocities after impact. [1]

(b) For a perfectly elastic collision, assuming all velocities lie in the same straight line, it can be shown that:
velocity of approach = velocity of separation i.e.
i.e. $u_{pellet} – u_{ball} = - (v_{pellet} – v_{ball})$
Use this equation together with the equation written in answer to (a) (ii) to calculate the velocity of the ball after impact. [3]

(c) Calculate the magnitude of the impulse exerted by the pellet on the ball. [2]

(d) The pellet and the ball are in contact for 0.001 seconds. Calculate the magnitude of the average force exerted by the pellet on the ball.

(a) (i) None ✓
(ii) $0.3 \times 20 = 0.3\ V_{pellet} + 0.1\ V_{ball}$ ✓
(b) $0.3 = - (V_{pellet} - V_{ball})$ ✓
so $V_{pellet} = V_{ball} - 0.3$
so $6 = 0.3\ (V_{ball} - 0.3) + 0.1\ V_{ball}$ ✓
$6 = 0.3\ V_{ball} - 0.09 + 0.1\ V_{ball}$
$6.09 = 0.4\ V_{ball}$
$V_{ball} = 15.22\ ms^{-1}$ ✓
(c) impulse = change in momentum ✓
$= 0.1 \times 15.22 = 1.522 Ns$ ✓
(d) impulse = force × time ✓
$1.522 = F \times 0.001 \Rightarrow F = 1522N$ ✓

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

(a) Define

(i) A perfectly elastic collision **[1]**

Energy is conserved 0/1

> Energy is always conserved – the student should have said "Kinetic energy is conserved"

(ii) A completely inelastic collision **[1]**

Energy is not conserved 0/1

> The student should be aware that the answer cannot be true. Learning definitions is vital

(b) A stationary uranium nucleus of mass 238 u (atomic units) disintegrates, emitting an α-particle of mass 4.00 u and creating a daughter nucleus. You may assume that mass is conserved in the decay.

(i) Find the mass of the daughter nucleus in atomic units. **[1]**

234 a.u. ✓ 1/1

(ii) Using the symbols v_α and v_d for velocities, write an equation for the conservation of momentum in this disintegration. **[1]**

$238v_u = 4v_\alpha + 234\,v_d$ 0/1

> The student clearly understands conservation of momentum, but has ignored the fact that the uranium nucleus was initially stationary. This is poor exam technique, firstly through not reading the question, and secondly through not appreciating that since the symbol v_u was not mentioned, it cannot be required in the answer.

(iii) Using the answer to (ii), calculate the ratio $\frac{v_\alpha}{v_d}$

$v_\alpha / v_d = 4/234$ 0/1

> The student's answer to ii) did not allow him/her to complete this part, so s/he has guessed. It would have been sensible to go back to check ii) at this stage.

(c) The α-particle then travels until it collides head on with another stationary uranium nucleus of mass 238u. The speed of the α-particle is 1.00×10^6ms^{-1} before the collision, and that of the uranium nucleus is 3.31×10^4ms^{-1} after the collision.
Calculate the speed and direction of the α-particle after the collision. You may assume the particles simply rebound with no nuclear reaction taking place. **[3]**

$4(1 \times 10^6) = 238\,(3.31 \times 10^4) + 4v$ ✓

$v = -969450$

So 969450 in opposite direction to original ✓ 2/3

> The student understands this well, and has carried out the calculation correctly, but has lost the final mark due to using too many significant figures and omitting the units.

Examiner's Answer

(a) (i) Collision in which kinetic energy is conserved ✓

(ii) A collision in which the bodies coalesce ✓

(b) (i) $238 - 4 = 234$ u ✓

(ii) $0 = 234\,v_d + 4\,v_\alpha$ ✓

(iii) $\frac{v_\alpha}{v_d} = -\frac{234}{4} = -58.5$ ✓

(c) $1 \times 10^6 \times 4 = v \times 4 + 3.31 \times 10^4 \times 238$ ✓

-969000 ms^{-1} $= v$

969000 ms^{-1} ✓ *in opposite direction to original motion* ✓

Practical Investigations

Momentum investigations are best carried out using a **linear air track**. This is a triangular tube with holes in it, through which air is blown. The "vehicles" are slides which fit over the track; they move over the cushion of air. This allows the effects of friction (an "external force") to be ignored. The track is first adjusted so that it is horizontal – so that the vehicles have no tendency to move in either direction on the track.

Each vehicle has a card of a set length on it. This allows **light gates** to be used. These contain a beam of light falling on a photodiode; when a vehicle goes through the gate, the card on it interrupts the beam of light. The light gate records the time for which the beam of light is interrupted; this allows the speed of the vehicle to be calculated using speed = length of card ÷ time for which light beam is interrupted.

The vehicles can have various masses added to them to verify conservation of momentum for different masses. The vehicles can be made to stick together during a collision by adding a pin to the front of one vehicle and a piece of plasticine to the other – this will allow **inelastic collisions** to be investigated. The vehicles can be made to undergo almost **perfectly elastic collisions** by attaching a stretched rubber band to the front of each, which allows them to "bounce off" each other.

Collisions can be investigated by ensuring that each moving vehicle passes through a light gate before collision and after collision. Conservation of momentum can then be verified by calculating the momentum of each vehicle before and after the collision; this can be repeated using different masses, and elastic and inelastic collisions. Experiments could also show whether the "elastic" collisions really are perfectly elastic.

Questions

1. Explain what is meant by:
   a) the principle of conservation of linear momentum
   b) an elastic collision
   c) a totally inelastic collision.
   d) the impulse of a force
2. State Newton's Second Law
3. Outline how you would verify experimentally the principle of conservation of linear momentum.
4. A car of mass 950kg is moving at 72kmh^{-1}.
   a) Calculate its momentum
   The car comes to rest in a period of 15 seconds
   b) Calculate the average force acting on it during this period.
5. In an explosion, an object splits into two parts whose masses are in the ratio 2:3. Assuming that the object was motionless before the explosion, and that the larger mass moves with speed 16ms^{-1} after the explosion, calculate the speed of the smaller mass.
6. A gun of mass 2kg fires a bullet of mass 30g with speed 200ms^{-1}.
   a) Calculate the recoil speed of the gun
   The gun comes to rest in 2 seconds.
   b) Find the average force exerted on it by the person firing it.
7. A train of mass 2000kg is moving with speed 10ms^{-1} when it couples with a stationary truck of mass 300kg. The two move off together. Find the velocity with which they move.

Answers

Answers to questions 1 – 3 can be found in the text.

4. a) 19000kgms^{-1} b) 1270N 5. 24ms^{-1} 6. a) 3ms^{-1} b) 3N 7. 8.70ms^{-1}

Acknowledgements: *This Factsheet was researched and written by Cath Brown* ***Curriculum Press, Unit 305B The Big Peg, 120 Vyse Street, Birmingham B18 6NF.*** ***Physics Factsheets*** *may be copied free of charge by teaching staff or students, provided that their school is a registered subscriber. They may be networked for use within the school. No part of these Factsheets may be reproduced, stored in a retrieval system or transmitted in any other form or by any other means without the prior permission of the publisher. ISSN 1351-5136*

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Momentum - Macroscopic & Microscopic

Momentum features in many sections of the A-level Physics specifications. It links with vectors and energy, and is important in the study of collisions and explosions (Conservation of Momentum) with both large objects (cars, rockets, etc) and small objects (atoms, alpha and beta particles, etc).

We will begin with the general idea of momentum, and then revise the more familiar large-scale events, before finishing with a look at momentum properties at the atomic and subatomic levels.

Definition

Momentum = mass × velocity $(p = mv)$

> *Momentum is a vector. In linear events, '+' and '-' are often used to represent momentum towards the right and towards the left. However you must always state the direction to make the answer complete.*

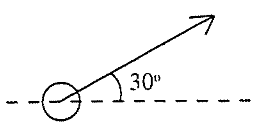
Example1 : Find the momentum of a 500kg car travelling with a velocity of 0.005kms^{-1} towards the right.

Answer: $p = mv = 500 \times 5 = 2500$ *kgms^{-1} towards the right.*
(Remember to use the standard units and state the direction.)

Components

It is often necessary to separate vectors into horizontal and vertical components. Momentum is a vector.

Example 2: A body has a momentum of 250 kgms^{-1} directed as shown in the diagram.



Find the horizontal component of its momentum.

Answer: $p_x = 250 \times \cos 30 = 217$ *kgms^{-1} towards the right.*

Impulse and Change in Momentum

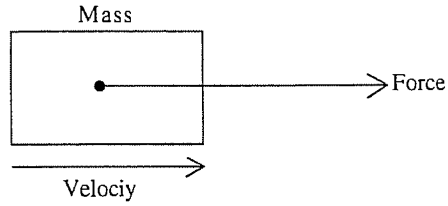
The momentum of an object can be changed if a force acts on it for a period of time.

We define **impulse** as $F \times t$. *(Ns)*

The impulse exerted equals the change in momentum.

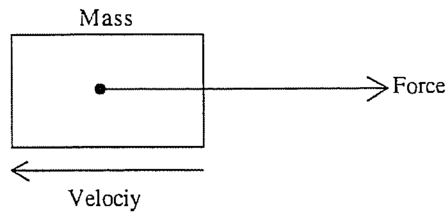
$F \times t = \Delta mv = mv - mu.$

Example 3: A force of 10N towards the right is applied for 2s to a 3kg mass already travelling at 4ms^{-1} towards the right. Find the final velocity.



Answer: $F \times t = mv - mu$, $10 \times 2 = 3v - (3 \times 4)$,
$v = 10.7$ *ms^{-1} towards the right.*

Example 4: A force of 10N towards the right is applied for 2s again to a 2kg mass travelling at 2ms^{-1} towards the left. Find the final velocity,



Answer: $Ft = mv - mu$, $10 \times 2 = 2v - 2 \times (-2)$,
$v = 8.0$ *ms^{-1} towards the right.*

NB - the convention that positive stands for motion towards the right was used. (The initial velocity was towards the left.)

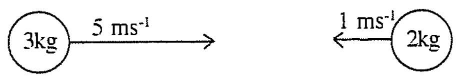
Conservation of momentum

During a collision or explosion event, the total momentum of the system is conserved. We will confine ourselves to linear events, but this law also holds in 2 or 3 dimensions.

> *Remember that Kinetic Energy is **not** usually conserved in a collision. Some KE is usually converted to **heat** and **sound**.*

One standard collision situation involves the two objects sticking together after the collision. In this case, kinetic energy is **never** conserved.

***Example 5 : The two balls shown stick together when they* collide.**



Find the velocity after the collision, and the KE lost.

Answer
Momentum before: $3 \times 5 + 2 \times (-1) = 13$ *kgms*$^{-1}$.
Momentum after: $(3+2)v$.

Momentum conserved: $5v = 13$, $v = 2.6$ *ms*$^{-1}$ *towards the right.*

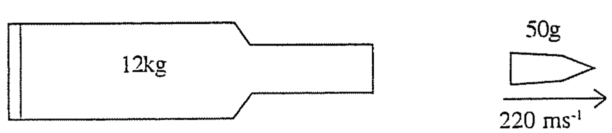
KE before: $(\frac{1}{2} \times 3 \times 5^2) + (\frac{1}{2} \times 2 \times 1^2) = 38.5$ *J.*
KE after: $(\frac{1}{2} \times 5 \times 2.6^2) = 16.9$ *J.*

KE lost to heat and sound: 21.6 J.
This collision is said to be ***inelastic.***

If KE is conserved in a collision, the collision is said to be ***elastic.***

An "explosion" event involves an action-reaction pair pushing each other apart. The recoil of a gun is an example.

Example 6: A 50g shell is fired from a gun of mass 12 kg. The initial velocity of the shell is 220 ms^{-1} in the direction shown.

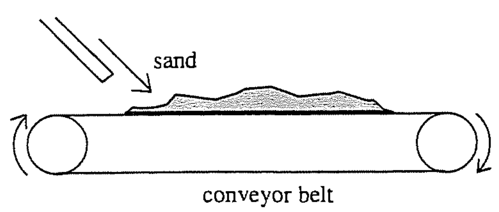


Find the recoil velocity of the gun.
Answers
Momentum before = 0 *kgms*$^{-1}$
Momentum after = $(0.050 \times 220) + 12v$.
$12v + 11.0 = 0$, $v = -0.92$ *ms*$^{-1}$.
So the recoil velocity is 0.92 ms^{-1} *towards the left.*

Exam Hint: *Conservation of momentum calculations are fairly standard. Use a simple system such as stating momentum before, momentum after, and then equating the two. Take care with units and direction.*

Changing mass
Remember that momentum depends on both mass and velocity. Momentum changes can be caused by a change in mass or a change in velocity. Sand pouring onto a moving conveyor belt would slow the belt down (if the motor were turned off) as mass times velocity must stay the same for the sand/conveyor belt system.

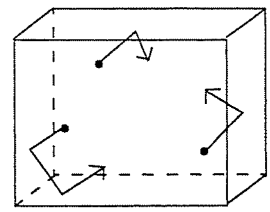
These systems are less common than changes in velocity, but you should be aware that changes in mass cause changes in momentum.



Microscopic events
We will finish with a quick look at momentum considerations in atomic and subatomic Physics.

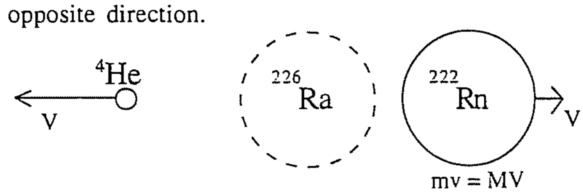
Gas Pressure (Kinetic Theory)
The pressure of a gas on its container depends on the total force per second exerted by the molecules as they collide with the walls of the container.

A molecule changes momentum when it rebounds from the wall. Using the relationship $Ft = \Delta mv$, we can work through to our normal Gas Law equations.



Question 6 at the end of this Factsheet helps you to follow through a set of calculations leading to the determination of the pressure of a gas sample in a container.

Alpha decay
When an alpha particle is emitted from a nucleus, conservation of momentum insists that the remainder of the nucleus must recoil in the opposite direction.



Example 7: For the decay reaction illustrated,
$\frac{v}{V} = \frac{M}{m} = \frac{222}{4} = 56$

To find the ratios of the kinetic energies of the two masses:
$\frac{\frac{1}{2}mv^2}{\frac{1}{2}MV^2} = \frac{m}{M} \times \frac{v^2}{V^2} = \frac{1}{56} \times \left(\frac{56}{1}\right)^2 = 56$
So the alpha particle ends up with almost all of the kinetic energy produced in the decay.

Electron diffraction
When an electron is accelerated through a voltage, V, it gains kinetic energy which can be found from $\frac{1}{2}mv^2 = eV$.

As momentum $p = mv$, we can replace $\frac{1}{2}mv^2$ with $p^2/2m$.
Setting $p^2/2m = eV$, we find that $p = \sqrt{(2meV)}$, where m is the mass of the electron (9.1×10^{-31} kg).

Then $\lambda = h/p$ lets us calculate the de Broglie wavelength for the electron.

For low energies (small voltages) this all works. But for large energies, calculations show that the electron speed is greater than the speed of light.

As the electron speed approaches the speed of light, the electron mass increases from the rest mass value up to relativistic values, and the Newtonian equations no longer work.

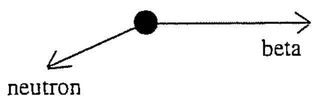
Example 8: find the speed of an electron accelerated through (a) 1000V, (b) 1MV.

Answer:
(a) $v^2 = (2eV) / m$, $v = 1.9 \times 10^7$ ms^{-1}.
(b) $v = 5.9 \times 10^8$ ms^{-1}. (greater than the speed of light in a vacuum)

For very high voltages, we must use the equation for the momentum of photons (**p = E/c**) to work out an approximate value for the de Broglie wavelength of the electron.

Beta decay and neutrinos

Finally, cloud chamber photographs can show a decayed neutron (a proton) recoiling in a non-linear situation from the beta particle it has just emitted.

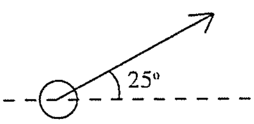


Conservation of momentum predicts that a third particle must be involved. This is the neutrino, which must be heading in an upwards direction to satisfy our conservation law.

Questions

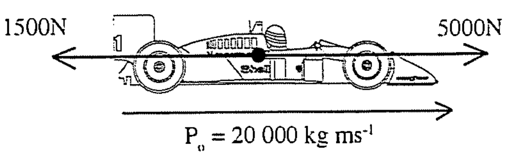
1. Find the momentum of a ball of mass 82g moving towards the left with a speed of 30ms^{-1}.

2. A 20 kg projectile is launched with a speed of 15 ms^{-1} at an angle of 25° above the horizontal.



   Find its initial vertical momentum.

3. A car has a momentum of 20 000 kgms^{-1} towards the right. Its engine provides a thrust of 5000N for 10 seconds, also towards the right. All resistive forces add to 1500N. Fine the momentum of the car at the end of the 10s period.

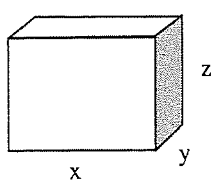


4. Explain why an egg is more likely to break when dropped onto a concrete floor rather than onto a carpet.

5. A child of mass 30kg jumps forward off a stationary skateboard of mass 1.5 kg. The child's forward speed is 0.10 ms^{-1}. what is the recoil velocity of the skateboard?

6. There are 3.0 × 10^{6} molecules of a gas moving randomly in a cubic box of sides 0.5 m at a speed of 500 ms^{-1}.

   (a) How many molecules are effectively moving in the x-plane?



   (b) How often will each of these collide with the shaded wall?

   (c) How many collisions per second with this wall will there be in total?

   (d) What is the momentum of each molecule before and after each collision with the shaded wall? (Assume elastic collisions, and assume that the mass of each molecule is 2.5 × 10^{-26} kg.)

   (e) Use **Ft = Δmv** to find the force of all the collisions with this wall in one second.

   (f) Find the pressure the gas exerts on this wall, and comment on the result.

Answers

1. $p = mv = 0.082 \times 30 = 2.46$ kgms^{-1} towards the left.

2. $p_y = 20 \times 15 \times \sin 25 = 127$ kgms^{-1} **upwards.**

3. $\Delta p = Ft = 3500 \times 10 = 35\,000$ kgms^{-1}
   Final p = 55 000 kgms^{-1} towards the right.

4. Change in momentum as the egg comes to rest is the same for each surface, but the time taken to decelerate is greater with the carpet. **Ft = Δp** predicts that for Δp fixed, a greater time of contact will mean a smaller force applied to the egg.

5. $(30 \times 0.10) + 1.5v = 0$, $v = 2.0$ ms^{-1} backwards.

6. (a) One-third of them, or 1.0× 10^{6} molecules.

   (b) $t = s / v = (2 \times 0.5)/500 = 0.002$ s.

   (c) $1 / 0.002 = 500$ s^{-1} by each molecule.
   Total = $500 \times 1.0 \times 10^{6} = 5.0 \times 10^{8}$ collisions s^{-1}.

   (d) Before: $mv = 2.5 \times 10^{-26} \times 500 = 1.25 \times 10^{-23}$ kgms^{-1}
   After: $mv = -1.25 \times 10^{-23}$ kgms^{-1}

   (e) $F = \Delta mv / t = 2.5 \times 10^{-23} \times 5.0 \times 10^{8} = 1.25 \times 10^{-14}$ N

   (f) $p = F / A = (1.25 \times 10^{-14}) / 0.25 = 5.0 \times 10^{-14}$ Nm^{-2}.
   This is a very small pressure, because this is a very small number of molecules.

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Physics Factsheet

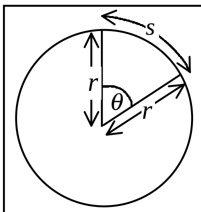
September 2001 Number 19

Circular Motion – Basic Concepts

This Factsheet will:
- **explain how to use radian measure for angles**
- **explain the nature of the resultant force involved in circular motion at a uniform speed**
- **explain how to use the equations for centripetal force and acceleration in questions**
- **describe the forces involved in vertical circular motion**

Later Factsheets will cover specific examples of circular motion in connection with gravitation and magnetic fields.

Radian measure
Radians are a way of measuring angles, other than using degrees.
To see how they work, we will look at a circle of radius r.



In this diagram, the arc (=*part of circumference*) subtended by angle θ is equal to *s*.

Then the size of angle θ, in radians, is $\frac{s}{r}$

From this, we can conclude

> The arc subtended by an angle θ radians is of length rθ.

Since the whole circumference is 2πr, we can also conclude:

> There are 2π radians in a full circle, so 2π radians is equivalent to 360°

It is sometimes necessary to convert between radians and degrees. The rules are:

> To change radians to degrees, multiply by $\frac{180}{\pi}$
>
> To change degrees to radians, multiply by $\frac{\pi}{180}$

Radians on your calculator
You need to learn to use radian mode on your calculator. On standard scientific calculators, you will probably be able to reach it using a button labelled "DRG" (the R stands for radians and the D for degrees - you never need to use the G!). In some calculators, you need to change to the mode number for radians instead. Some graphical calculators work in the same way; in others you have to go via the set-up menu.

Once you are in radians mode, the calculator "thinks" any number you put in is an angle in radians. So to find the sine of 2.03 radians, you change the calculator into radians mode, then put in SIN 2.03 (or 2.03 SIN, depending on your calculator). Check this now – you should get the answer 0.896…

> *Exam Hints:*
> 1. ***Do not try to do circular motion calculations without going into radians mode – it will slow you down a lot. Find out how to get into this mode now, and make sure you remind yourself of this before the exam.***
> 2. ***Always check which mode you're in before doing any calculation. One easy way is to find sin30 – if you're in degrees mode, this will be a nice number (0.5), if you're in radians , you'll get a long decimal.***
> 3. ***Be alert to "silly" answers – for example, if you come up with an answer of 137 radians, or 0.2 degrees, check you are in the right mode!***

Why bother with radians?
Working in radians actually makes some calculations easier - for example, we have already seen that the formula for the length of an arc is simple when the angle is in radians. Other formulae encountered later in circular motion and in simple harmonic motion (Factsheet 20) automatically assume any angles measured are in radians, and will not work otherwise! Exam boards also specify that candidates must know how to work in radians.

Angular speed
Angular speed (symbol ω; unit rad s^{-1}) is an important concept in circular motion.

> Angular speed (ω rad s^{-1}) = $\frac{\text{angle turned } (\theta \text{ radians})}{\text{time taken } (t \text{ seconds})}$

In most cases of horizontal circular motion (i.e. where the body is staying at the same horizontal level), ω is constant; however in many cases of vertical circular motion (eg a ball tied to the end of a string being swung in vertical circles), ω is not constant. If the angular speed is not constant, then there must be some **angular acceleration**; this is the rate of change of angular speed.

> ***Exam Hint: Do not confuse angular acceleration with linear (or "normal") acceleration. <u>All</u> bodies moving in a circle are accelerating (see later) but they only have <u>angular</u> acceleration if their angular velocity changes.***

Instead of giving angular speed in rad s^{-1}, questions may give you angular speed in revolutions per second; you may also be given the **period** of the motion – the time required to complete one full circle. **You need to convert to radians per second before using any formulae!**

> To convert:
> - revs/sec to radians/sec: multiply by 2π
> - revs/min to radians/sec: divide by 60 & multiply by 2π
> - degrees/sec to radians/sec: divide by 180 & multiply by π
>
> $\omega = \frac{2\pi}{T}$ where T = period in seconds and ω is in rad s^{-1}

Relationship between angular speed and linear speed
We will consider a body travelling in a circle of radius r with constant angular speed. As it turns through an angle θ radians, it will travel through an arc of length θr (since $\theta = \frac{s}{r}$).

So speed = $\frac{\theta r}{t} = \frac{\theta}{t} \times r = \omega r$.

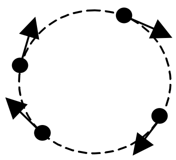
$v = \omega r$
where v = speed (ms^{-1})
ω = angular speed (rad s^{-1})
r = radius of circle (m)

Forces in circular motion
To understand forces in circular motion, first recall Newton's first law:

Every body continues at rest or with constant velocity unless acted upon by a resultant force

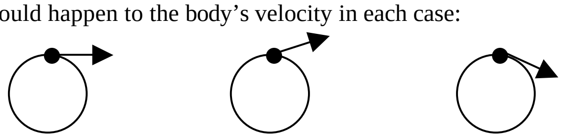
Since velocity is a vector quantity, possessing both magnitude (the speed) and direction, if velocity is constant, then both the speed and direction of motion must be constant.

In circular motion, although the speed may be constant, the velocity is continually changing direction. At any point on the circular path, the velocity is along the **tangent** to the circle at that point – it is at right angles to a line joining the moving particle to the centre of the circle



Since the velocity of a body performing circular motion is changing, the body must be subject to a force. If the force acting on the body had a component parallel to the velocity, this would increase (or decrease) the speed of the body. So, **if the speed is constant, the force must be perpendicular to the velocity at all times**.

Since the force is perpendicular to the velocity, it could be either inwards – towards the centre of the circle – or outwards, away from the centre of the circle. To see which of these is correct, imagine what would happen to the body's velocity in each case:



Starting from this velocity | An outward force would produce this | An inward force would produce this

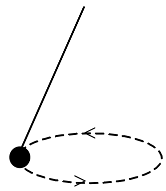
Therefore we can conclude that:

A body moving in a circular path experiences a force directed towards the centre of the circle.
This is known as **centripetal force**.

Where does the centripetal force come from?
It is important not to consider the centripetal force as an "extra" force – it must be produced by the forces you know are acting on the body. Some examples of forces producing circular motion are shown below; others will be encountered in other contexts later in the course.

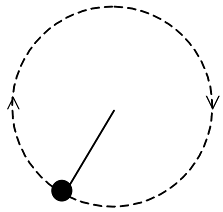
The **conical pendulum –** a mass on a string, moving in horizontal circles.

The centripetal force is produced by the horizontal component of the tension in the string



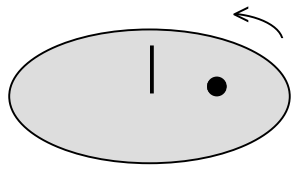
Vertical circles – produced by a mass on a string.

The centripetal force is produced by resultant of the tension in the string and the weight



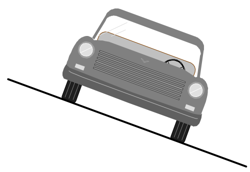
Mass on a turntable – so the mass moves in horizontal circles.

The centripetal force is provided by the friction between the mass and the turntable



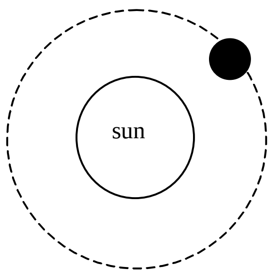
Banked track – racing tracks are banked (=sloped) at bends, which assists the cars in turning. The car turns through part of a horizontal circle.

The centripetal force is provided by the horizontal components of the friction acting on the car and of the reaction of the track on the car



Planetary motion – although the planets do not move in exactly circular orbits, for many of them this is a good approximation.

The centripetal force is provided by the gravitational force between the planet and the sun.



Typical Exam Question
(a) Explain why a body moving in a circle with a constant speed has acceleration [2]
(b) In a question about a spacecraft in orbit, a student writes: "The gravitational pull of the Earth is balanced by the centripetal force of the space craft and so it stays in orbit." Explain why this statement and is incorrect and write a corrected version [4]

(a) An object moving in a circle is continually changing the direction of its motiony ✓ Since velocity is a vector, change of speed or direction means that it is accelerating ✓
(b) Centripetal force is not a separate force due to circular motion ✓ It is an unbalanced force, producing the motion ✓
Statement could read: "The gravitational pull of the Earth provides ✓ the centripetal force required ✓ to keep the space craft in orbit."

Formulae for centripetal force

The centripetal force on a body is given by:

$$F = \frac{mv^2}{r}$$

F = centripetal force (N)
m = mass of body (kg)
v = speed (ms^{-1})
r = radius of circular path (m)

Since $v = \omega r$, another formula can also be obtained:

$$F = \frac{mv^2}{r} = \frac{m(\omega r)^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r$$

The centripetal force on a body is also given by:

$$F = m\omega^2 r$$

F = centripetal force (N)
m = mass of body (kg)
ω = angular speed (rad s^{-1})
r = radius of circular path (m)

Calculations on horizontal circular motion

To answer circular motion questions successfully, you will need to draw a **force diagram** as well as know the appropriate circular motion formulae.

For horizontal circular motion, the key ideas to use are:

- resultant horizontal force = centripetal force = $\frac{mv^2}{r}$ or $m\omega^2 r$
- resultant vertical force = 0

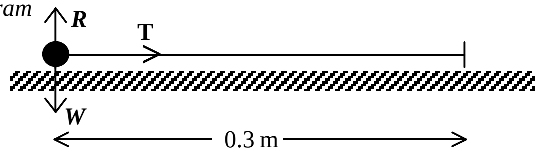
The procedure when answering questions is:

1. Draw a diagram showing all forces acting on the body. **Do NOT put in the centripetal force as an "extra" force.**
2. Work out from the question what information you are given, and what you want – in particular, whether you are given/want ω or v. **Remember: convert ω into rad s^{-1} or *v* into ms^{-1}.**
3. Resolve horizontally, and set the resultant inward force equal to $\frac{mv^2}{r}$ or $m\omega^2 r$, depending on what you have decided in 2.
4. If you need to, resolve vertically and equate to zero.
5. Combine the equations from 3. and 4. to get the answer.

Example 1
A small mass is attached to one end of a string of length 0.3m. The other end of the string is fastened to the centre of a smooth, horizontal table. The mass is made to move in horizontal circles at constant speed on the table, with the string taut.

Given that the mass makes 30 revolutions per minute, find the tension in the string

1. *Diagram*



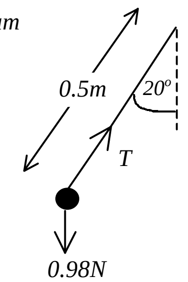
Note:
i) Since it is a "smooth" table, there is no friction
ii) All units changed to SI

2. *We want T*
We are given that the mass makes 30 revolutions per minute – so we have to convert this into rad s^{-1} to find ω.
$30 \text{ rev min}^{-1} = 30 \div 60 = 0.5 \text{ rev s}^{-1}$
$0.5 \text{ rev s}^{-1} = 0.5 \times 2\pi = \pi \text{ rad s}^{-1}$
So $\omega = \pi \text{ rad s}^{-1}$

3. $T = m\omega^2 r$ *(we use this form as we know ω)*
Substituting in:
$T = 0.05 \times \pi^2 \times 0.3 = \mathbf{0.15N}$

Example 2
A particle of mass 0.1kg is attached to the end of a string of length 0.5m, and is swung in a horizontal circle at constant speed, so that the string makes an angle of 20º with the vertical.
(a) Calculate the tension in the string (take g = 9.8 N kg^{-1})
(b) Calculate the speed of the particle

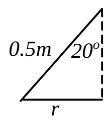
1. *Diagram*



2. *We need T and v*
3. *Resolving:* → $T\sin 20 = mv^2/r$
4. *Resolving:* ↑ $T\cos 20 = 0.98N$
5. *Use the second equation first, since this has only one unknown:*
$T\cos 20 = 0.98N$
$T = 0.98 / \cos 20 = 1.04N$

We now need to use the first equation to find v:
$T\sin 20 = mv^2/r$
$1.04 \times \sin 20 = 0.1v^2/r$

So we need to find r before we can find v



So $r = 0.5\sin 20 = 0.171m$

So $v^2 = 1.04 \times \sin 20 \times r/0.1 = 0.61 m^2 s^{-2}$
$\mathbf{v = 0.78\ ms^{-1}}$

NB: Make sure you understand why r is not 0.5m. The particle is moving in a horizontal circle, so the radius must be measured horizontally.

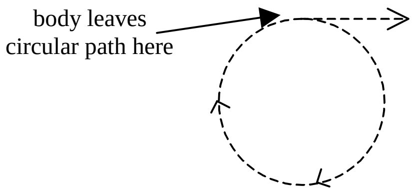
Exam hint:-It is sensible to leave the 20º in this question as degrees, rather than change to radians, because it is not directly related to the angular speed

Leaving the circular path

Sometimes a body that is moving in a circle leaves the circular path; examples include a car skidding when going around a bend and the string breaking when an object is being swung on it. This happens because the force(s) providing the centripetal force vanish (as with the string breaking), or become insufficient to hold the body to the circular path.

What happens after leaving the circular path

After leaving the circular path, the body will initially go off at a **tangent**:



Its motion will then, of course, be governed by the forces still acting on it (eg gravity).

Why bodies may leave the circular path

To see why this happens, we need to return to centripetal force:

- For a body being swung on a string, a component of the tension provides the centripetal force. If the tension is too great, the string will break. The higher the speed of the body, the higher the centripetal force is required to be. So the string may break if the body is being swung too fast.
- For a car going round a bend, the centripetal force is provided by friction. So if the speed is too high, the friction may not provide sufficient centripetal force, and the car will skid. If conditions arise that lower friction – such as rain or ice – then the maximum possible safe speed will need to be decreased.

Typical Exam Question

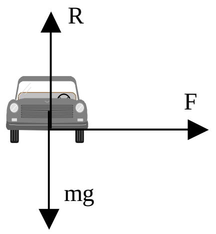
A car travels along a level road round a curve of radius 500m.

(a) Draw a diagram to show the forces acting on the car. [2]

(b) If the maximum frictional force between the tyres and the road is 70% of the weight of the car, find the greatest speed at which the car can travel round the curve. Take g = 9.8Nkg^{-1} [4]

(c) Describe the path the car will take if it exceeds this speed. [1]

(a)



R = normal reaction
F = friction
mg = weight of car

✓✓

(b) Resolving horizontally: $F = mv^2/r$
vertically: $R = mg$ ✓ *(both)*
Maximum speed occurs when friction is limiting, so F = 0.7mg ✓
So: $0.7mg = mv^2/r$
$0.7g = v^2/r$ ✓
$v^2 = 0.7g \times r = 0.7 \times 9.8 \times 500 = 3430$
$v = 58.6\text{ms}^{-1}$ ✓

(c) It will leave the road, at a tangent to the curve ✓

Vertical Circular Motion

Generally, a body moving in a vertical circle will not be moving at constant speed unless there is some mechanical device – such as the motor for a "big wheel" – forcing it to do so. This means that the size of the centripetal force on the body will also vary.

Here, the key approach is:

1. Draw a diagram, showing all the forces.
2. Use conservation of mechanical energy to find the speed at any point.
3. If necessary, set the resultant force towards the centre of the circle equal to the centripetal force.
4. If necessary, combine the equations from 2. and 3. to obtain the answer.

Example 1

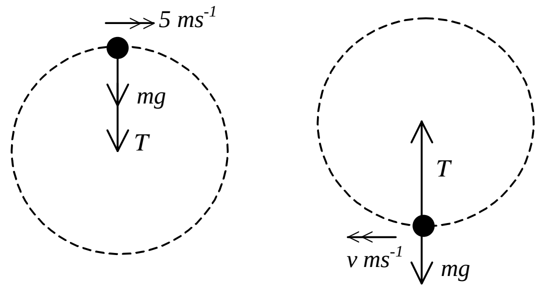
A ball of mass 0.30kg is attached to a string of length 0.50m, and swung so that it moves in a vertical circle. When the ball is at its highest point, it moves with speed 5.0 ms^{-1}. Taking g = 9.8Nkg^{-1}:

a) Find the speed of the ball at its lowest point

b) Find the maximum and minimum values of the tension in the string.

a)

1. *At highest point* *At lowest point*



2. Taking potential energy as zero at lowest point:
At highest point: $p.e. = 0.3 \times 9.8 \times 1 = 2.94J$
$k.e. = ½ \times 0.3 \times 5^2 = 3.75J$

At lowest point: $p.e. = 0$
$k.e. = ½ \times 0.3 \times v^2 = 0.15v^2$

Using conservation of mechanical energy:
$2.94 + 3.75 = 0.15v^2$
$v^2 = 6.69/0.15 = 44.6\ m^2s^{-2}$
$\mathbf{v = 6.7\ ms^{-1}}$

b) The tension will be the largest at the lowest point, since it has to supply the centripetal force and overcome the weight of the particle It will be smallest at the highest point, since here the weight is contributing to the centripetal force.

Exam Hint: - The points at which the tension (or other force on the object) is largest and smallest is commonly examined. Always ask yourself whether the weight of the object is helping or hindering.

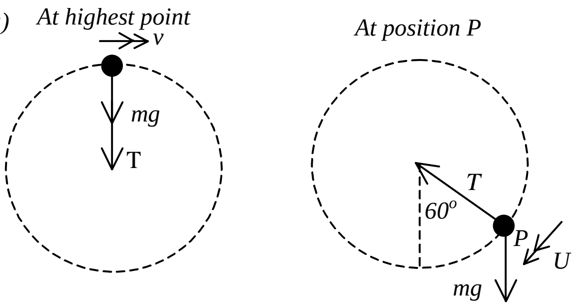
At lowest point: resolving inwards gives
$T - mg = mv^2/r$
$T = m(g + v^2/r)$
$T = 0.3(9.8 + 44.6/0.5)$
$T = \mathbf{30N}$

At highest point: resolving inwards gives:
$T + mg = mv^2/r$
$T = m(v^2/r - g)$
$T = 0.3(25/0.5 - 9.8)$
$T = \mathbf{12N}$

Example 2
A small object of mass 200g is attached to the end of a light string of length 40cm. The string is fastened at its other end, and the object is set in motion so that it moves in a vertical circle. When the string makes an angle of 60^o with the downward vertical, the object moves with speed U ms^{-1}. (g = 9.8Nkg^{-1})

(a) Find the speed of the object at the highest point in terms of U
(b) Find the tension at the highest point in terms of U
(c) Hence give the minimum possible value of U

(a)



Taking p.e. = 0 at the lowest point
Height above lowest point when string makes 60^o to vertical
$= 0.4 – 0.4\cos60 = 0.2$
So, using conservation of mechanical energy:
Total energy at top = total energy at P
(k.e. + g.p.e) at top = (k.e. + g.p.e.) at P
$mg \times 0.8 + ½ mv^2 = ½ mU^2 + mg \times 0.2$
$0.8g + ½v^2 = ½U^2 + 0.2g$
$7.84 + ½ v^2 = ½U^2 + 1.96$
$v^2 = U^2 – 11.76$
$v = \sqrt{(U^2 – 11.76)}$

(b) Resolving inwards at highest point:
$T + mg = mv^2/r$
$T + 1.96 = 0.2\times((U^2 – 11.76)/0.4)$
$T + 1.96 = 0.5U^2 – 5.88$
$T = 0.5 U^2 – 7.84$

(c) The tension must be ≥ 0 at all times, or else the particle would not continue to move in a circle.

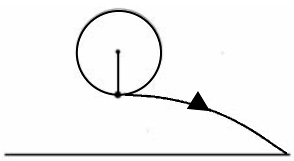
So $½ U^2 – 7.84 \geq 0$
$U^2 \geq 15.68$
So minimum value of U is $\sqrt{15.68} = 4.0$ ms^{-1}

Typical Exam Question
An object of mass 2.0kg is rotated in a vertical circle on a cord of length 1.0m. The cord will break if the tension in it becomes 500N. The speed of rotation is gradually increased from zero.
(a) Find the angular velocity at which the string breaks. [4]
(b) Draw a diagram to show the position at which the string breaks and the subsequent motion of the object. [4]

(a) Max tension in cord at the bottom of the circle, $T = m\omega^2 r + mg$ ✓

$$\omega = \sqrt{\frac{T - W}{mr}} \checkmark = \sqrt{\frac{500 - 2\times 9.8}{2\times 1}} \checkmark = 15.5 \text{ rad s}^{-1} \checkmark$$

(b) Diagram ✓

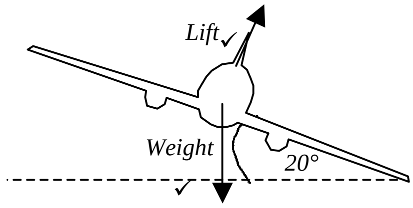


String breaks at lowest point ✓
Object has horizontal (tangential) velocity at first ✓
Follows parabolic path under gravity ✓

Exam Workshop
This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

A plane flying in a horizontal circle of radius 1.0km is banked at 20^o.

(a) Show the forces acting on the plane and explain how it maintains circular motion. [3]



The plane flies in a circle because there's a centripetal force ✗ *2/3*

The student needed to explain where the centripetal force came from, not just state that one existed.

(b) Taking *g = 9.8Nkg^{-1}*, calculate:
(i) the speed of the aircraft. [4]

$mv^2/r = L\cos20^o$ $\quad mg = L\sin20^o$ ✗
$L = mg/\sin20^o$ ✓ $= 29m$
$mv^2/1 = 29m\cos20^o$ ✗
$v^2 = 29\cos20^o = 27$
$v = 5.22$ ms^{-1} ✗ *1/4*

The student's method is fundamentally correct, but s/he has scored only one mark due to careless errors. The most serious ones were that the initial resolving was incorrect and the radius was taken as 1, rather than 1000. In addition, the student has rounded prematurely (taking g/sin20^o = 29) and then used an inappropriate number of SF in the final answer. The answer obtained should have alerted the student to problems – planes do not move so slowly! This answer appears to have been rushed.

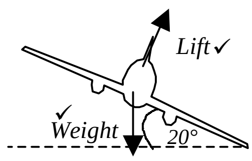
(ii) the time it takes to complete one circle. [2]

$T = 2\pi/\omega$. ✓
$\omega = v/r = 5.22$ rad s^{-1}
$T = 1.2$s ✓ *2/2*

Although the student's value for v was incorrect, s/he is not penalised again for this, nor for taking the radius as being 1 a second time. Full marks are therefore awarded for this part of the question.

Examiner's answer

(a)



When the aircraft banks there is a horizontal component of the lift, which provides the centripetal force ✓

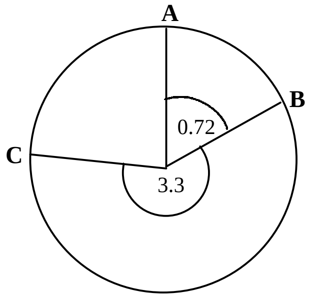
(b) (i) Vertically: $W = mg = L\cos20^o$, *horizontally:* $mv^2 / r = L\sin20^o$ ✓
$L = mg / \cos 20^o$ ✓ so $mv^2/r = mg \tan20^o$ ✓
$v^2 = gr\tan20^o = 3640$ m^{2}s^{-2}
$V = 60$ms^{-1} ✓

(jj) $T = 2\pi/\omega = 2\pi r/v$ ✓ $= 2000\pi/60.3 = 104$s ✓

Questions

1. (a) Convert to radians:
   (i) 60^o (ii) 111^o (iii) 249^o
   (b) All the angles in this part of the question are in radians. Use your calculator to find:
   (i) sin(1.2) (ii) cos(0.789) (iii) tan(2.85)
   (c) x, y, and z are angles in radians between –1 and 1. Use your calculator to find them, given that
   (i) sinx = -0.3 (ii) cosy = 0.8 (iii) tanz = 1.5

2. In the diagram below, the circle has radius 5cm and O is its centre. The angles marked are in radians.



Find the lengths of: (i) arc AB (ii) arc BC (iii) arc CA

3. Explain what is meant by angular speed, giving its symbol and unit.

4. Find the angular speeds of each of these bodies
   (i) A is moving at 13 revolutions per second
   (ii) B is moving at 240 revolutions per minute
   (iii) C is moving at 40^o per second
   (iv) D takes 5 seconds to make one revolution

5. State the relationship between angular speed and linear speed for a body moving in a circle.

6. Explain why a body moving at a constant speed in a circle must have a resultant force acting on it, and state the name given to this force.

7. What provides the centripetal force in each of the following cases:
   (i) the Moon orbiting the Earth
   (ii) an aircraft banking to turn
   (iii) a mass on a string moving in vertical circles
   (iv) a car going round an unbanked corner

8. Write down two formulae for centripetal acceleration

9. A particle is attached to a string of length 20cm. It is swung so that it moves in a horizontal circle at constant speed, with the string making an angle of 30^o to the vertical.
   (a) Calculate the speed of the particle, ($g = 9.8Nkg^{-1}$)
   (b) The string suddenly snaps. Given that the particle is 40cm from the ground, find the time it takes to hit the ground and the horizontal distance it travels in this time.

Answers

1. (a) Multiply by π and divide by 180^o in each case:
   (i) 1.05 (ii) 1.94 (iii) 4.35
   (b) Change the calculator to radians mode first:
   (i) 0.932 (ii) 0.705 (iii) –0.300
   (c) Calculator in radians mode; use $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$
   (i) –0.305 (ii) 0.644 (iii) 0.983

2. arc length = radius × angle in radians = $r \times \theta$
   (i) $5 \times 0.72 = 3.6$cm
   (ii) $5 \times 3.3 = 16.5$cm
   (iii) arc length = circle – other two arcs
   $= 2\pi \times 5 - 3.6 - 16.5 = 11.3$cm

3. $$\text{Angular speed} = \frac{\text{angle turned } (\theta\,/\,\text{radians})}{\text{time taken } (t\,/\,\text{seconds})}$$
   Symbol: ω; unit: radians per second

4. (i) $13 \times 2\pi = 81.7$ rad s^{-1}
   (ii) 240 rev min^{-1} = 4 rev s^{-1}
   $4 \times 2\pi = 25.1$ rad s^{-1}
   (iii) $40 \times \pi \div 180 = 0.698$ rad s^{-1}
   (iv) $\omega = 2\pi/T = 2\pi/5 = 1.26$ rad s^{-1}

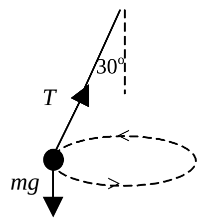
5. $v = \omega r$, where r = radius of the circle, ω = angular speed; v = linear speed

6. Velocity is a vector. Since the direction of motion of the body is always changing, its velocity is always changing. Since the body is not moving with constant velocity, there must be a resultant force acting on it. This force is called centripetal force.

7. (i) Gravitational force of the Earth on the Moon.
   (ii) Horizontal component of lift.
   (iii) The resultant of the components of tension and weight acting towards the centre of the circle.
   (iv) Friction

8. $a = \omega^2 r$, $a = v^2/r$ or $a = \omega v$

9. (a)



$T\cos30^o = mg$ (i)
$T\sin30^o = mv^2/r$ (ii)
(ii) ÷ (i): $\tan30^o = v^2/(gr)$
$v^2 = gr\tan30^o$
$v^2 = 0.20 \times 9.8 \times \tan30^o$
$v^2 = 1.13\ m^2s^{-2}$
$v = 1.06\ ms^{-1}$

(b) When string snaps, particle moves under gravity.
So for time taken: $\frac{1}{2} gt^2 = 0.4 \Rightarrow t = \sqrt{0.08} = 0.28$s
Horizontal distance travelled = $0.76 \times 0.28 = 0.21$m

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Physics Factsheet

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Circular Motion and Vectors

The purpose of this Factsheet is to help you to understand and handle vector quantities, to explain the ideas of circular motion, to consider important equations from those ideas and to give practice at handling the equations.

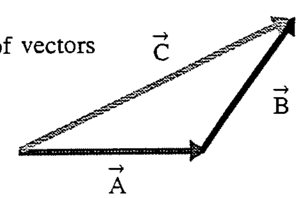
Vectors

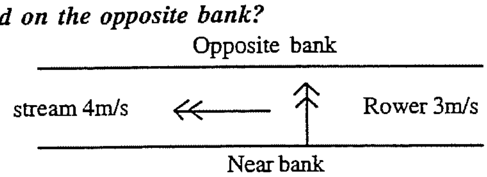
You may remember from your GCSE work, that **speed** is a **scalar** quantity (it has magnitude, but no direction), whereas **velocity** is a **vector** (it has direction as well as magnitude). So "3m/s" is a speed, but "3m/s due east" is a vector.

> ***Exam Hint:*** *To quickly decide if a quantity is a vector or a scalar, determine whether it has a direction associated with it. Sometimes this can be represented by plus or minus values.*

Often it does not matter whether a quantity is a vector or a scalar, but it **does** matter when it comes to adding them up.

Scale drawing of vectors



> ***Worked example 1***
>
> ***A man rows across a river at 3ms^{-1}, but the stream is flowing past at 4ms^{-1}. What is his effective speed across the river and in what direction is he traveling with respect to the bank? If the river is 6m wide, how far down from his starting position will he land on the opposite bank?***
>
> 
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>
> *The resultant speed could be worked out by doing a scale drawing of the two velocities, but since they are at right angles to each other they can be combined by using Pythagoras' Theorem.*
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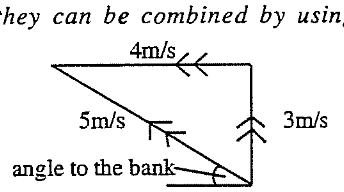
> 
>
>
> *Resultant speed^2 = $3^2 + 4^2 = 25$*
> *Resultant speed = 5m/s*
>
> *So the rower is effectively travelling along the path shown on the diagram at a speed of 5m/s.*
> *The direction can be worked out by trigonometry. The tangent of the angle is ¾, so the angle is 36.9°.*
>
> *If the river is 6m wide, the distance down the river is given by:*
> *6/distance = tan 36.9°*
>
> *Distance = 6/ tan 36.9° = 6 × 4/3 = 8m*

Other important vectors

A velocity is an important vector quantity, but you are likely to be even more concerned with **force** as a vector quantity, since particles are often influenced by more than one force at a time and so the effects have to be added up. Acceleration and momentum are also vector quantities.

> *Acceleration, momentum and force are all vector quantities.*

An important special case is of a force acting at right angles to the velocity of an object travelling in a straight line. As we shall see, this gives rise to circular motion.

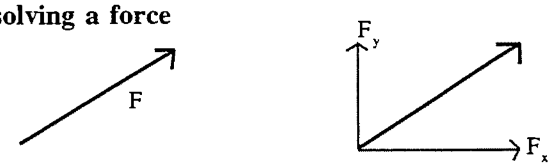
Forces as vectors

For AS and A2 you will only be required to deal with vector addition by scale drawing or calculations for vectors at right-angles to each other, so you only need to be able to perform calculations by using Pythagoras' theorem.

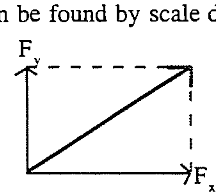
If all vectors can be added by vector addition to give a resultant, then it follows that **any** force can be expressed as the sum of two other forces at right angles to each other. This process is called **resolving** the force into two **components** at right angles to each other, and it is a very useful tool for working out problems involving forces.

> *Any force can be resolved into two components at right angles to each other.*

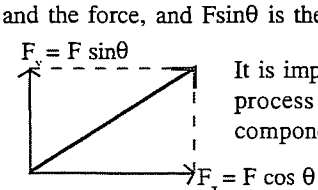
Resolving a force



The components can be found by scale drawing, by completing the rectangle:



or by calculation from the angle between the force and the horizontal and vertical. If θ is the angle with the horizontal, then simple trigonometry shows F_x is $F\cos\theta$. If the angle between the force and the horizontal is θ, then the angle with the vertical is 90 - θ, so the vertical component, F_y is $F\cos(90 - \theta)$, which is $F\sin\theta$, so it is easiest to remember the two components as $F\cos\theta$, and $F\sin\theta$, where $F\cos\theta$ is the component with the known angle between it and the force, and $F\sin\theta$ is the other one.



It is important to become familiar with this process of resolving a force into its two components, $F\cos\theta$ and $F\sin\theta$.

Any force may be replaced by its two components: $F\sin\theta$ and $F\cos\theta$, where θ is the angle between the force and one of the components.

Exam Hint: It is sometimes possible to use either Pythagoras or trigonometry to determine the magnitude of vectors. Use whichever method you are happiest with.

Study the worked example and use the questions at the end of the Factsheet to get some practice.

Worked Example 2
A force of 6N acts at an angle of 40° to the horizontal.
(a) What is the horizontal component of the force?
$F_x = 6\cos40^o = 4.6N$
(b) What is the vertical component?
$F_y = 6\sin40^o = 3.86N$

Using the resolution of forces to solve problems

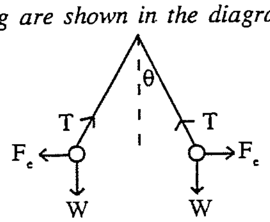
If an object is in equilibrium, then two conditions apply:
1. The components of the forces in any two directions at right angles to each other are balanced.
2. The sum of the moments about any point equals zero (Clockwise moments can be counted as positive and anticlockwise moments negative.)

We are not, here, concerned with the idea of moments, but the other condition requires us to resolve forces into components.

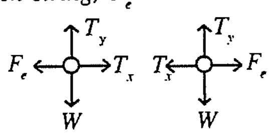
Study the worked example to help you to understand how the ideas can be used.

Worked example 3
Two charged pithballs are suspended from light strings. They exert a force of 1×10^{-6}N on each other. If the balls each have a mass of 0.0005g:
(a) Calculate the angle of the strings when they are in equilibrium.
(b) Calculate the tension in the strings.

The forces acting are shown in the diagram:



Where T is the tension in each string, F_e is the electrical force and W is the weight.
If we resolve the T's we get:



(a) $T_x = T\sin\theta$ and $T_x = T\cos\theta$. Since the balls are in equilibrium, $T_x = F_e$ and $T_y = W$
So $T\sin\theta = 1 \times 10^{-6}$
$T\cos\theta = 5 \times 10^{-7} \times 9.81$
Dividing these two equations gives $\tan\theta = 0.2$
So $\theta = 11.5^0$

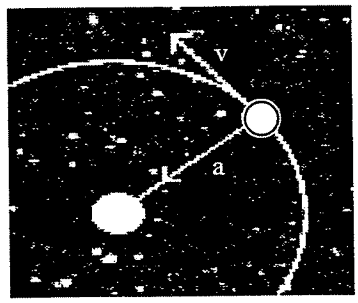
b) $T\sin\theta = 1 \times 10^{-6}$, $\theta = 11.5$, *so* $\sin\theta = 0.20$
and $T = 1 \times 10^{-6}/0.2 = 5 \times 10^{-6}N$

Exam Hint: *One of the most common errors on these questions is getting the sine and cosine reversed. Take great care to ensure you have these the right way around.*

Circular Motion

If a force acts on an object which is moving with constant speed in a straight line, at right angles to its velocity, then the force will change the direction of the object. Since a change in the direction constitutes a change in the **velocity**, (though not the **speed** of the object), this gives an **acceleration**, (since acceleration is the change in velocity, not just a change in speed). If the direction of the force changes continuously to be always at right angles to the velocity, then the object will move in a circle and the acceleration will be always towards the centre of the circle.

Velocity and acceleration vectors for circular motion



You will remember Newton's 1st law from your GCSE work, which states, basically, that an object will continue in a straight line at the same speed unless a force acts on it. We can now see, from the argument above, that in order for an object to be constrained to move in a circle, at constant speed, it must have a force acting at right angles to its velocity. This force is known as the **centripetal force** and if there is nothing to provide the centripetal force, then the object will no longer move in a circle, but continue in a straight line.

A little geometry allows the value of the necessary centripetal force to be worked out. It turns out to be **mv^2/r**, where m is the mass of the object, v its speed, and r the radius of the circle.

In order to move in a circle at constant speed, an object must have a force of value mv^2/r, acting towards the centre of the circle. This is the necessary centripetal force.

Equations of circular motion

For an object moving in a circle, we can define its **angular speed, ω** as the angle in radians (θ turned through per s. So: $\omega = \theta / t$

This combined with the definition of the angle in radians as the arc/radius, gives:

$$v = r \times \omega$$

Consideration of a full circle, 2π radians, shows that T (the **time period**, the time for a complete revolution) is **$2\pi/\omega$**, and the **frequency**, f ($1/T$) is given by $f = \omega/2\pi$ or $2\pi f$

The key equations for circular motion are:
$v = r\omega$
$\theta = \omega t$
$T = 2\pi/\omega$
$\omega = 2\pi f$

If you are not accustomed to dealing with angles in radians, it would be worth while spending some time becoming familiar with the idea. It is also worthwhile learning these equations, so that they spring to mind immediately when you are faced with problems on circular motion.

Exam Hint: These equations are based on the expression of angles in radians, not degrees. Remember that 360deg = 2π radians.

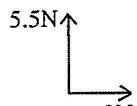
Since $v = r\omega$, the necessary centripetal force, mv^2/r, can also be expressed as $mr\omega^2$ or mvw. Which is the most convenient to use in any question depends on which combination of v, r or ω you are given.

The necessary centripetal force to constrain an object to move in a circle may be expressed as mv^2/r or as $mr\omega^2$ or $mv\omega$

Worked example 4
A girl standing at the equator is in circular motion about the Earth's axis.
(a) Calculate the angular speed of the girl.
For the Earth's rotation, the time period T = 24hours = 24 × 60 × 60s, so $\omega = 2\pi/T = 7.27 \times 10^{-5}$ rad/s.
(b) The radius of the Earth is 6400km. The girl has a mass of 60kg. Calculate the resultant force necessary for this circular motion.
Since we are given m, r and ω use centripetal force as $mr\omega^2$.
Force = $mr\omega^2 = 60 \times 6400000 \times (7.27 \times 10^{-5})^2 = 2.03$N

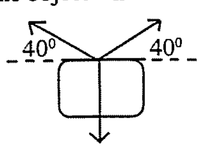
Practice Questions

1. The diagram shows two forces acting on a body.



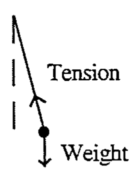
(a) Calculate the resultant force acting on the body.
(b) What name is given to physical quantities which add by the same rule as forces?
(c) Name two other examples of such physical quantities.

2. A container has a weight of 600N. The diagram shows the forces acting on the object when it is being carried by two people..



(a) Show that the tension in each rope is about 467N.
(b) Calculate the force in the each rope if the angle is increased to 60°.

3. A simple pendulum has a length l and a mass m on the end. The diagram shows the pendulum when it is at an angle of 3° to the vertical.



(a) Redraw the diagram, showing the components of the weight resolved in the direction of the string and at right angles to it.
(b) Express the components in terms of m, g and the angle.

4. (a) An object is moving in a circle with constant speed. Explain why it is said to have an acceleration towards the centre of the circle.

(b) A mass of 1kg is being whirled on a string in a horizontal circle with constant speed.
(i) What provides the necessary force towards the centre of the circle?
(ii) If the string is of length 0.5m and the maximum tension which the string can take before it breaks is 10N, what is the maximum speed of the mass?
(iii) What will happen to the mass if its speed exceeds this maximum?

5. A satellite orbits the Earth once every 74 minutes.
(a) Show that its angular speed is approximately 1.4×10^{-3} radians per s.

(b) The radius of the satellite's orbit is 6540km. Calculate the magnitude of its acceleration.

(c) A geostationary orbit is one in which the satellite remains over the same spot on the Earth.
(i) What is the time period for a geostationary orbit?
(ii) Calculate the speed for a geostationary satellite, where the radius of its orbit is 41500km.

6. In a tumble drier, clothes are placed inside a drum with small holes in it. The drum rotates at high speed and water escapes through the holes. A particular drum has a radius of 0.25m and rotates at 900 revolutions per minute.
(a) (i) Calculate the speed of the rim of the drum.
(ii) Calculate the force necessary to allow a piece of clothing of mass 0.4kg to perform circular motion.

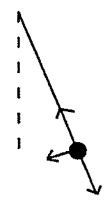
(b) (i) In the regions without holes, what provides the centripetal force to cause the clothes + water to perform circular motion?
(ii) Explain why the water escapes through the holes.

Answers

1. (a) Resultant ^{2} = $5.5^2 + 3^2$ = 39.25
   Resultant = 6.26N
   (b) The name given to such quantities is vector
   (c) Velocity, acceleration and momentum are also vector quantities.

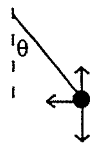
2. (a) Since the vertical components of the forces must balance, $2 \times F\sin40 = 600$
   $F = 300/\sin40 = 466.7N$ approx 467N
   (b) If the angle is 60^0, then $2 \times F\sin60 = 600$, $F = 300/\sin60 = 346.4N$

3. (a)



   (b) Component along the line of the string = weight × cos3 = mgcos3
   Component at right angles to the string = mgsin3

4. (a) Acceleration is a vector quantity. It is the change of the vector quantity velocity. Thus it is not just a change of speed which constitutes a change of velocity, it can also be a change in direction. The direction can be shown to be towards the centre of the circle by simple geometry.
   (b) (i) The horizontal component of the tension in the string provides the force towards the centre of the circle. The vertical component of the tension in the string balances the weight.



   (ii) The vertical component of the tension is $T\cos\theta$ and this = mg = 9.81N,
   so if T cannot be greater than 10, $10\cos\theta = 9.81$, $\cos\theta = 0.981$, $\theta = 11.2^0$
   The horizontal component is $T\sin\theta$ and this provides mv^2/r,
   so $v^2 = rT\sin\theta/m = 0.97$ $v = 0.98ms^{-1}$

   (iii) If the speed of the mass exceeds this figure, then the required tension in the string will be greater than 10N, so the string will break. There will then be no tension at all and the mass will initially continue in a straight line (i.e. it will move off at a tangent to the circle)

5. (a) $T = 74 \times 60s$, $T = 2\pi/\omega$, so $\omega = 2\pi/T = 1.41 \times 10^{-3}$ rads^{-1}, approx 1.4×10^{-3}rads^{-1}
   (b) Acceleration = $r\omega^2$ = $6540 \times 10^3 \times (1.41\times10^{-3})^2$ = 13 ms^{-2}
   (c) (i) For a geostationary orbit, T = 24 hours = 24 × 60 × 60s = 86400s
   (ii) $v = r\omega = \frac{r2\pi}{T} = 4.15 \times 10^7 \times \frac{2\pi}{864000} = 3.02 \times 10^3 ms^{-1}$

6. (a) (i) v = r? , 900 revolutions per s is $900 \times 2\pi/60$ rads^{-1} = 94.2rads^{-1}
   $v = 0.25 \times 94.2 = 23.6$ ms^{-1}
   (ii) Force required = mv^2/r = $0.4 \times 23.6^2/0.25$ = 891N

   (b) (i) It is the normal reaction force between the drum and the clothes + water which provides the centripetal force.
   (ii) At the holes, there is no reaction force, so there is nothing to provide the necessary centripetal force, so the water continues on in a straight line, i.e. tangentially to the drum, at the same speed.

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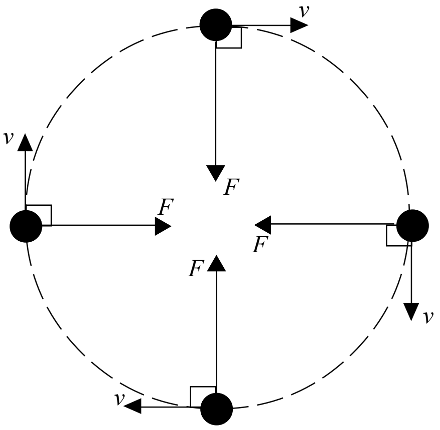
Applications of Circular Motion

Circular motion requires a resultant force in order to keep an object moving through a circular path at constant speed. It is accelerated motion as the velocity changes even though the speed remains the same; the reasoning for this is laid out below:
- Velocity is a vector (it has magnitude and direction).
- As the object travels around the circle its direction changes.
- Acceleration is rate of change of velocity.
- Therefore an acceleration is present.
- There must be a resultant (unbalanced) force acting to cause the acceleration ($F = ma$)
- This force acts towards the centre of the motion and is known as the *centripetal force.*

Object in circular motion



The time period (T) is the time, in seconds, it takes for the body to complete one revolution.

The frequency (f) is the number of revolutions the body will complete in one second.

*The resultant force that acts towards the centre of the circle is called the **centripetal force** and the acceleration it causes also acts towards the centre and is called the **centripetal acceleration**.*

There are several equations that maybe required when answering questions on circular motion; these are listed below:

$$v = \omega r \qquad a = \frac{v^2}{r} = \omega^2 r \qquad F = \frac{mv^2}{r} = m\omega^2 r$$

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \omega = 2\pi f \qquad \omega = \frac{2\pi}{T} \qquad f = \frac{1}{T}$$

ω – angular velocity (rads^{-1})	*θ – angle moved through (rad)*
t – time taken (s)	*f – frequency (Hz)*
v – linear speed (ms^{-1})	*T – time period (s)*
r – radius (m)	*a – centripetal acceleration (ms^{-2})*
F – centripetal force (N)	*m – mass (kg)*

The centripetal acceleration is always at **right angles** to the velocity - otherwise the speed would increase. This means that, as the acceleration is always directed towards the centre of the circle, the velocity is **tangential**..

The key equation is $a = \frac{v^2}{r}$ - this means that for a given force, the radius of the circle determines the speed at which circular motion can be performed.

***Exam Hint:** The centripetal force is always provided by one or more of the forces already present - it is not an "extra" force. Look for any forces present that have a component along the line joining the moving object to the centre of the circle.*

There are several types of exam question on this topic; the simplest form will involve a horizontal circle. This involves no resolving and the route to the answer is normally reasonably obvious if you are familiar with the equations listed earlier. In this case the centripetal force is normally equal to one of the forces present, rather than a component.

Example: Horizontal circular motion

A mass of 8.00 kg is attached to a piece of inelastic string of length 4.00m, and rests on a smooth horizontal plane. The other end of the string is fastened to the plane. The mass is set in motion so that it performs horizontal circles on the plane. The maximum tension that the string can provide is 700N.

(a) Draw a diagram showing the forces that acts on the mass. (Air resistance is negligible).

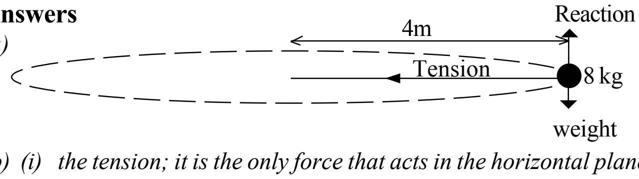
(b) (i) Which force supplies the centripetal force?
(ii) Why does the weight make no contribution to the centripetal force?

(c) (i) Calculate the maximum linear speed the mass can move at without breaking the string.
(ii) What maximum angular velocity does this equate to?

(d) Why is it important that the plane is smooth?

Answers

(a)



(b) (i) the tension; it is the only force that acts in the horizontal plane, towards the centre of the motion
(ii) it acts at right angles to the centripetal force (vertically) so has no effect on the centripetal force (which is horizontal).

(c) (i) Using the formula $F = \frac{mv^2}{r}$ *so* $v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{700 \times 4}{8}} = 18.7\text{ms}^{-1}$

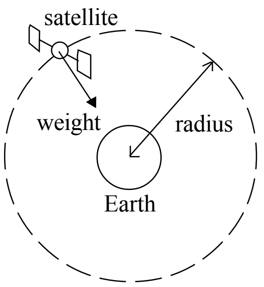
(ii) Using $v = \omega r$: $18.7 = \omega \times 4$
$\omega = 18.7/4 = 4.68 \text{ rad s}^{-1}$

(d) Because otherwise friction would also act on the mass

A similar type of question involves the **orbits of satellites or planets**. Here the weight of the object provides the centripetal force and the acceleration is equal to the acceleration due to gravity at that point in space – this is given using Newton's Law of Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

F = gravitational force between objects
G = gravitational constant
m_1, m_2 = masses of objects
r = distance between centre of masses of objects



When the only force on an object is its weight the object is said to be in *freefall*. Objects in freefall accelerate at the same rate as the value of g at that point; i.e. $a = g$. An object in orbit is considered to be in freefall.

So we have $\frac{Gm_Em}{r^2} = \frac{mv^2}{r}$, where m=mass of satellite, m_E = mass of Earth.

This simplifies to $Gm_E = v^2r$

> ***Exam Hint:*** *Remember that "r" is the radius of the orbit - which is the distance of the satellite from the **centre** of the Earth, not its surface.*

A similar approach applies to planets orbiting the sun; in this case the two masses concerned are those of the planet and the sun.

> ***Exam Hint:*** *Questions often involve **geostationary** satellites - these stay in the same position relative to the earth, and hence have T = 24 hours.*
> *Remember in questions like this to use the correct **units** - for example, a period of 24 hours must be converted into seconds.*

Example: Satellites
A geostationary satellite remains above the same point on the earth as it orbits. It remains a constant distance *R* from the centre of the earth
(a) Write down an expression, in terms of *R*, for the distance it travels in 24 hours
(b) Write down, in terms of *R*, an expression for its speed in ms^{-1}
(c) Find the value of *R*.
($G = 6.67 \times 10^{-11}\ Nm^2Kg^{-2}$; mass of earth = 5.98×10^{24} kg)

(a) It travels through a circle, radius R, so distance is $2\pi R$

(b) 24 hours = $24\times 60\times 60$ = 86400 s

So speed = $\frac{2\pi R}{86400}$ ms^{-1}

(c) We have $\frac{Gm_E}{R^2} = \frac{v^2}{R}$

So $Gm_E = v^2R$

$$\text{So } (6.67 \times 10^{-11})(5.98\times 10^{24}) = \left(\frac{2\pi R}{86400}\right)^2 R$$

$$3.99 \times 10^{14} = \frac{4\pi^2R^3}{86400^2}$$

$$3.99 \times 10^{14} \times 86400^2/(4\pi)^2 = R^3$$

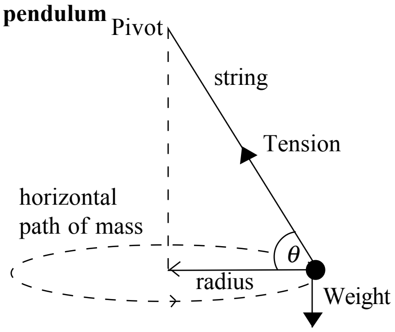
$$1.89 \times 10^{22} = R^3$$

$$2.66 \times 10^7 = R$$

More complicated examples of circular motion occur when the centripetal force is caused by only a **component** of a force rather than all of it. A typical example in this style is the **conical pendulum** as shown below.

We resolve the relevant force into two components, one directed towards the centre of the circle and one perpendicular to it. Provided the body is moving in a circle at **constant speed**, the **only** resultant force will be towards the centre of the circle.

A conical pendulum



Remember the centripetal force is the resultant force directed towards the centre. So in this case we resolve all forces horizontally and vertically. All of the weight acts vertically so this is simple. The tension however has a component both horizontally and vertically.

Considering forces vertically we know that the weight must be balanced by the vertical component of tension as the bob does not accelerate vertically - it moves in a horizontal circle.

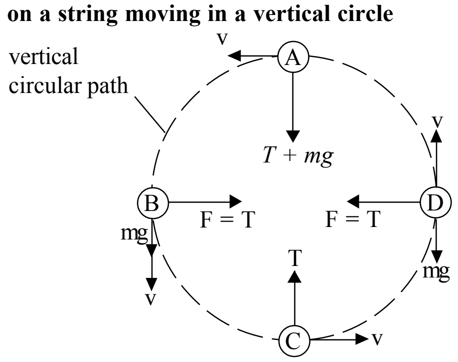
$mg = T\sin\theta$

Horizontally the only force that acts is the horizontal component of tension. As this is the only force it is unbalanced and therefore is the resultant or centripetal force:

> *For a conical pendulum* $F = \frac{mv^2}{r} = T\cos\theta$

The most difficult example you are likely to meet is the type of example where the centripetal force is provided by a combination of more than one force. These examples normally involve objects travelling in **vertical circles** e.g. rollercoaster cars, masses on strings or buckets of water.

A mass on a string moving in a vertical circle



The diagram represents a mass on a string moving in a vertical circle. As it moves in a circular path we know there is a resultant force acting towards the centre of the circle. This force will involve both the weight of the mass and tension in the string as the mass moves around the circle.

Position A: $\frac{mv^2}{r} = T + mg$ $T = \frac{mv^2}{r} - mg$

Position B & D: $\frac{mv^2}{r} = T$

Position C: $T - mg = \frac{mv^2}{r}$ $T = \frac{mv^2}{r} + mg$

> ***Remember**: The maximum tension occurs at the bottom of the circle as the tension has to overcome the weight of the mass **and** provide the centripetal force. This is where the string is most likely to break.*

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's mark scheme is given below.

A child ties a 500g mass to a length of string spins it in a horizontal circle with radius 0.50m. The string makes an angle of 60° to the horizontal. Calculate:

(a) The tension in the string. **[2]**

$T_{vertical} = T\cos 60° = mg$

$T = \frac{mg}{\cos\theta} = \frac{500 \times 9.81}{\cos 60^o} = mg$ 0/2

The candidate has made two main mistakes, forgetting to convert from grams to kilograms and using the wrong component of tension. Perhaps the candidate had thought the angle was with the vertical.

(b) The centripetal force acting on the mass. **[2]**

The centripetal force will be the component of the tension acting towards the centre of the circle. ✓

$F = T_{horizontal} = T\cos 60° = 9800\cos 60° = 4900N$ ✓ *ecf* 2/2

The student uses their wrong answer from the previous section but as they have already been penalised and they have used the correct component this time then they gain error carried forwards marks.

(c) The mass on the string is now changed to 0.7kg and the centripetal force required to keep it travelling at the same radius is found to be 4N. Find the angular velocity it must be travelling at. **[3]**

$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{4 \times 0.5}{0.7}} = 1.7ms^{-1}$ 1/3

The student has confused (linear) speed, v, with angular velocity, ω. One mark is awarded, as this could be the first step in finding ω.

Examiner's Answers

(a) $T\sin\theta = mg,\ T = \frac{mg}{\sin\theta} = \frac{0.5 \times 9.8}{\sin 60} = 5.7\ N$

(b) $F = T\cos 60° = 5.7\cos 60° = 2.85N$

(c) $F = m\omega^2 r \quad \omega = \sqrt{\frac{F}{mr}} = \sqrt{\frac{4}{0.7 \times 0.5}} = 3.4\ rad\ s^{-1}$

Practice Questions

1. A 12g stone on a string is whirled in a vertical circle of radius 30cm at a constant angular speed of $15rads^{-1}$.
   (a) Calculate the speed of the stone along its circular path.
   (b) Calculate the centripetal force acting on the stone.
   (c) Why is the string most likely to break when the stone is nearest the ground?
2. A proton enters of region of magnetic flux travelling at $6.0 \times 10^6 ms^{-1}$. If the field strength is a constant 0.70T calculate the radius of the path the proton travels thorough and state its direction relative to the field. $M_{proton} = 1.7 \times 10^{-27}$kg, $Q_{proton} = 1.6 \times 10^{-19}$C.
3. For an object travelling with circular motion at constant speed, state what direction the centripetal force and velocity act in relative to the circumference of the circular path. What happens if the speed of the object increases without a corresponding increase in centripetal force?

A microscopic example of applying the principles of circular motion comes when considering charges moving in magnetic fields. When a charge enters a region of magnetic field at right angles to the field lines it experiences a force, as given by Fleming's Left Hand Rule (FLHR). FLHR tells us the force acts at right angles to both the velocity and the field lines, a force at right angles to a velocity causes circular motion.

Exam Hint: *FLHR represents direction of current by using the second finger. Remember that we say current flows in the same direction as positive charge, so for negative particles, such as electrons, you point your second finger in the opposite sense to their direction of travel.*

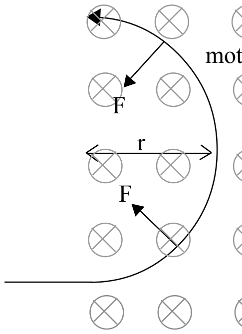
The force on a charge moving through a magnetic field is given by the equation below:

$F = BQv$ where: F = the magnitude of force on the charge
Q = the charge on the particle
v = the speed of the charged particle.

BQv is the only force so it must be supplying the centripetal force, a particle of mass m will move in a circle of radius r.

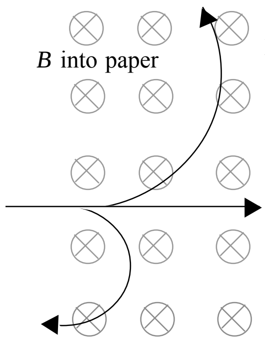
$$BQv = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{BQ}$$

Motion of a charged particle moving at rigth angles to a magnetic field



This shows that faster, more massive particles will follow paths with greater radii and that particles with higher charge will follow paths with smaller radii. The direction the particle curves is determined by the charge on the particle.

Motion of particles at identical velocities moving through a magnetic field



Answers

1. (a) $v = \omega r = 15 \times 0.30 = 4.5ms^{-1}$
   (b) $F = mv^2/r = 0.012 \times 4.5^2 0.30 = 0.81N$
   (c) When the stone is nearest the ground, T = Centripetal Force + W. This means that the tension in the string must be a maximum in order to provide the centripetal force and oppose the weight.
2. $r = mv/BQ = (1.7 \times 10^{-27} \times 6.0 \times 10^6)/(0.70 \times 1.6 \times 10^{-19}) = 0.091m$
3. The velocity is tangential, the force at right angles – towards the centre. The object starts to spiral away from the centre of the motion.

*Acknowledgements: This Physics Factsheet was researched and written by **Alan Brooks**. The Curriculum Press, Bank House, 105 King Street, Wellington, Shropshire, TF1 1NU. Physics Factsheets may be copied free of charge by teaching staff or students, provided that their school is a registered subscriber. No part of these Factsheets may be reproduced, stored in a retrieval system, or transmitted, in any other form or by any other means, without the prior permission of the publisher.*

Physics Factsheet

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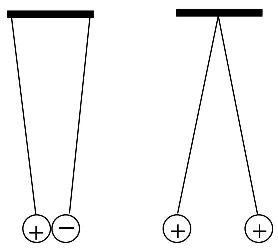
Coulomb's Law

You probably already know that electrically charged particles can exert a force on each other even when they are not actually touching. You may have carried out experiments with electrostatics which have shown you that:

- there are two kinds of electric charge – called positive (+) and negative (-);
- opposite charges (+ and -) attract one another;
- like charges (+ and +) or (- and -) repel one another.



Physicist Charles Coulomb came up with a law which described not only the direction, but also the size, of this force between charged particles. It is very similar in form to Newton's law of gravitation, but of course, gravitational forces are only ever in one direction – always attractive, never repulsive.

> ***Coulomb's law** states that the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.*
>
> $F = k\frac{Q_1Q_2}{r^2}$ *where Q_1 and Q_2 are the two charges (in **coulombs**)*
> *r is the distance (in **metres**) between them.*
> *F is the force in **newtons***
> *k is the constant of proportionality, and it has an equation of its own:*
>
> $k = \frac{1}{4\pi\varepsilon_0}$ *where ε_0 is called the permittivity of free space.*

The force between the two charged particles depends on what is between them. If anything other than empty space (a vacuum) comes between them, the force between the charges is reduced (which means that the permittivity is increased).

The permittivity of air at standard temperature and pressure is $1.0005 \times \varepsilon_0$ and so we can usually take ε_0 as the value for air as well as for free space. The value of the constant $\varepsilon_0 = 8.85 \times 10^{-12}$ N m^2 C^{-2} or Fm^{-1} (**farads** per metre)

So, we can combine all of this into the original mathematical form of Coulomb's equation:

> **Coulomb's equation:** $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
>
> You need to remember this equation and be able to use it to answer questions. (You won't have to remember the value of ε_0 - it will be given you in any exam.)

An approximate numerical value for $\frac{1}{4\pi\varepsilon_0}$ is 9×10^9,

so a useful form of Coulomb's law for making *rough* calculations is:

$$F = 9 \times 10^9 \frac{Q_1Q_2}{r^2}$$

If you have already done any work on Newton's law of gravitation, you will probably see the similarities. The force in each case depends on both of the objects causing it (charges for Coulomb, masses for Newton) and in each case it is an *inverse square* law.

This means for example that if the distance is doubled, the force gets four times smaller (because the inverse of the square of 2 is ¼ or, mathematically: $\frac{1}{2^2} = 0.25$)

> **Typical Exam Question**
> **(a) Calculate the force between two point charges of +100μC placed 125cm apart in a vacuum. ($\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1})**
> **(b) State what difference it would make to the force calculated if:**
> **(i) one of the charges was negative.**
> **(ii) the space between the charges was occupied by air.**
> **(iii) the space was occupied by paraffin (which has high permittivity)**
>
> (a) $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
> $= \frac{(100\times 10^{-6})(100\times 10^{-6})}{4\pi(8.85\times 10^{-12})(1.25)^2} = 57.5$ N ✓
>
> *(b) (i) the force would be attractive rather than repulsive* ✓
>
> *(ii) there would be a very slight reduction in the size of the force (or, no change)* ✓
> *(because the permittivity of air is only very slightly higher than that of free space).*
>
> *(iii) there would be a large reduction in the force (because the permittivity of paraffin is many times higher than that of free space).* ✓

> ***Exam Hint**: Be very careful with the units and the powers of 10 in this sort of calculation. It is most sensible to convert everything to SI units at the beginning (metres, kilograms, seconds, amps etc) and work with them; for example, put 100μC and 125 cm into your equation as 100×10^{-6} C and 1.25 m at the very beginning. Also, be very careful about how you enter these values into a scientific calculator – make sure you know exactly how to enter powers of 10. It is especially important in this sort of question where you don't usually have a 'common-sense feel' for what a reasonable final answer ought to be.*

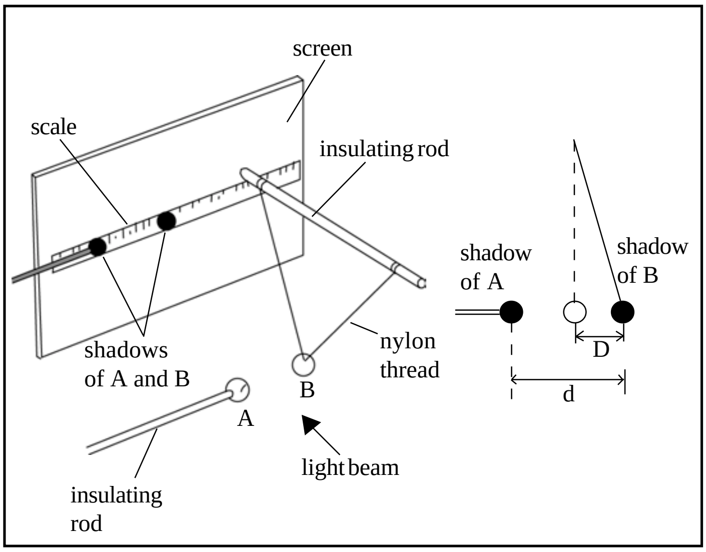
Practical considerations

A considerable proportion of the matter which makes up everything in the universe consists of charged particles and a knowledge of the forces between these particles is needed if we are to understand the structure of matter and the structure of the atom. The law as we have stated it applies to *point charges*, that is, charges which are concentrated in a single point. Sub-atomic particles – the protons and electrons within an atom – are so tiny that to all intents and purposes we can regard them as point charges. In fact, any uniformly charged conducting sphere (with a total charge Q) behaves – as far as external effects are concerned – as if it were a single point charge Q concentrated at the centre of the sphere. This only holds true if there are no other charges nearby to disturb the distribution of charge on the surface of the sphere. When these conditions are met, then Coulomb's law gives an acceptable approximation of the forces experienced by such a sphere and of the forces exerted by it on other charged particles.

Any experimental attempt to investigate Coulomb's law by measuring the forces between charged spheres is fraught with practical difficulties. The amount of charge we can investigate is difficult to measure with accuracy, the forces involved are small and the charge has a tendency to leak away into the air.

A popular method involves light spheres (usually polystyrene balls) coated with a conducting material such as graphite paint, one mounted on an insulating **perspex** handle and the other suspended from insulating nylon thread. The diagrams below show a suitable arrangement. The balls may be charged by touching each in turn with, for example, the plate of an electrophorus or some other electrostatic charging device (Fig1).

Fig 1. Coulomb's law apparatus



One of the charged balls is glued to the bottom of a 'V'-shaped swing of nylon thread so that it can only swing backwards or forwards in one direction. As the second charged ball, mounted on the insulating rod, is moved closer to the swinging ball, the amount of displacement can be measured by shining a beam of light from the front and marking the position of the shadows of both balls on a screen behind the apparatus. The deflection is proportional to the force causing it. If a few values of d (distance between the two balls) and D (deflection of the swinging ball from its start position) are measured, then it can be shown that

$$F = \frac{1}{r^2}$$

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

a) State Coulomb's law for the force between two point charges [2]

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$ ✓ ✗

- One of the two marks available is awarded for explaining what the symbols (Q, r and ε_0) stand for. Easy to do, easy to forget to do!

b) Calculate the force on an electron placed at a point 3.3×10^{-13}m from a spherical nucleus whose charge is 1.12×10^{-18}C. (Take the charge on an electron (e) to be -1.6×10^{-19}C and ε_0 = 8.85×10^{-12} Fm^{-1}) [3]

Field strength = 1.31×10^{-13} ✗ ✗ ✗

- Disaster! The wrong answer. However – there are *three* marks available for this part of the question, and only one of them is for the correct final answer. This student has thrown away all three marks by not remembering the golden rule of all physics exams – **show your working!**
- The *only* error the student has made is to forget 'ε_0' when calculating the answer – an easy sort of mistake to make, especially when concentrating on entering all the powers of 10 correctly into the calculator. If the working had been shown, this would have still allowed two of the three marks for using the correct equation and making the substitution of numbers for symbols correctly.
- Final error – no units for the answer. Again, easy to forget at the end of a complex calculation – but the units are just as important a part of the answer as the numerical value.

c) Explain how an atom could be ionised by the application of a suitable external electric field [1]

An atom can be ionised by the application of a suitable external electric field because when something is ionised it means making it into ions (charged particles) and the electric field can do this. ✗

- It is very common for students to rewrite the question when they don't know the answer. This student has remembered something about ions from GCSE (that they are charged particles) but it doesn't make up to an adequate answer at this level – the fact remembered is not sufficient for this question. Repeating the same thing twice doesn't show any greater understanding.

Examiner's answers

a) *Coulomb's Law:* $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$ ✓

where: Q_1 & Q_2 are the sizes of the charges
r the distance apart
ε_0 the permittivity of the medium separating them ✓

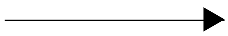
b) $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ ✓
$= (1.12 \times 10^{-18}) \times (-1.6 \times 10^{-19}) / (4\pi\, 8.85 \times 10^{-12} (3.3 \times 10^{-13})^2)$ ✓
$= -0.0148$ N ✓

c) *The force on the electron due to the external field must be greater than the attractive force between the electron and the nucleus for ionisation to occur.* ✓

Questions

1. A potential difference of 5.5 kV is applied to the electron gun of a cathode ray tube, in order to accelerate the electrons from rest. The charge on an electron is -1.6×10^{-19} C, and the mass of an electron is 9.1×10^{-31} kg. Calculate:
   (i) the kinetic energy of the electrons as they leave the electron gun. [2]
   (ii) the speed of the electrons [2]

2. (a) The arrow below illustrates a beam of electrons leaving the electron gun of a cathode ray tube. Complete the diagram to show what you would need to add to make the beam deflect downward using a d.c. supply. Draw and label the path of the beam. [3]



electrons

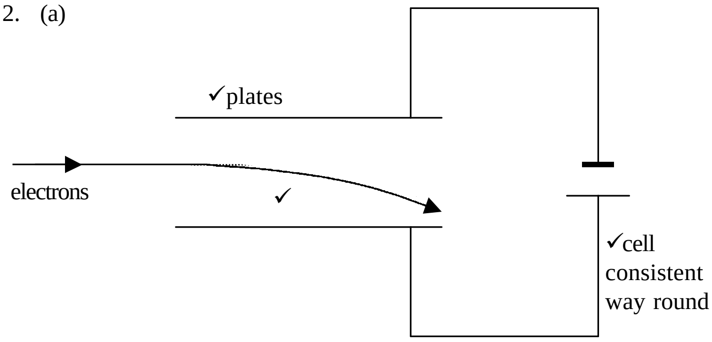
   (b) What difference would increasing the accelerating voltage make? Explain your answer. [2]

Answers

1. (i) E_k, the kinetic energy lost by each electron equals E_p, the potential energy lost as it falls through a voltage, V. ✓
   $= eV = 1.6 \times 10^{-19} \times 5.5 \times 10^{3} = 8.8 \times 10^{-16}$ J ✓ [2]

   (ii) $E_k = ½ mv^2$ ✓
   $v = (2E/m)^{1/2}$
   $= [(2 \times 8.8 \times 10^{-16})/(9.1 \times 10^{-31})]^{1/2}$ ✓ $= 4.4 \times 10^{7}$ ms^{-1} ✓ [2]

2. (a)



[3]

   (b) If the accelerating voltage is increased the electrons would leave the electron gun at a greater velocity than before. ✓ [1]

Acknowledgements:
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Physics Factsheet

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Electric Field Strength and Potential

Field theory - electric, magnetic or gravitational - is a model for explaining forces that act at a distance - that is, forces between two bodies that are not in physical contact. Electric fields are used to explain the forces that act between charges.

Forces on charges

There are two types of charge: positive charge (carried by particles such as protons) and negative charge (carried by particles such as electrons). The unit of charge is the coulomb (*C*). Experiments strongly suggest that charge comes in discrete packets - i.e. it is **quantised**. The smallest unit of charge is 1.6×10^{-19}C - the size of the charge on an electron or proton - and all other amounts appear to come in integral multiples of this.

Any object carrying a charge will experience a force when in the presence of another body also carrying a charge - like charges repel and unlike charges attract. This force is given by **Coulomb's Law** (Factsheet 32)

> $F = \frac{kQ_1Q_2}{r^2}$
> *F is the force on either charge (N)*
> *Q_1 and Q_2 are the two charges (C)*
> *r is the seperation of the **centres** of the two charges (m)*
> *k is a constant; its value is 9×10^9 Nm^{2}C^{-2}*
> *$k = 1/(4\pi\varepsilon_0)$, where ε_0 is the permittivity of free space.*

- A **attractive** force is **negative** (Q_1 and Q_2 are of opposite signs)
- A **repulsive** force is **positive** (Q_1 and Q_2 are of the same sign)
- The force acts upon the line joining the centres of the two particles

Electric fields

> *Any charged body generates an electric field*
> *Electric fields only act on charged particles.*

An electric field is a useful concept to help us describe electrical forces - forces that act on charges. This may seem pointless if there are just two charges - Coulomb's law tells us the size and direction of the force. But electric fields allow us to describe the effect of a whole collection of charges, by combining their electrical fields.

> *The electric force on a charge in an electric field is:*
> $F = qE$
> *F = force (N)*
> *q = charge (C)*
> *E = electric field strength (NC^{-1})*

Note that since force is a vector and charge is a scalar, **electric field strength must be a vector.**

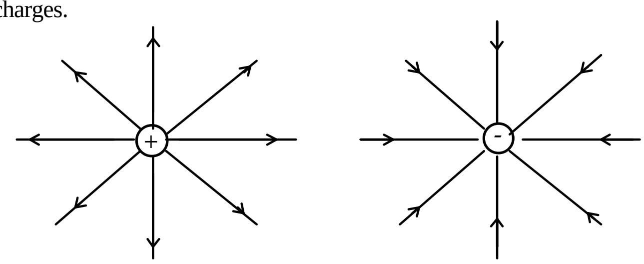
> *Electric field strength (NC^{-1}) is defined as the force (N) per unit charge (C) experienced by a test charge at that point.*
>
> *The direction of the electric field at any point is the direction of the force experienced by a positive charge at that point.*

Representing electric fields

Fields are represented by drawing field lines (lines with arrows).

- The closer together the lines are the stronger the field.
- Field lines point in the direction a positive test charge would move.

The diagrams below show electric fields for isolated positive and negative charges.



Notice in both cases the spacing of the field lines is not constant. As the distance from the point charge increases the field lines diverge, showing that the field becomes weaker.

Types of electric field

Point charges

A "point charge" - i.e. a body with charge, but no size - does not really exist, but it is a useful approximation for small charged particles such as electrons.

The electric field for such a charge can be found from Coulomb's law. The force on a test charge q in the presence of a charge Q is given by:

$$F = \frac{kqQ}{r^2}$$

But from the definition of electric field, we also have:

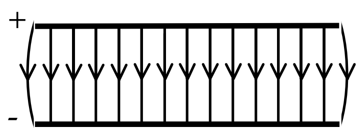
$$F = qE$$

So: $qE = \frac{kqQ}{r^2}$. This gives:

> $E = \frac{kQ}{r^2}$ for a point charge Q

Parallel plate capacitor

A parallel plate capacitor has a uniform field between the plates, except for close to the edges (edge effects). Accordingly, the field lines are constantly spaced.

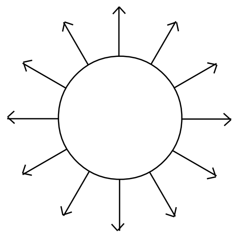


Note the non-uniformity of the field at the edges. For the uniform part of the field:

> $E = \frac{V}{d}$ V is the potential difference between the two plates. d is the distance between plates.

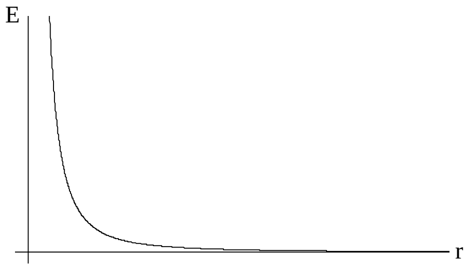
Note: This shows that electric field strength can also have units of Vm^{-1}

Hollow charged sphere
- The net field strength **inside** a hollow charged sphere is **zero**.
- **Outside**, the field behaves as if all the charge creating it is at a point in the centre of the sphere.



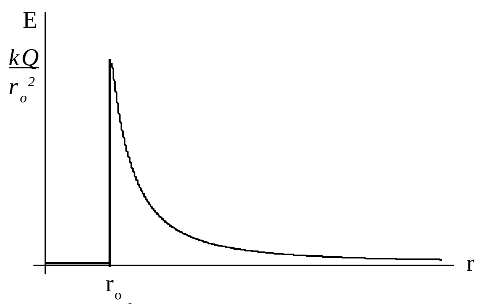
Variation of field with distance
The graphs below show how field strength varies with distance for a point charge, a capacitor and a hollow charged sphere. Negatively charged point charges and spheres would give negative versions of the same graph.

Point charge
The decline of E with distance follows an **inverse square law**.($E \propto \frac{1}{r^2}$)



Parallel plate capacitor
E is **constant** whatever the position between the plates

Hollow charged sphere
E is **zero** within the sphere, and declines following inverse square law outside.



(= radius of sphere)

Exam Hint: - The variation of field strength with distance is commonly tested. You need to be able to recognise and reproduce the above graphs.

Electric potential
You will already have met the idea of **potential difference** - a potential difference between two points means there is a voltage drop between them. Accordingly, electrical potential is measured in **volts**.

The potential at a point in an electric field is defined as the work done in bringing a test unit positive charge from infinity to that point.

The potential of a charge at infinity is always defined to be zero.

If the charge creating an electric field is positive, then since "like charges repel", work must be done to bring a test positive charge in from infinity.

However, if the charge creating the field is negative, then as "unlike charges attract", the work done to bring in a test positive charge from infinity is **negative** - the charge would tend to be attracted in, so work would actually be required to stop it.

The potential due to a positive charge is positive.
The potential due to a negative charge is negative.

The definition of potential in terms of work tells us that electric potential is a measure of the **potential energy per unit charge**.

The potential energy on a charge due to a potential V is:
$$P.E. = qV$$
P.E. = potential energy (J)
q = charge (C)
V = electric potential (V)

Since energy is a scalar and charge is a scalar, **potential** must also be a **scalar** quantity.

If two points have different electrical potentials, then the potential energy of a charge must change if it moves from one point to the other. For this to happen, work must be done **on** or **by** the charge.

The work done when a charge moves through a potential difference is given by $W = Q\Delta V$
W = work done (J) Q = charge (C)
ΔV = potential difference (V)

*The **electrical potential energy increases** if:*
- *A **positive** charge moves to a point of **higher** potential*
- *A **negative** charge moves to a point of **lower** potential*

Note that the work done depends **only** on the potential of the starting and finishing point - the route that the charge follows does not matter.

This tells us that if a charged particle is taken around a **closed** loop in an electric field, **no work is done** - since the potential of the starting point is the same as the potential of the finishing point.

Typical Exam Question
a) Explain the difference between electric potential and electric field strength [3]
b) A parallel plate capacitor has 100V across plate 2mm apart.
(i) Find the strength of the field between the plates. [2]
(ii) What force would be exerted on an electron if it were positioned between the plates? (Charge on an electron = 1.6×10^{-19}C)[2]

a) EFS is the force ✔ per unit charge on a test charge at a point in the field whereas potential is the work done ✔ per unit charge on a test charge brought from infinity ✔ to that point in the field.
b) (i) $E=V/d$ ✔ $= 100/(2\times10^{-3}) = 50kVm^{-1}$ ✔
(ii) $F=qE = 1.6\times10^{-19}\times50\times10^{3}$ ✔ $= 8\times10^{-15}N$ ✔

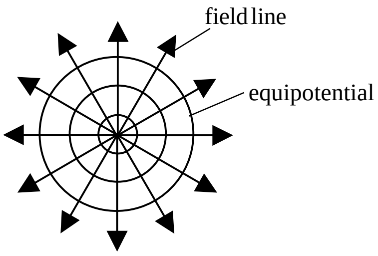
Electric potential and the electric field

> *Electric field = – gradient of electric potential*

Lines connecting points of equal potential - called **equipotentials** - can be drawn on field line diagrams.

> *Equipotentials are at right angles to field lines*

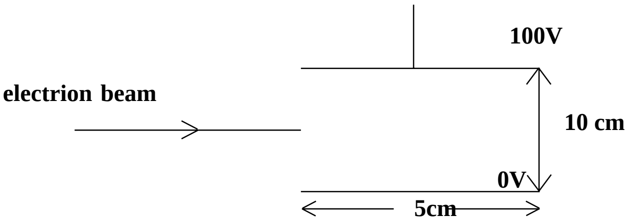
Equipotentials are like contour lines on a map (in fact, contour lines are gravitational equipotentials - they link points with the same height, and hence the same gravitational potential energy)
The diagram below shows the field lines and equipotentials for a point positive charge:



In fact, the equipotentials around a point charge will be **spheres**, centred on the point charge. This is because all points on the sphere are the same distance from the point charge - so the work required to bring a charge to any point on the sphere will be the same.

Equipotentials tell us about **energy changes** - the energy required to move from one equipotential to another. Field lines tell us about **forces** - the force on an electric charge at a particular point.

Typical Exam Question
Electrons are accelerated and then directed between two parallel plates as part of a cathode ray tube, as shown below.



a) For the region between the plates find:
(i) the electric field strength; [2]
(ii) the force on an electron. [2]

(b) Given that the velocity of the beam is $1\times10^7ms^{-1}$ find:
(i) the time spent between the plates; [2]
(ii) the acceleration of the electron; [2]
(iii) the vertical displacement of the electron as it just leaves the plates. [3]

Mass of an electron = 9.11×10^{-31}kg
Charge on an electron = 1.6×10^{-19}C

a)i) $E = V/d = 100$ ✓ $/10\times10^{-2} = 1kVm^{-1}$ ✓
(ii) $F = qE = 1.6\times10^{-19}\times1\times10^3$ ✓ $= 1.6\times10^{-16}N$ ✓

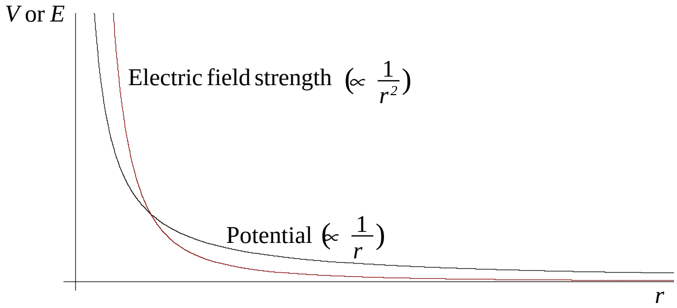
(bi) $t = d/v = 5\times10^{-2}/1\times10^7 = 5\times10^{-9}$ s ✓
(ii) $a = F/m$ ✓ $= 1.6\times10^{-16}/9.11\times10^{-31} = 1.76\times10^{14}ms^{-2}$ ✓
(iii) Using equations of motion and applying them vertically
$a = 1.76\times10^{14}ms^{-2}$ $u = 0$
$t = 5\times10^{-8}s$ $s = ?$
$s = ut + ½at^2$ ✓ $= ½\times1.76\times10^{14}ms^{-2}\times(5\times10^{-9})^2$ ✓ $= 2.2mm$ ✓
as measured from its "weight" on entering the field.

Potential for point charge, parallel plate capacitor, hollow sphere

Point charge

> *For a point charge Q,* $V = \frac{kQ}{r}$

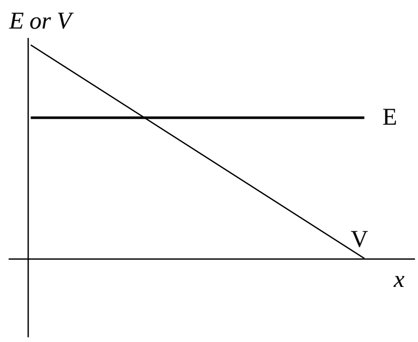
The graph below shows how potential and electric field strength decrease with distance for a point charge.



Parallel plate capacitor

> *For a parallel plate capacitor with spacing d and potential difference V_0 between the plates, the potential decreases linearly between the positive and negative plates*

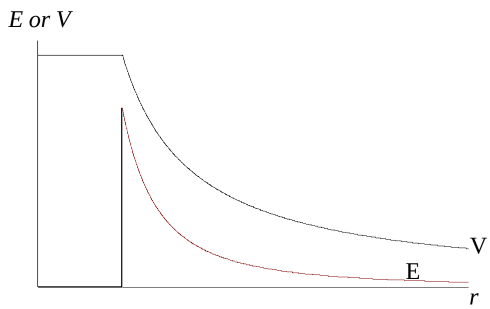
The graph below shows how potential and electric field strength vary with distance for a parallel plate capacitor



Hollow charged sphere

> *The potential inside the sphere is constant, and equal to the potential on the surface. Outside, the potential is as if all the charge creating it is at the centre of the sphere.*

The graph below shows how potential and electric field strength vary with distance for a point charge.



Potential and field for any conductor

In a conductor, there are charge carriers (electrons) that are free to move. If there was a difference in potential between any two points on the conductor, charges would move from one to the other until the difference was removed.

Accordingly, the surface of a conductor is an **equipotential**. So the electric field is perpendicular to the surface of the conductor.

Combining Electric Fields and Potentials

The electric field and potential due to several charges can be found by adding up their individual electric fields and potentials. For potentials, this involves just adding the values numerically. Since electric fields are vectors, these must be combined using vector addition.

Worked Example 1.
Points A, B, C are in a straight line, with AB = BC = r.

A charge 2Q is placed at point A. A charge -3Q is placed at pointC.

Find the potential and field strength due to these charges at point B.

For charge at A:
Potential at B $= \dfrac{2kQ}{r}$

Electric field at B $= \dfrac{2kQ}{r^2}$ *away from A, (as charge + at A)*

For charge at C:
Potential at B $= \dfrac{-3kQ}{r}$

Electric field at B $= \dfrac{3kQ}{r^2}$ *towards C, (as charge - at C)*

So altogether, potential at B $= \dfrac{2kQ}{r} + \dfrac{-3kQ}{r} = \dfrac{-kQ}{r}$

For electric field at B, both fields are in the same direction, so overall field is $\dfrac{5kQ}{r^2}$ *towards C.*

Questions

In the questions below, assume $1/4\pi\varepsilon_0 = 9.0 \times 10^9$ m F^{-1}

1. State Coulomb's Law.

2. Define (i) electric field strength (ii) electrical potential, and state whether each is a vector or a scalar.

3. Sketch graphs to show the variation of electric field strength and potential with distance from a point charge.

4. Explain why all points on a conductor must be at the same potential.

5. In a parallel plate capacitor, the distance between the plates is 1mm, and the potential difference across the plates is 3V.
Calculate the electric field strength between the plates.

6. A hollow sphere of radius 2cm carries a charge of 7.0 μC.
A point charge of 0.1μC is placed 1cm away from the surface of the sphere.
Calculate the force on the point charge due to the sphere.

7. ABC is a right-angled, isosceles triangle with AB = BC = 2cm.
A charge of 1μC is placed at A, and a charge of -1μC is placed at C.
(a) State, giving your reasons, the potential at point B.
(b) Calculate the magnitude and direction of the electric field at B

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

(a) Calculate the electric potential midway between two identical charges, +20 μC, placed 30 cm apart in vacuum. Take the value of ε_0, the permittivity of free space, to be 8.9×10^{-12} Fm^{-1}. **[3]**

$\dfrac{✓\quad 20 ✗}{4\pi\,(8.9 \times 10^{-12})\ 30} \times 2 = 1.19 \times 10^{10}$ ✗ 1/3

> Incorrect units used; candidate did not notice that the charge was in μC rather than C, and that the distance was in cm rather than m. Also, units omitted from final answer. Note:- charges will almost invariably be in μC , since the coulomb itself is a very large unit.

(b) (i) Explain what is meant by an *equipotential surface* **[1]**

All points have the same electric field ✗ 0/1

> Exam technique! The question mentions equi**potential** surface, so the answer must refer to potential. The direction of the electric field will always be at **right-angles** to the equipotential surface, so all the points on it will certainly not have the same field (as electric field is a **vector**, direction must be taken into account)

(ii) Explain why a metal plate placed anywhere in an electric field will be an equipotential surface. **[2]**

If it wasn't the electrons would move.✓ 1/2

> This answer is correct, but it needs to also say that movement of electrons would lead to the potential differences disappearing.

Examiner's Answers

(a) $V = 2 \times 20 \times 10^{-6}/(4\pi 8.9 \times 10^{-12} \times 15 \times 10^{-2})$ ✓✓
$V = 2.38 \times 10^6$V ✓

(b) (i) A surface on which all points are at the same electrical potential. ✓
(ii) If there were any differences in potential over the surface of the metal, then its conduction electrons would move under the influence of the potential gradient. ✓ *This electron movement would eventually reduce the potential differences to zero.* ✓

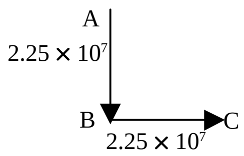
Answers

1 - 4: answers may be found in the text

5. $E = V/d = 3/10^{-3} = 3000$ Vm^{-1}

6. $F = kQ_1Q_2/r^2 = (9.0 \times 10^9) \times (7 \times 10^{-6}) \times (0.1 \times 10^{-6})/(0.03)^2$
$= 7$N

7 (a) Zero. It is the same distance from positive and negative charges of the same magnitude.
(b) Electric field due to A is $(9.0 \times 10^9) \times (10^{-6})/(0.02)^2$ in direction AB
Electric field due to C is $(9.0 \times 10^9) \times (10^{-6})/(0.02)^2$ in direction BC



So magnitude of field$^2 = (2.25 \times 10^7)^2 + (2.25 \times 10^7)^2$
magnitude of field $= 3.18 \times 10^7$ NC^{-1}
Direction is parallel to AC (or 45° to AB or BC)

Acknowledgements: *This Physics Factsheet was researched and written by Alan Brooks. The Curriculum Press,Unit 305B, The Big Peg,120 Vyse Street, Birmingham, B18 6NF*
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Physics Factsheet

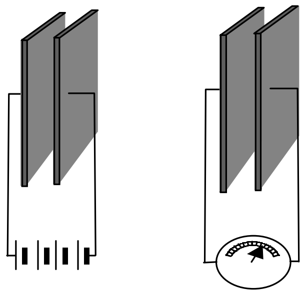
January 2002 Number 29

Capacitors

Introduction

Capacitors are widely used in electrical engineering and electronics. They are important in any physics course because of the variety of uses they have.

A very simple capacitor consists of two parallel metal plates.

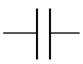


The capacitor is first connected to a d.c. supply and then to a sensitive ammeter (or galvanometer).When the capacitor is connected to the sensitive ammeter, a *momentary* deflection is observed. This deflection is a brief pulse of *charge* and illustrates an important idea with capacitors.

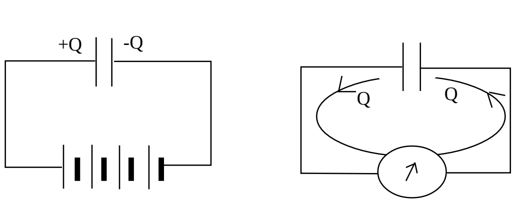
We say ***"a capacitor stores charge"***. We use the symbol Q to represent the amount of charge involved. In this case, charge Q is taken from the d.c. supply, stored on the capacitor plates and then the same charge Q discharged, through the ammeter.

> ***Remember*** *that the unit of charge is the coulomb (abbreviation C). Although one coulomb is a small amount in current electricity, it is an enormous amount in static electricity. In capacitors, the charge stored is static and we use much smaller units, typically microcoulombs μC.*

Although a capacitor can be made from any two conductors close to each other, we have considered the simplest case where the conductors are two parallel metal plates. You should also note that the plates are separated by an insulator, in this case air. The insulating material is called the *dielectric*. Because the dielectric is an insulator it is clear that a steady d.c. current cannot pass through a capacitor and this is why we only get the brief pulse of charge referred to above.

The symbol for a capacitor is simply: 

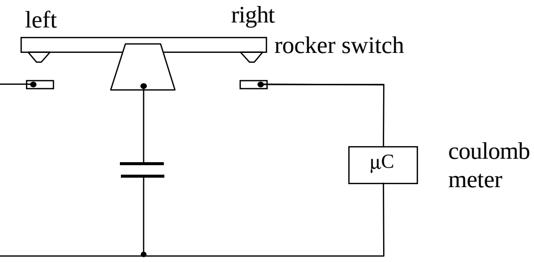
so the above diagram can be redrawn with this symbol.



Notice that the plates are marked + and – and that these signs correspond to those on the d.c. supply.

You might think that the charge stored is 2Q. This is *not* so. Ask yourself how much charge is flowing through the ammeter during discharge. Only the amount Q flows from the positive plate, through the ammeter, and 'neutralises' the charge on the negative plate.

To investigate the charge stored on a capacitor we can use a 'coulombmeter'. When a coulombmeter is connected to a charged capacitor, it will take all the charge from the capacitor, measure it and display the result on a digital readout.



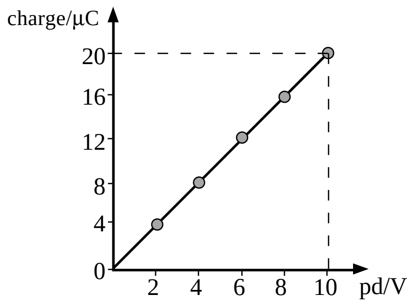
By depressing the left hand key of the rocker switch, the capacitor is charged to a potential difference V which can be adjusted by the variable d.c.supply. Upon depressing the right hand key, the stored charge Q is measured and displayed by the coulombmeter.

Table 1 shows the charge stored for various potential differences up to 10 volts.

Table 1

pd	*(Volt)*	2	4	6	8	10
charge	*(μC)*	4	8	12	16	20

The results are displayed in the graph below. You can see that there is a linear relationship.



The ratio $\frac{Q}{V}$ is the gradient of the straight line, so $\frac{Q}{V}$ = constant

> This constant is called the *capacitance* of the capacitor.
>
> $$\frac{\text{Charge stored}}{\text{potential difference}} = \text{capacitance} \quad \text{or} \quad \frac{Q}{V} = C$$
>
> **The unit of capacitance is the farad *(F)*.** To calculate the capacitance, Q must be in coulombs and V in volts. Because the coulomb is a large unit so also is the farad.

In practice you will use submultiples as shown in table 2.

Table 2

factor	prefix name	symbol
F × 10^{-3}	millifarad	*mF*
F × 10^{-6}	microfarad	*μF*
F ×10^{-9}	nanofarad	*nF*
F × 10^{-12}	picofarad	*pF*

To calculate the capacitance of the capacitor in the graph, we can use the last point (20 μC, 10V)

$$C = \frac{Q}{V}$$

$$C = \frac{20 \times 10^{-6}}{10} \quad F$$

$C = 2 \times 10^{-6}$ F
$C = 2$ μF (2 microfarads)

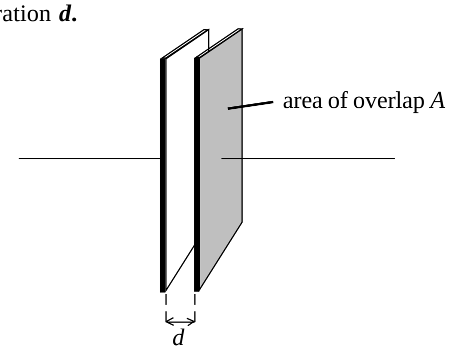
Any pair of readings in table 1 could have been used giving the same answer, try using one or two yourself. In each case you are finding *the gradient* of the line on the charge against potential difference graph.

If you do a real experiment, your readings will probably not increase uniformly as in table 1. In this case you plot all the points and then draw the best straight line passing through the origin. The *gradient* of this line gives the average value for the capacitance.

> ***Exam Hint:*** *If you are working out the gradient of a line, or doing any calculation, always look carefully at the units to see if metric prefixes are being used.*

Capacitance

The size and separation of the plates affects the capacitance.
The two quantities you need are area of overlap of the plates ***A*** and the plate separation ***d***.



Experiments show that:
- capacitance *varies directly* with the area **A , $C \propto A$.**
- capacitance *varies inversely* with the separation $C \propto \frac{1}{d}$

Combining these two gives: $C \propto \frac{A}{d}$

To change from a proportionality to equality we introduce a constant of proportionality, in this case ε_0.

We can now write the equation $\mathbf{C = \varepsilon_0 \frac{A}{d}}$ We use the subscript 0 when there is nothing between the plates. (Strictly there should be a vacuum between the plates but the presence of air makes almost no difference).

The term ε_0, epsilon nought, is called 'the permittivity of free space; its value is given by (ε_0 = 8.85 × 10^{-12} Fm^{-1} (farads per metre)).

The units for ε_0 can be found by rearranging the equation above and then putting in the known units:

$$\varepsilon_0 = \frac{Cd}{A}$$

units for $$\varepsilon_0 = \frac{F \times m}{m^2} = Fm^{-1}$$

So far we have been thinking that the space between the plates is air or a vacuum. What will happen if an insulator, *the dielectric*, is introduced between the plates ? The answer is that the capacitance will be *increased* by several times. The factor by which it is increased is between 2 and 10 for most dielectrics and is called **the relative permittivity ε_r** . Note that ε_r does not have any units, it simply 'multiplies up' the capacitance (see table 3).

Table 3

Material	Relative permittivity ε_r
Air	1.00053
Paper	3.5
Mica	5.4
Wax paper	2.2

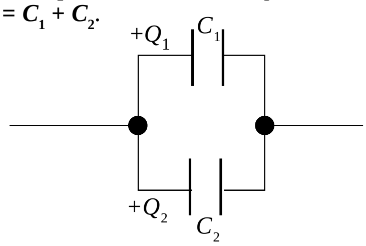
> *Expression for capacitance*
>
> $C = \varepsilon_r \varepsilon_0 \frac{A}{d}$ *where* **A** = *common area of the plates* (m^2)
> **d** = *separation between the plates (m)*
> ε_r = *relative permittivity (no units)*
> ε_0 = *constant of proportionality* (8.85×10^{-12}) (Fm^{-1})

> **Worked example**
> A capacitor is made from two parallel metal plates with a common area of 1m^2 and a separation of 1mm ($\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1})
> (a) calculate the capacitance
>
> $C = \varepsilon_0 \frac{A}{d}$, *where* $A = 1m^2$ *and* $d = 10^{-3}m$.
> *So,* $C = 8.85 \times 10^{-12} \times \frac{1}{10^{-3}} = \mathbf{8.85 \times 10^{-9}}$ ***F.***
>
> (b) If the plates are now held apart by a thin sheet of paper 0.1 mm thick, calculate the new capacitance. (Relative permittivity for paper = 3.5).
>
> $C = \varepsilon_0 \varepsilon_r \frac{A}{d}$, *so* $C = \frac{3.5 \times 8.85 \times 10^{-12}}{10^{-4}} = 3.1 \times 10^{-8}$ F (=310 nF)

> ***Remember:*** *In the expression for capacitance,* $C = \varepsilon_0 \varepsilon_r \frac{A}{d}$
> *The area A must be in* m^2 *(normally small) and separation d must be in m.*

Combining capacitors in parallel and series

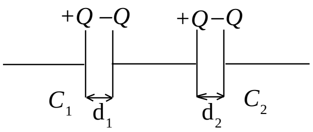
The following are not proofs but aim to help understanding.
The diagram below shows two capacitors C_1 and C_2 **in parallel**. You can see that **the area** of C_2 is **added** to that of C_1. So, you can see that the total capacitance $\mathbf{C_T = C_1 + C_2}$.



Another way to look at capacitors in parallel is to look at their *charge*. In this case the total charge is found by **adding** Q_1 and Q_2.

> *For capacitors in parallel* $\mathbf{C_T = C_1 + C_2}$

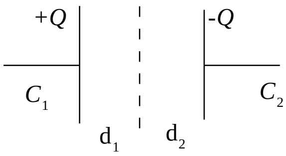
The diagram below shows two capacitors C_1 and C_2 **in series**. The values of the capacitors may be different but the charge on each is *the same.*



You can see this by looking at the two inner plates, one from each capacitor, and remembering that they are *insulated* from the rest of the circuit. The charge lost by one plate *must* equal that gained by the other. The total charge stored here is Q.

Look at diagram below showing the *equivalent* capacitor with an increased separarion and (because $C \propto \frac{1}{d}$), the **total capacitance C_T is reduced.**

It is found using the formula $\frac{1}{C_T} = \frac{1}{C_1} = \frac{1}{C_2}$

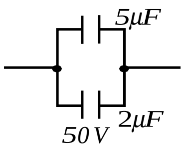


For capacitors in series $\frac{1}{C_T} = \frac{1}{C_1} = \frac{1}{C_2}$

Worked example
Two capacitors of 5µF and 2µF are connected in parallel and a d.c. supply of 50 volts applied to the combination. Calculate:
(i) the charge on each,
(ii) the total charge stored,
(iii) the total capacitance of the combination,
(iv) the charge stored on the combination.

Answer: From the definition for capacitance, $Q = C \times V$.

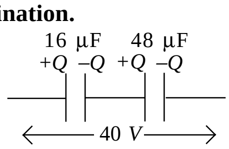
(i) $Q_5 = 5 \times 10^{-6} \times 50 = \mathbf{250\mu C}$ $\quad Q_2 = 2 \times 10^{-6} \times 50 = \mathbf{100\mu C.}$
(ii) $Q_{total} = (250 + 100)\ \mu C = \mathbf{350\mu C}$
(iii) $C_T = C_1 + C_2$ so, $C_T = (5 + 2)\mu F = \mathbf{7\mu F}$
(iv) *Again use* $Q = C \times V$ *with* $C = 7\mu F$
$Q = (7 \times 50)\mu C = \mathbf{350\mu C}$



Note that the answers to ii) and iv) are the same.

Remember: *for capacitors in parallel, the potential difference across each capacitor is the same.*

Typical Exam Question
Two capacitors of 16µF and 48µF are on connected in series and a d.c. supply of 40 volts applied to the combination.
Calculate
(i) the total capacitance
(ii) the total charge stored
(iii) the charge on each capacitor
(iv) the potential difference across each capacitor.



Answer:

(i) *Using* $\frac{1}{C_T} = \frac{1}{C_1} = \frac{1}{C_2}$, *we have,* $\frac{1}{C_T} = \frac{1}{16} + \frac{1}{48}$ $\therefore C_T = 12\ \mu F$
(ii) *Using* $Q = C \times V$, *we have stored charge* $Q = 12\mu \times 40 = \mathbf{480\ \mu C}$
(iii) *The charge on each capacitor (in series) must be the same*
Hence $Q_{16} = 480\ \mu C$ *and* $Q_{48} = 480\ \mu C$
(iv) *Rearrange* $C = \frac{Q}{V}$ to give $V = \frac{Q}{V}$

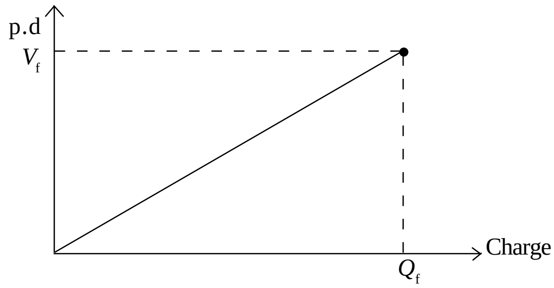
So, $V_{16} = \frac{480\ \mu C}{16\ \mu F} = 30V$ *and* $V_{48} = \frac{480\ \mu C}{48\ \mu F} = 10V$

(Note that 30V + 10V = 40V)

Remember: *for capacitors in series, the charge on each capacitor must be the same and the applied potential is divided.*

Energy stored in a capacitor.

As well as storing charge, a capacitor must store energy. You can see this is true because work has to be done to charge the capacitor and energy is released during discharge. For a capacitor, the energy stored is the area under the graph of voltage against charge.



Here, this is a triangle, so $w = ½\, Q_f \times V_f$

If you use $C = \frac{Q}{V}$ and substitute for Q and then V , two other expresions are found:

Energy stored in a capacitor $= ½\ QV$
$= ½\ CV^2$
$= \frac{Q^2}{2C}$

Typical Exam Question
A 10µF capacitor, initially uncharged, is connected to a 2 volt supply. Calculate
(i) the charge transferred from the supply to the capacitor
(ii) the energy taken from the supply
(iii) the energy stored in the capacitor.

Answer:

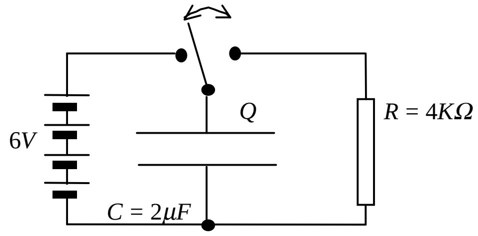
(i) *From the definition of capacitance,*
$Q = C \times V$. *So,* $Q = 10\mu \times 2 = \mathbf{20\ \mu C}$
(ii) *The supply provides 20µC at a steady p.d. of 2 volts.*
The energy taken from the supply, W , is given by $W = ½QV$.
So, $W = 20\mu \times 2 = \mathbf{40\mu J}$
(iii) *Any of the three expressions can be used to find the energy in the capacitor: suppose we use* $W = ½QV$ *This gives*
$W = ½ \times 20\mu \times 2 = \mathbf{20\mu J}$

Note that because we are using $W = ½QV$ the energy is in joules when the charge is in coulombs and the p.d. in volts. With capacitors, it is quite common to have microjoules.

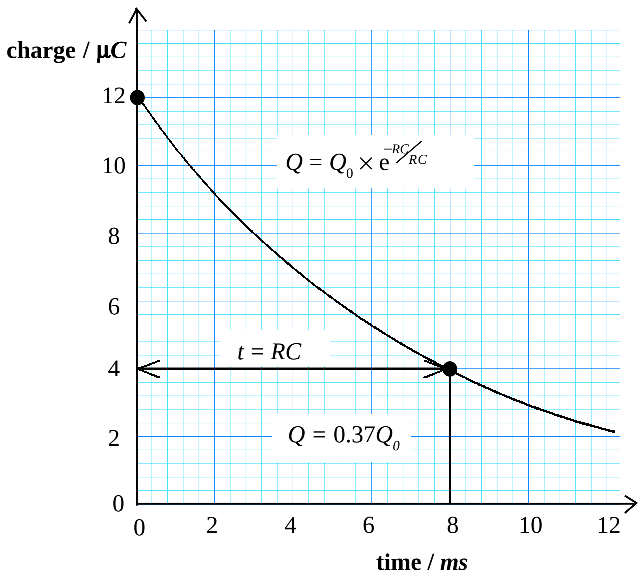
When a charge Q coulombs moves through a potential difference of V volts, the work done is QV. Only half of this stored in the capacitor! What has happened to the other half? It is hard to believe, but the answer is that the 'missing' half is lost as heat in the connecting leads. In charging the capacitor there is a current for a short time. This current passes through the resistance of the leads and gives the joule heating effect ($I^2 \times R$). Note that the resistance of the leads is usually very small and consequently is ignored in most cases.

Time constant

In the diagram the capacitor is first charged from the 6 *V* supply and then discharged through the 4*kΩ* resistor *R*. The question is 'how will the charge leak away from the capacitor through the resistor *R*?'



The answer is displayed in graph below . It shows the charge Q remaining on the capacitor. The initial charge is found by using $Q = CV$: $Q_0 = 2\mu \times 6 = 12\mu C$ as shown on the graph.



All other values are found from the equation $Q = Q_0 \times e^{-t/RC}$

In this equation, e means exponential, and may be found on a calculator. Look for the button marked e^x and check for yourself that $e^1 = 2.718$. and $e^{-1} = 0.37$ (to 2s.f.) (see Factsheet 10 Exponentials and Logarithms) The equation and graph describe an *exponential decay*. All you need to do is to let $t = RC$ in the equation. You will then have:

$Q = Q_0 \times e^{-t/RC}$
$Q = Q_0 \times e^{-1}$
$Q = Q_0 \times 0.37$

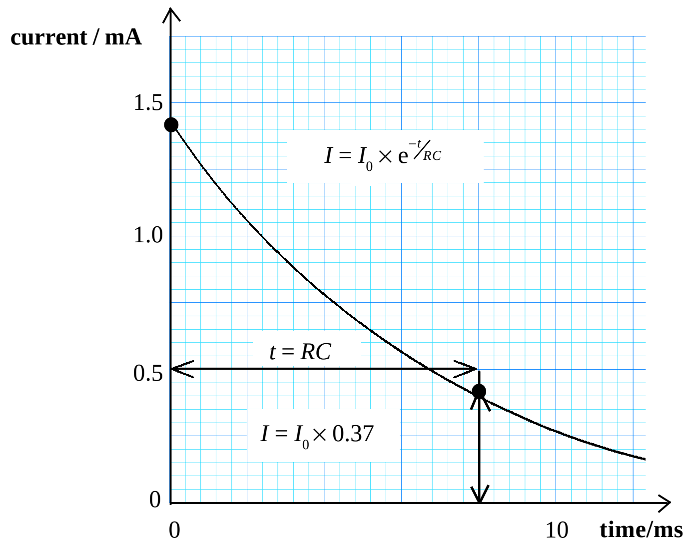
The term *RC* is called the **time constant,** it is the time taken for the charge stored to fall to **0.37** or to **37%** of the original charge.

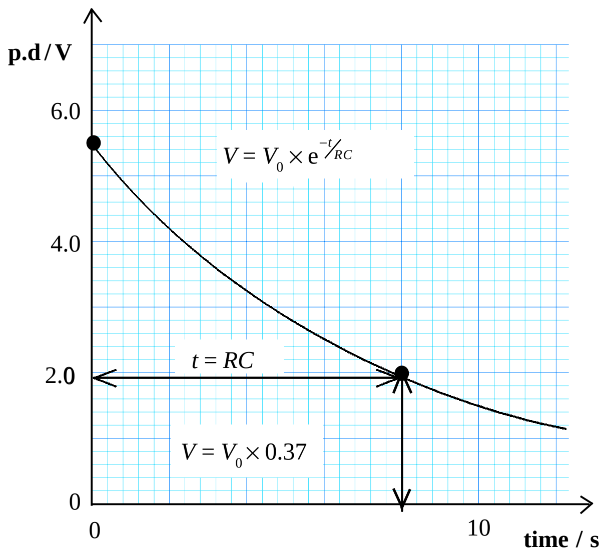
The time constant for the circuit is:

$t = RC$
$t = 4 \times 10^3 \times 2 \times 10^{-6}$ seconds
$t = 8 \times 10^{-3} s$ (8 *ms*)

The graph above it shows the initial charge $Q_0 = 12\mu C$ as calculated above. The time constant is 8*ms*, so the charge remaining at that time should be $12\mu C \times 0.37 = 4.44\mu C$.

Not only does the charge fall exponentially but the current and pd decrease exponentially as shown in the two graphs below and they have the same time constant of 8 *ms*.





After the time constant, $t = RC$, *Q*, *V*, and I all fall to 0.37 of their original value.

Sometimes we may need to know when these quantities fall to a *half* of their original value. This time is obviously slightly less than RC and is approximately $0.7 \times RC$ seconds. (The exact value is found by using the *natural logarithm* of 2, ln2 = 0.6931. You will meet ln2 in calculations on half life in radioactivity).

The time for *Q*, *V* and *I* to fall to half their original values is given by: $T_{½} = RC \times \ln 2$

$T_{½} = RC \times 0.69$ approximately

Exam Workshop

In this question, take the permittivity of free space $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$.

A student has to design a parallel plate capacitor of value 13 μF.

(a) Estimate the common area of this capacitor if the dielectric used is air of uniform thickness 0.1 *mm*. [3]

$$C = \frac{A\varepsilon_0}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0} ✓ \Rightarrow A = \frac{13\times10^{-4}}{8.85\times10^{-12}} ✓ \Rightarrow A = 1.47\times10^{8}\,m^2 \quad 2/3$$

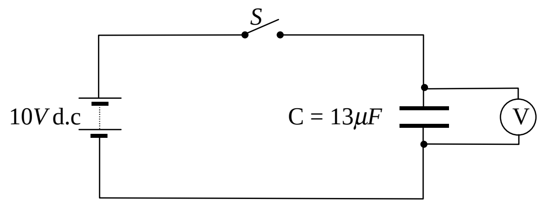
This candidate has quoted the correct formula and substituted the given values but failed to write C in farads (13×10^{-6}). The high value produced should have given a hint that something was wrong.

(b) In an improved design, the student fills the space between the plates with an insulator which is only 0.01 *mm* thick with a relative permittivity $\varepsilon_R = 1.5$. Estimate the new common area. [2]

$$C = \frac{A\varepsilon_0\varepsilon_R}{d} \Rightarrow A = \frac{Cd}{\varepsilon_0\varepsilon_R} ✓ \Rightarrow A = 1.47\times10^{8}\times\frac{10^{-1}}{1.5} \approx 1\times10^{7}\,m^2 \quad \text{ecf}✓ \quad 2/2$$

Has correctly recognised and used relative permittivity. For a given capacitor reducing d will reduce A in same ratio. Note the candidate has been awarded full credit for using the wrong answer from 1 a)

(c) A capacitor is conected in parallel with a high resistance voltmeter V in a circuit with a 10 volt supply and switch S.



Upon closing the switch, calculate; the pd across the capacitor and the charge stored in it. [2]

Pd across capacitor = supply pd = 10 V ✓

Q = CV Q = 13 × 10^{-6} × 10 =130 micro-coulombs ✓ 2/2

Has now correctly worked in micro-coulombs.

(d) When the switch has been open for 4 seconds, the voltmeter reads 6 volts. Calculate:

(i) the charge remaining in the capacitor. [1]

e, at t = 4s, Q = CV, so $Q_4 = 13\mu \times 6 = 78\mu C$ ✓ 1/1

(ii) the time constant for the capacitor voltmeter combination. [4]

$$Q = Q_0 e^{-t/RC} \quad 78 = 130e^{-t/RC} ✓ \Rightarrow \frac{78}{130} = e^{-t/RC} ✓ \Rightarrow 0.51 = RC ✗ \quad 2/4$$

Candidate has calculated $\ln(\frac{78}{130})$ correctly, but then equated it to RC. Common sense should have told him/her that a negative answer was wrong. S/he has also ignored the t.

Examiner's Answer

(a) $C = \dfrac{A\varepsilon_0}{d}$

$A = \dfrac{Cd}{\varepsilon_0}$ ✓ $= \dfrac{13\times 10^{-6}\times 0.1\times 10^{-3}}{8.85\times 10^{-12}}$ ✓

$= 147\ m^2$ ✓

(b) $A = \dfrac{Cd}{\varepsilon_0\varepsilon_R}$ ✓

$= \dfrac{13\times 10^{-6}\times 0.01\times 10^{-3}}{8.85\times 10^{-12}\times 1.5}$

$= 9.79\ m^2$ ✓

(c) *P.d = 10 V* ✓

Q = CV Q = 13 × 10^{-6}× 10 =130 μC

(d) (i) *as in answer*

(ii) $Q = Q_0 e^{-t/RC}$ ✓

$7.8 = 130e^{-t/RC}$

$\ln(78/130) = -t/RC$ ✓

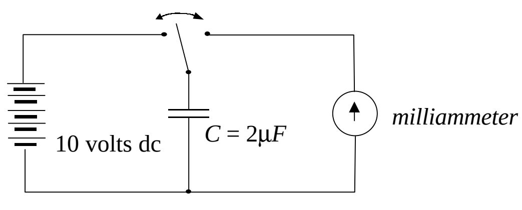
$t = 4$: $RC = \dfrac{-4}{\ln(78/130)}$ ✓

$= 7.8\ s$ ✓

Typical Exam Question

a) Two capacitors are available, one of 2μF and one of 5μF. Each capacitor is given a charge of 300μC. Calculate the potential difference across each.

b) The 2μF capacitor is now in the circuit shown below.



Calculate:

(i) the charge which flows through the milliammeter when the switch is moved from the left to the right;

(ii) the current in the milliammeter when the switch oscillates with a frequency of 200 *Hz*;

(iii) the frequency at which the switch should vibrate in order to produce a current of 10*mA*.

a) *By definition,* $\text{capacitance} = \dfrac{\text{Charge stored}}{\text{potential difference}}$ or $C = \dfrac{Q}{V}$ ✓

For 2μF, $V = \dfrac{300\times10^{-6}}{2\times10^{-6}}$ ***= 150 Volt*** ✓

for 5μF, $V = \dfrac{300\times10^{-6}}{5\times10^{-6}}$ ***= 60 Volt*** ✓

b) (i) *From definition, Q = V × C,*
so charge stored Q = 10 × 2μ = 20μ C or 20 × 10^{-6}C ✓

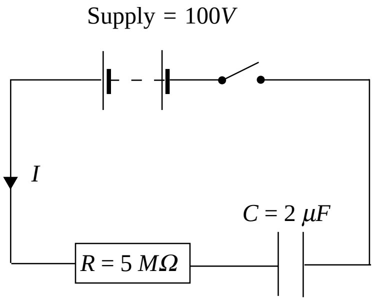
(ii) *We (should) know that current is the charge circulating in 1 second. Since the switch oscillates 200 times in one second, the charge circulating in this time is (20 × 10^{-6}) × 200 C.* ✓ *This is 4000 × 10^{-6} coulombs in 1 second* ✓ *= 4 × 10^{-3}coulombs in 1 second =4 millicoulombs in 1 second .Hence,* ***current = 4 mA*** ✓

(iii) *required current = 10 mA which is 10 × 10^{-3} coulombs in 1 second.* ✓
Working in microcoulombs, this becomes (10 × 10^{-3}) × 10^{6} microcoulombs in 1 second, = 10^{4}μC in 1 second.
Charge is still being 'delivered' in pulses of 20 μC ✓ *so the number of pulses per second or frequency will* be

$\dfrac{10^4\ \mu}{20\ \mu} = 500\ Hz$ ✓

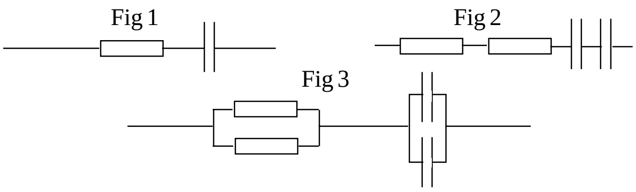
Questions

1. In the circuit shown below, the capacitor is initially uncharged and then the switch s is closed.

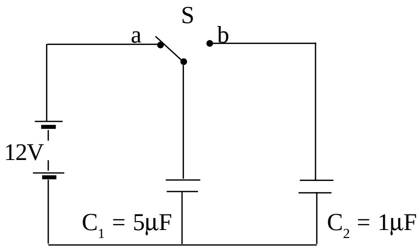


   (a) Find the initial current I_0 charging the capacitor.
   (b) Find the current charging the capacitor when the charge stored on the capacitor is
   (i) 40 μC
   (ii) 190 μC
   (c) Find the maximum charge stored by the capacitor.
   (d) Sketch a graph to show how the charge stored on the capacitor varies with the time from when the switch is closed.
   (e) What is the gradient at the origin on the graph you have drawn?

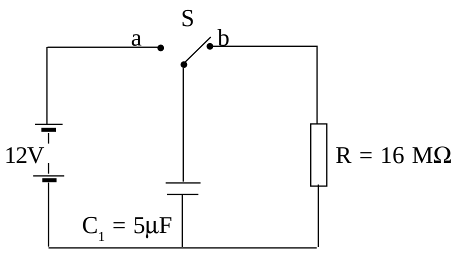
2. In the diagrams all the resistors are of equal value and all the capacitors are of equal value. The circuit in Fig 1 has a time constant of T. What are the time constants for Fig 2 and Fig 3 ?



3. The circuit below has a 12 volt supply and two capacitors C_1 and C_2. The switch S is connected to terminal a.



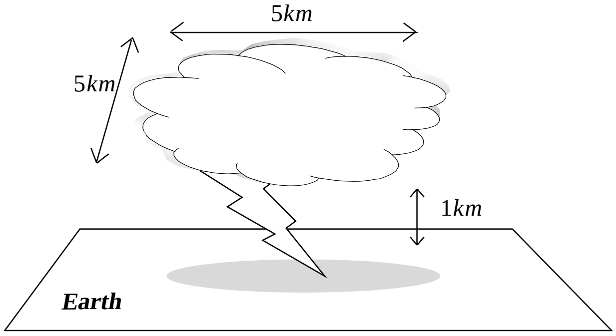
   (a) Calculate the charge stored on C_1.
   (b) The switch S is then connected directly to terminal b. Calculate:
   (i) the capacitance of the capacitor combination.
   (ii) the potential difference across the capacitors and,
   (iii) the charge on C_1.
   (c) The circuit is now modified by replacing C_2 with resistor R = 16 MΩ. The switch is again moved directly from a to b.



   Calculate the charge on C_1 60 seconds after closing S.

Typical Exam Question

The diagram below represents a thunder cloud 5.0 *km* × 5.0 *km* and 1.0*km* above the ground.



(a) Estimate the capacitance of the cloud-earth system stating any assumptions you make.
(b) If the maximum potential difference between the cloud and earth is 1.0 *GV* calculate the maximum charge stored and the corresponding energy. ($\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$)

Answer:
(a) Assumptions are

- *to treat the cloud - earth system as a parallel plate capacitor with dimensions given.*
- *air as dielectric so that ε_r is approximately 1. The capacitance is found from $C = \frac{\varepsilon_0 A}{d}$. Points to watch are the units.*

  So area $A = (5 \times 10^3) \times (5 \times 10^3) = 25 \times 10^6\ m^2$ and $d = 10^3\ m$

  $C = \frac{25 \times 10^6 \times 8.85 \times 10^{-12}}{10^3} = \mathbf{221 \times 10^{-9}}$ *F or,* $\mathbf{2.2 \times 10^{-7} F}$ *to 2s.f.*

In calculating charge, remember that $1GV = 10^9$ V. In this case,using $Q = CV$, $Q = 2.2 \times 10^{-7} \times 10^9 = \mathbf{2.2 \times 10^2}$ ***coulomb***

(b) To calculate the energy, any of the three expressions may be used. Using $W = ½QV$ we have $W = ½2.2 \times 10^2 \times 10^9 = \mathbf{1.1 \times 10^{11}\ J.}$

Note that in questions of this type, we are only estimating our final value. Hence it seems sensible to give answers to no more than 2 significant figures. As a rule, be guided by the figures supplied. In the above, ε_0 is given to 3 sig.fig. but the rest only to 2 sig, fig. So the answer must be to 2 sig. fig. You may wonder what the difference is between lengths of 5km, 5.0km, and 5.00km. The answer is that the calculation will show no difference but the information supplied is telling you the number of significant figures to which you should work.

Answers

1. (a) 20 μA
   (b) 16 μA
   (c) 1μA
   (d) see page 4
   (e) gradient = initial current = I_0 = 20 μA
2. all three have the same time constant T
3. (a) 60 μC
   (b) (i) 6 μF
   (ii) 10V
   (iii) 50 μC
   (c) $Q = Q_0 \times e^{-60/80} = 60 \times 0.472 = 28\mu C$

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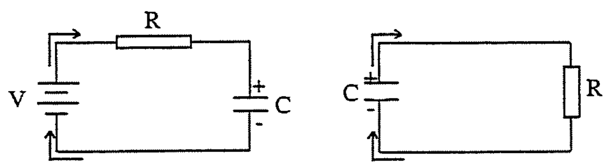
Uses of Capacitors

Factsheet 29 provided a thorough discussion of capacitor theory. It may well be worth looking through this again.

In this Factsheet we will be looking at the ways that capacitors can be used in electronic circuitry, and the types of capacitors that are in common use.

A brief summary of relevant theory

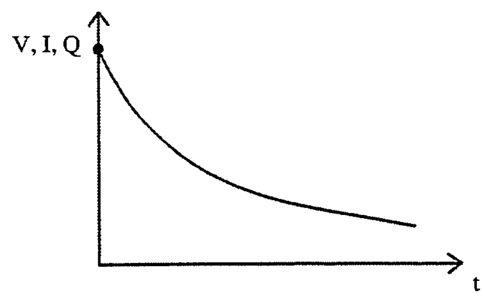
1. Charge and discharge rates:



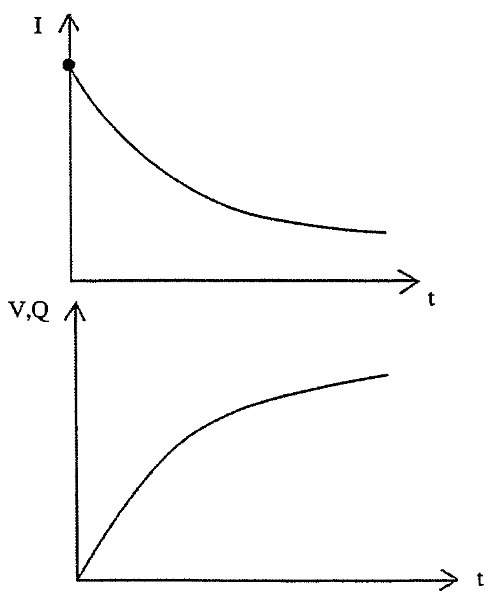
Charging Discharging

A capacitor charges or discharges through a resistor following an exponential curve. The time constant, **T**, is defined as: **T = RC** (s)

The discharge graphs resemble:



Things are slightly more complex for the charging graphs. The p.d. and charge stored increase as the charging current falls:



In each case the current drops to 0.37 of its initial value after one time constant: $e^{-t/RC} = e^{-1} = \mathbf{0.37}$

Example:

(a) To what fraction of its original voltage does a discharging capacitor drop after 3 time constants (T=3RC)?

(b) How many time constants does it take for the voltage to drop to 25% of its initial value?

Solution:

(a) $e^{-3T/RC} = e^{-3} = 0.050$

(b) $e^{-t/RC} = 0.25, \frac{-t}{RC} = \ln 0.25 = -1.39,\ t = 1.39RC = 1.39T$

Most uses of capacitors involve this time constant. It can be seen that increasing R or increasing C results in a larger time constant. The charge and discharge rates change more slowly.

2. Capacitance and Energy:

When a capacitor C is charged to a potential difference V, a charge Q is stored in it. $Q = CV$, or $C = Q/V$ (farads)

The energy stored on this capacitor can be written:

$$E = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV = \tfrac{1}{2}Q^2/C \quad (joules)$$

3. Reactance:

In an a.c. circuit, a capacitor C has a **reactance.**

$$X_C = \frac{1}{(\omega C)} \text{ (ohms)}$$

Where ω is the angular frequency ($\omega = 2\pi f$).

Reactance for a capacitor is equivalent to **resistance** for a resistor.

At very high frequencies the capacitor acts as a short circuit; at very low frequencies the capacitor blocks most of the current.

Example:

(a) A 1.5μF capacitor is placed in a mains circuit (f=50Hz). Find its reactance.

(b) It is then placed in a d.c. circuit (batteries). What is its reactance now?

Solution:

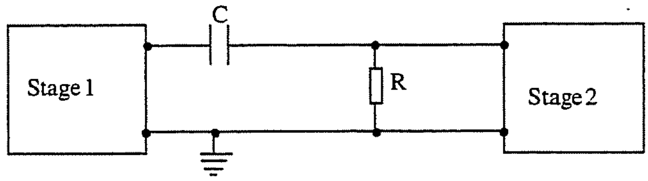
(a) $X_C = 1/(\omega C) = 1/(2\pi \times 50 \times 1.5\times10^{-6}) = 2.1k\Omega$

(b) $X_C = 1/(\omega C)$ = *Infinity, as the frequency is zero.*

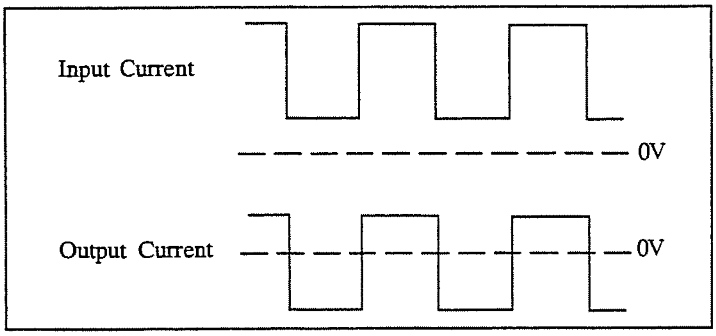
In a d.c. circuit, the capacitor does not let any current through.

Uses of Capacitors

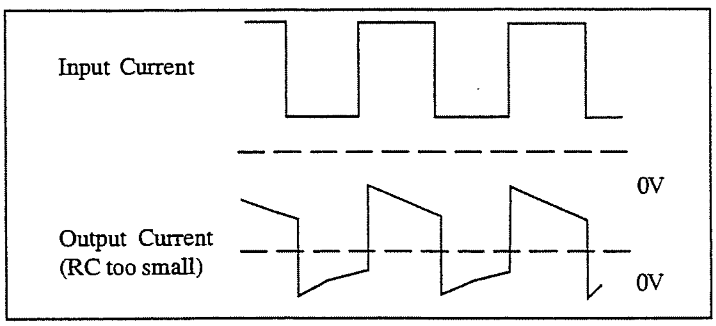
1. Coupling
Because a capacitor blocks d.c. signals, it can be used between stages of an amplifier, or to connect the amplifier to a loudspeaker. Only the a.c. signal can get through.



If the input signal is a square wave:



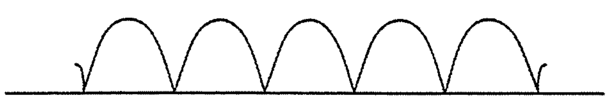
It is important that the time constant of the coupling circuit (*RC*) is much greater than the period of the input signal. If the capacitor were to become significantly charged (or discharged) during each half cycle, then the current flow would fall. This would cause distortion in the output.



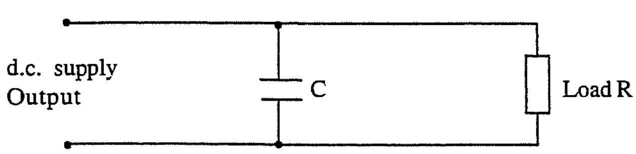
Obviously this would not be acceptable.

> Only the varying component of a signal can travel through a capacitor.

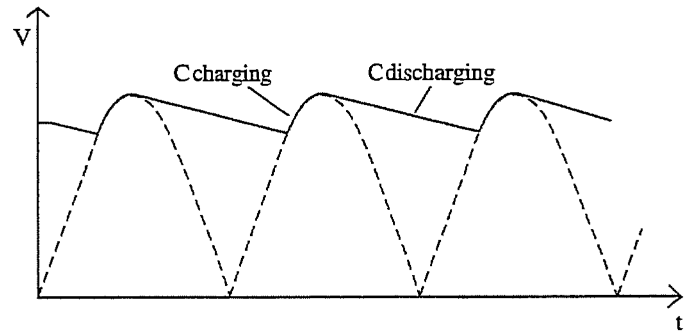
2. Smoothing
A diode bridge rectifier changes a.c. voltage (e.g. mains) into d.c. voltage. This is often required in electronic circuitry. However the d.c. output of the rectifier is not steady:



A large capacitance across the output stores charge, which can then be discharged through the load as the output voltage from the rectifier falls.



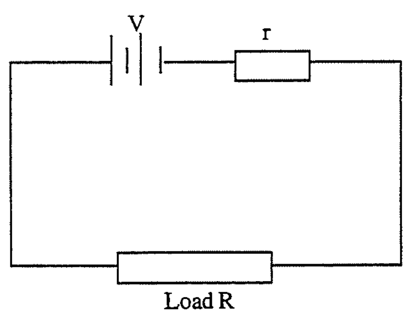
The voltage across the load is smoothed by this extra flow of charge.



There is still a slight **ripple** in the voltage. The larger the capacitance, the smaller the ripple will be. If the load only requires a small current flow (e.g. electronics), then the smoothing can be almost perfect.

> *A high value capacitor can be used to smooth d.c. voltage for many applications.*

3. Energy discharge
In certain devices, it is important that all of the energy is supplied in a sudden pulse. Examples are flashguns and some types of lasers. This requires a very high current flow for a very short period of time.
Power supplies cannot do this.

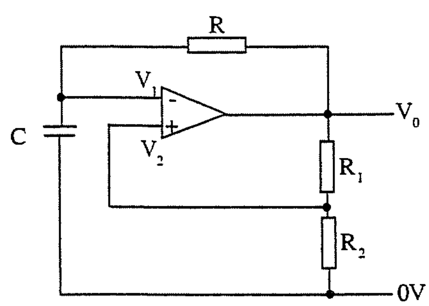


The internal resistance, *r*, limits the flow of current,
$I = \frac{V}{(R+r)}$, restricting the rate of energy transfer.

However, if energy is stored on a capacitor, then the discharge current can be very large, as long as the resistance of the load, *R*, is very small. The initial current flow is given by $I=V/R$. If the load is tiny, then the rate of energy transfer $P=V^2/R$ can be very large.

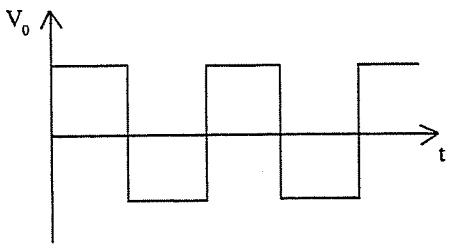
> *The lack of significant internal resistance means that a charged capacitor can deliver energy very quickly (for a short period of time).*

4. Oscillation
The charge and discharge rates for a capacitor circuit can be used to produce an alternating signal. The frequency depends on the components chosen.



This is an astable multivibrator. When V_1 becomes larger than V_2, the output V_o instantly becomes negative. Capacitor, C, discharges through resistor, R, until V_1 becomes less than V_2. The output V_o jumps to a positive value. The capacitor charges up (through R) until V_1 becomes greater than V_2, forcing V_o to go negative again.

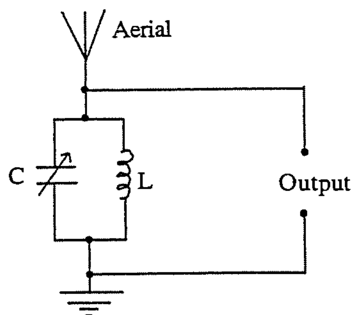
The output voltage is a square wave.



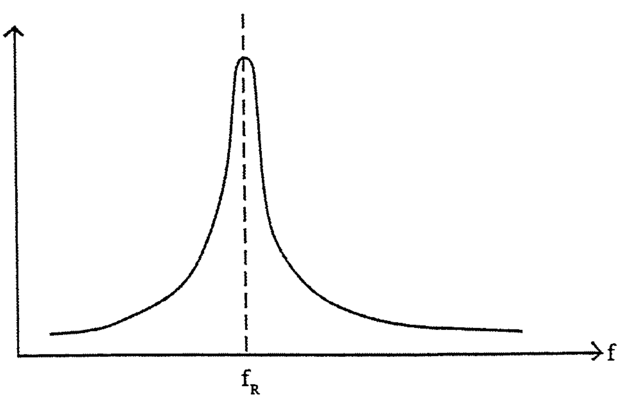
The frequency is controlled by the values of R and C, as time constant RC determines the rate of charging and discharging (and thus the rate at which V_1 changes). Often R is a variable resistor, allowing the operating frequency to be adjusted.

> *An RC circuit can be used to determine the rate of switching in many types of oscillating circuits.*

5. Tuners
A traditional and simple tuning circuit for radios, televisions, etc, relies on a capacitor and inductor in parallel.



All the carrier frequencies are picked up by the aerial. By choosing the correct values for C and L, the device can be tuned to a selected frequency.



The reactance of the inductor **increases** with frequency; the reactance of the capacitor **decreases** with frequency. The maximum voltage across the tuner occurs when:

$$f_R = \frac{1}{(2\pi\sqrt{LC})}$$

f_R is the resonant frequency.

> *An L-C parallel circuit allows us to tune to a selected frequency. A variable capacitor makes tuning through a range of frequencies possible.*

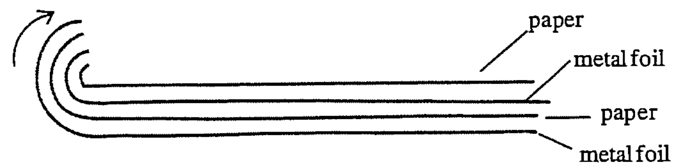
Some useful capacitors
The basic capacitor has two conducting plates separated by a dielectric material. The capacitance is given by:

$$C = \frac{A\varepsilon_o\varepsilon_r}{d}$$

A is the area of each plate
ε_o is the permittivity of free space
ε_r is the relative permittivity of the dielectric
d is the separation of the plates.

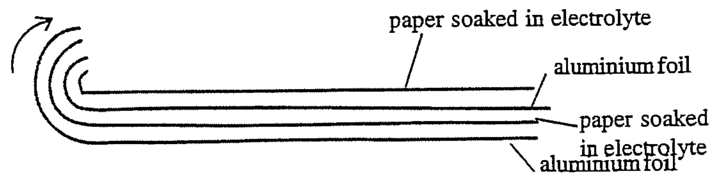
There are different ways that practical capacitors can be constructed.

1. Film capacitors



The paper and metal strips are rolled into a cylinder, forming large areas of metal separated by waxed paper as the dielectric. These capacitors are very cheap. They make good general-purpose capacitors.
Often plastic films such as polycarbonate and polystyrene are used instead of paper, as they improve the frequency response.

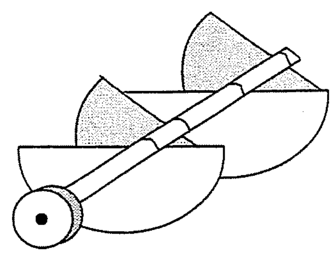
2. Electrolytic capacitors
Once again thin strips of material are rolled into a cylinder to form a large area in order to maximise capacitance. But this time the dielectric is just a thin film of aluminium oxide formed on the positive strip of aluminium foil (the anode).



A chemical reaction forms the oxide layer. It may be less than 10^{-6}m thick, greatly increasing the capacitance. Electrolytic capacitors routinely have capacitances of hundreds of microfarads, or even more.

Their high capacitance makes them very useful in smoothing and coupling circuits. However they have a constant leakage current (limiting their use in low power electronic circuits), and must be put in the circuit the correct way round to avoid damage (they are polarised).

3. Variable air capacitors
Tuning circuits require a capacitor whose value can be changed. The variable air capacitor accomplishes this by changing the effective area of the plates facing each other.



Turning the knob rotates one set of plates, changing the effective area. The actual capacitance is small. The important point is that it can be varied.

Questions

1. In a coupling circuit, sketch the output current (from a square wave input) if the time constant is much smaller than the period of the input signal.

2. In a smoothing circuit, the load requires a steady current of 0.5 amps. The smoothing capacitor holds a maximum charge of 2.4×10^{-4} coulombs. Is the smoothing likely to be very effective at mains frequency?

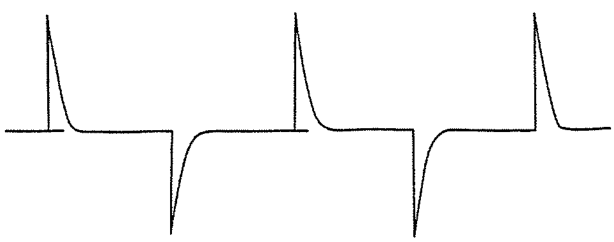
3. Two strips of paper and two strips of metal foil are rolled up to make a paper capacitor. Each strip is 50cm long, 2cm wide, and has a thickness of 0.1mm. Find the diameter of the cylinder formed.

4. Give one advantage and one disadvantage of electrolytic capacitors. Where are they used?

5. Two ways that capacitance can be maximised are by increasing the area of the plates or by decreasing the separation of the plates. What methods are used in film, electrolytic, and air capacitors to maximise capacitance?

Answers

1.



The capacitor becomes completely charged or discharged, and the current drops to zero.

2. For mains electricity, one-half of a cycle lasts 0.01s.
The charge required to provide the current in this time:
$Q = It = 0.5 \times 0.01 = 5 \times 10^{-3}$ coulombs.
Although the capacitor only has to supply the charge for part of the cycle, there is far too much charge required. The capacitor is much too small.

3. Volume cylinder = Volume rectangle
(choose cm as unit)
$\pi r^2 l = lwt$, $\pi r^2 = wt$, $r^2 = 4 \times 50 \times 0.01 / \pi$
$r^2 = 0.64\text{cm}^2$, $r = 0.80\text{cm}$ $d = 1.6\text{cm}$.

4. Advantage – large capacitance.
Disadvantage – leakage current, polarised.
Used in smoothing and coupling circuits.

5. Film capacitor – large area.
Electrolytic capacitor – large area, small separation.
Air capacitor – neither.

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Physics Factsheet

January 2003 Number 45

Magnetic Fields

This Factsheet will explain:
- what is meant by a magnetic field;
- the meaning of the term **magnetic flux density**;
- the shape of fields around wires, coils and solenoids;
- the factors affecting fields around wires, coils and solenoids;
- how to measure magnetic fields.

Before studying the Factsheet, it is helpful to revise GCSE work on magnetic fields i.e. how to investigate the shape of a magnetic field using iron filings or a plotting compass and the shape of the field around a bar magnet; also knowledge that a magnetic field can be caused by an electric current in a coil or a solenoid.

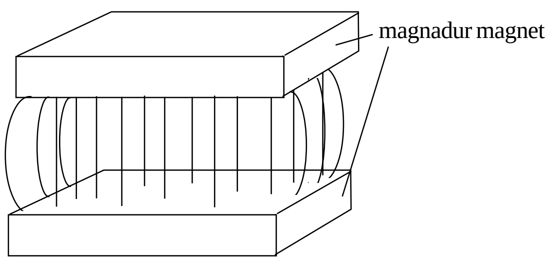
Factsheets 33 and 35 also contain related ideas about fields that would enhance understanding of this topic.

Fields
A field is the region around a mass, charge or magnet in which a force can be experienced. This definition is common to the three types of field – gravitational, electric and magnetic. Any of these fields can be described by "lines of force". The line of force shows the direction of the force on a unit mass, positive charge or North magnetic pole and the density of the lines of force shows the strength of the field. Another name for "lines of force" is "flux", so the flux density gives the strength of the field.

*The **strength of the field** is given by the "**flux density**"*

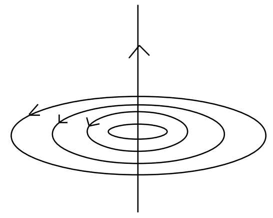
Field Strength
The strength of a gravitational field is defined as the force on a unit mass, and the strength of an electric field as the force on a unit positive charge. The strength of a magnetic field **can** be defined as the force on a unit N pole, but this is not very helpful, since unit poles do not actually exist and magnetic fields can also have an effect on charges in the field. So we shall see that magnetic fields are generally defined in terms of the force they exert on a current-carrying conductor.

Field of Magnadur magnets
A less common, but for some applications, more important shape of magnet than the bar-magnet is the "magnadur". The shape of the field between two magnadurs is shown in below. Since the lines of force (the flux) are parallel, then the field is constant (since the density of the lines shows the field strength) for most of the centre of the region between the magnets.



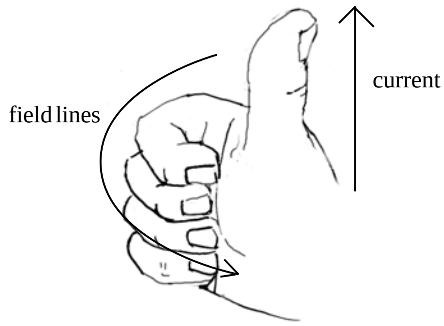
The field of two magnadurs is constant in the centre.

Field of a long, straight wire carrying a current
The lines of force of a long, straight wire carrying a current are concentric circles, in the plane at right angles to the wire, with the separation of the lines inversely proportional to the distance from the wire.

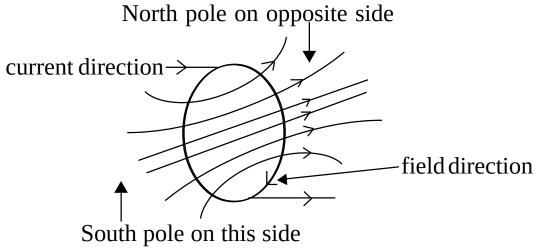


The field of a long straight wire consists of concentration circles in the plane at right angles to the wire, dropping off as $\frac{1}{r}$, where r is the distance from the wire.

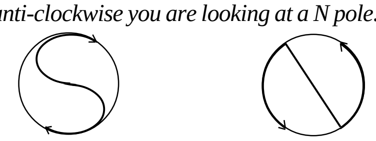
The direction of the lines can be remembered by using the right hand as shown below.



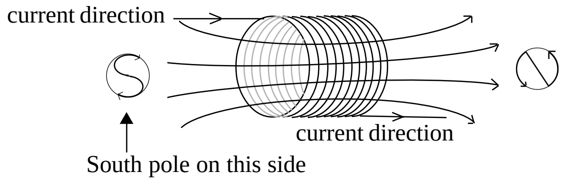
Field of a coil
A coil is really a long straight wire coiled into a flat circle. The field of a coil can be worked out by adding the contributions from each of the sections of the wire.



The direction of the field - If you look at the end of the coil and the current is clockwise, then you are looking at a S pole, conversely, if the current is anti-clockwise you are looking at a N pole.



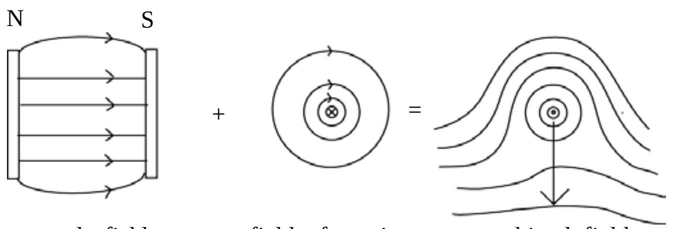
Field of a solenoid
A solenoid is a long coil (though it is often wrongly called a coil; a coil is flat.) The field is the addition of the fields from lots of coils.



Again the direction of the field can be worked out. When you look at the end of the solenoid, the current is clockwise, you are looking at a S Pole.

You should recognize this field as identical to that of a bar magnet, except that it continues inside the solenoid. It has the advantage over the bar magnet in that the strength of the field can be varied, whereas a bar magnet has a fixed strength.

Force on a current-carrying conductor in a magnetic field
The force arises out of the interaction of the two magnetic fields: the one due to the wire and the one due to magnadur field. Imagine the two fields superimposed on each other. Lines of force cannot cross, since they show the direction of the force at that point in the field, so the resultant of the two forces becomes the new line of force and a combined field results.



magnadur field — field of a wire carrying a current away from you — combined field gives a net force downwards

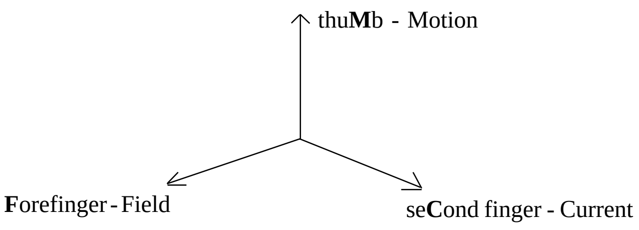
Sometimes the combination results in a "neutral point" – a point where the two fields exactly cancel each other out.

> *A neutral point is a point where two or more magnetic fields cancel each other out to give no resultant force.*

In the case of the long straight wire in a magnadur field, there is a null point on one side of the wire, and a stronger field on the other side of the wire, so there is a net force on the wire, which is at right angles to both the magnadur field and to the wire.

> A current-carrying conductor in a magnetic field experiences a force which is at right-angles to the field and to the wire, due to the combination of the two magnetic fields

The direction of the force on the wire is always at right angles to the field and to the wire, but to decide the exact direction, use the diagrams above to work out which side the field due to the magnadurs, and the field due to the wire reinforce each other, or use the left-hand rule:



Expression for the strength of a magnetic field
It seems likely that the force on the current-carrying conductor should depend on the strength of the field, the length of the wire in the field and the size of the current. Each of these factors may be investigated one at a time, keeping the other factors constant, by using magnadurs mounted on a C-yoke and passing a current through a wire placed in the field. If the whole arrangement is mounted on a sensitive balance, the force can be recorded. It is indeed found that the force depends on the current *I*, the length *l* of the conductor **in the field** and the field strength *B*, so we may write:

$$F \propto BIl$$

In fact we use this expression to define the field strength of a magnetic field and choose the units (the **tesla**) so that the constant of proportionality is one. So:

$$F = BIl$$

> *One tesla is that field, which in a wire carrying a current of 1amp produces a force of 1 newton for each metre of the wire's length.*

> ***Exam Hint**: -You will be expected to be able to use this expression linking force, flux density, current and length of the wire in the field.* $F = BIl$

Typical Exam Question
A uniform magnetic field of 40 mT acts at right angles on 3 m of a long straight wire carrying a current of 3.4 A. Calculate the force on the wire.
$F = BIl$, *therefore* $F = 40 \times 10^{-3} \times 3.4 \times 3 = 0.41N$

> *You should know the definition of the tesla T, the unit of magnetic field strength (flux density) as:*
> ***That field, in which a wire carrying a current of 1 amp, produces a force of 1 N for each 1metre of the length of the wire in the field, at right angles to both the wire and the field.***

Measuring magnetic fields
It is not easy to measure magnetic fields directly, since apparatus to measure the force on a wire is cumbersome. A **Hall probe** uses the effect of the magnetic field on electrons in a semi-conductor to give a p.d. across a slice of semi-conductor, which is proportional to the field. This gives a measure of the field strength. If an absolute value of the B-field is required the Hall probe must be calibrated in a known field, but often it is only relative values that are needed so the calibration is not always necessary.

> ***Exam Hint**:- Although you are expected to know about the use of a Hall probe, you are not expected to understand its internal workings.*

Field of a long straight wire
If the field of a long straight wire is investigated using a Hall probe it is found to depend on the current through the wire and to drop off as:
$\frac{1}{r}$ where *r* is the distance from the wire.
The constants of proportionality are such that we can write:

> $$B = \frac{\mu_0 I}{2\pi r}$$
> Where μ_0 = permeability of free space ($4\pi \times 10^{-7}$ NA^{-2})
> B = magnetic field strength, flux density (Tesla, T)
> I = current (A)
> r = distance from the wire (m)

> ***Exam Hint**: Although you will be given the expression* $B = \frac{\mu_0 I}{2\pi r}$ *you will be expected to be able to carry out calculations using it*

Typical Exam Question
Calculate the field strength at a point 60cm away from and in the plane at right angles to a long straight wire carrying a current of 1.3A

$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1.3}{2\pi \times 0.6} = 4.3 \times 10^{-7} T$

Field of a solenoid
If the field of a solenoid is investigated using a Hall probe, it is found to be constant for most of the length of the solenoid, but drops off to about ½ its value at the ends.

It is also perhaps surprising that the field is found to be **independent of the area of cross-section.** It is found to be proportional to the **current** and to the **number of turns per unit length.** Again the units chosen allow the constant of proportionality to be 1. So:

$B = \mu_0 nI$ Where n = number of turns per 1metre length
μ_0 = permeability of free space ($4\pi \times 10^{-7}$ NA^{-2})
B = flux density (Tesla, T)
I = current (A)

Exam Hint : *Although you will be given the expression $B = \mu_0 nI$, you should be confident in using it for calculations.*
*Remember that **n** is the **number of turns per 1metre length**, not the total number of turns.*

Typical Exam Question
A solenoid is 45cm long and has 100 turns. Calculate the magnetic field strength inside it when there is a current of 3.6A
n = 100/0.45 = 222.2 (Remember n is turns per metre)
$B = 4\pi \times 10^{-7} \times 222.2 \times 3.6T = 1.0mT$

Total Flux
You have learned that the strength of the magnetic field is the flux density, *B* in tesla. When you study electromagnetic induction, it is the total flux, not just its density that is important. The total flux is given the symbol Φ, and is the product of the flux density and the area.

$\Phi = B \times A$ where Φ = total flux (weber, Wb)
A = cross-sectional area (m^2)
B = flux density (Tesla, T)

Typical Exam Question
a) A solenoid is formed by winding 300 turns of wire onto a hollow cardboard tube of length 0.15m. Show that when there is a current of 0.4A in the solenoid, the magnetic flux density at the centre is 1.0×10^{-3}T. [2]
The solenoid has a cross-sectional area of 5×10^{-3} m^2 The magnetic flux emerging from one end of the solenoid is 2.7×10^{-6} Wb. Calculate the magnetic flux density at the end of the solenoid.[2]

b) Why is the flux density at the end not equal to the flux density at the centre? [1]

a) $n = \frac{300}{0.15} = 2000$ *turns/metre*
$B = \mu_0 nI = 4\pi \times 10^{-7} \times 2000 \times 0.4 = 1.0\ mT$

b) $B = \frac{\Phi}{A} = \frac{2.7 \times 10^{-6}}{5 \times 10^{-3}} = 0.54\ mT$

c) Through the centre of the coil the lines of force are parallel lines so the flux density is constant, but at the ends the same lines of force spread out over a greater area, so their density is less.

Exam Workshop
This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answers are given below.

(a) Explain what is meant by field strength of a magnetic field. [3]

It is a measure of how strong the field is. 0/3

The candidate has merely restated the question. Field strength is a precise definition. S/he should have realized that more was required for 3 marks.

(b) Express the tesla in base units [2]

F =BIl, therefore B is force/current × length so units are N/Am 1/2

The candidate has reduced the current and length to base units, but N is not a base unit.

(c) Calculate the current in the cable for a household electric fire rated at 3 kW (Take mains voltage to be 240V). [2]

P =V × I therefore I = 240/3 = 80A **0/2**

The candidate has forgotten that 3kW is 3000W and has the equation upside down. S/he should have realized that 80A is not a sensible household current.

(d) A child stands 50cm from the cable, what will be the value of the field in the region of the child? [2]

$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 80}{2\pi \times 50} = 3.2 \times 10^{-7} T$ 1/2

The candidate would not be penalized for carrying forward the error from (c), but s/he has also forgotten to change 50cm into 0.5m

Examiner's Answers
(a) The field strength is the flux density in Tesla, where a Tesla is that strength of field which causes a force of 1N on each 1metre length of a wire at right angles to the field, carrying a current of 1A. ✓✓✓
(b) B is force/current × length so is $kgs^{-2}A^{-1}$ ✓✓
(c) P = V × I therefore I = P/V = 3000/240 = 12.5 A ✓✓
(d) $B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 12.5}{2\pi \times 0.5} = 5 \times 10^{-6} T$ ✓✓

Questions

1. Explain what is meant by a neutral point in a field.

2. (a) Sketch the magnetic field pattern of a long straight, current-carrying wire, indicating the directions of both the current and the field lines.
   (b) Sketch the magnetic field pattern between two magnadur magnets. Indicating the direction of the field lines.
   (c) Sketch the combination of the fields of (a)and (b) and explain why it results in a force on the wire which is at right angles to both the wire and the field.

3. The coil in a microphone has an average radius of 5.0 cm and consists of 250 turns.
   (a) Calculate the total length of the wire.
   The microphone magnet produces a field strength, at right angles to the coil, of 250 mT.
   (b) Calculate the force on the coil when it carries a current of 4mA.
   (c) What is the effect on the coil when the current through the coil alternates?

4. Two long straight wires, each carrying a current of 1.5 A, are placed 2 m apart in air.
   (a) Calculate the force which each wire exerts on each 1metre length of the other.
   (b) If the current in each wire is in the same direction, which direction is the force?
   (c) If the current in one wire is in the opposite direction to that in the other wire, which direction is the force?

Answers

1. *A neutral point is a point in the resultant field of two or more magnetic fields where the field stregths cancel(in magnitude and direction) out so that the net field strength is zero.*
2. *See text.*
3. *(a) Length = $2\pi r \times 250 = 2\pi \times 5 \times 10^{-2} \times 250 = 78.6$m*
   (b) $F = BIl = 250 \times 10^{-3} \times 4 \times 10^{-3} \times 78.6 = 7.86 \times 10^{-2}$N
   (c) The force will reverse direction each time the current changes direction, i.e. the coil will vibrate.
4. *(a) Field due to one wire at the other wire*
   $= \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1.5}{2\pi \times 2} = 1.5 \times 10^{-7}$T
   Force exerted = $BIl = 1.5 \times 10^{-7} \times 1.5 \times 1 = 2.25 \times 10^{-7}$N
   (b) attracts wires
   (c) repels wires

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Physics Factsheet

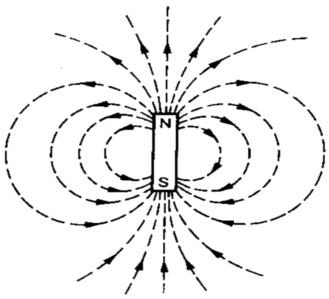
January 2002 Number 26

Electromagnetic Induction

If an electrical conductor and a magnetic field are moved relative to each other, an electric current or voltage is *induced* (or *generated* – it means the same thing) in the conductor. Most of the electricity we use is generated by **electromagnetic induction.**

Magnetic flux

A simple bar magnet creates a magnetic field in the space around it. We can represent the field by lines of force (you have probably seen the shape of the field revealed by iron filings or, more laboriously but less messily, with a plotting compass).



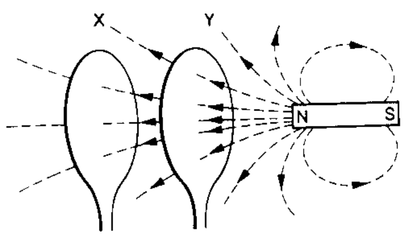
We call these lines of force 'magnetic flux lines'. The word 'flux' means 'flow' – and the lines of magnetic flux flow from the north pole of the magnet around to the south pole.

If a length of wire is moved into the magnetic field (or out of it) it must cut across the lines of magnetic flux. It is this cutting of flux by a conductor which causes a current to be induced in the wire. Remember, it doesn't matter whether it is the wire or the magnet which moves– in either case, the wire cuts the magnetic flux lines and a current will flow. (Actually, the current will only flow of course if the wire is part of a complete circuit – you don't have to forget everything you knew about electricity!)

If a coil of wire is used instead of a single length, each turn of the coil cuts across the magnetic flux and contributes to the size of the current induced. So if a coil has five turns, there will be five times as much current induced as in a single loop of wire.

Flux linkage

A slightly different way of thinking about this is to say that the lines of flux which pass through the middle of the coil are 'linking' the coil. When the coil is inside the magnetic field, flux 'links' the coil by flowing through it. When the coil is outside the magnetic field, no flux links it. We can then say that *changing the flux linkage* by moving the coil into the field (from X to Y in the diagram below) or out of the field (from Y to X) or through the field induces a current.



How much magnetic flux?

There are some similarities between electric fields, gravitational fields and magnetic fields. In each case we are able to define the strength of the field – how much force is applied at any given point to a given charge, a mass or a current. So, the strength (E) of an electric field at a particular point is the force on a unit charge (1 coulomb) at that point. The strength (g) of a gravitational field is the force on a unit mass (1 kilogram). In the same way, the strength (B) of a magnetic field is the force on a unit current length; that is, the force acting on a conductor 1 metre long carrying a current of 1 amp at right angles to the direction of the magnetic field (OK, just a bit more complicated than electric or gravity fields).

Another, more useful, name for this magnetic field strength B is **flux density.** It is the number of magnetic flux lines in unit area (i.e. number of flux lines per m^2.) You can picture flux density as a measure of how close together the lines of magnetic flux are.

We can now say that the total amount of flux (which we call Φ) is equal to the flux density B multiplied by the area A through which it is flowing:

$$\Phi = BA$$

(magnetic flux = flux density × area)

(Remember – if there are N turns in the coil, then the magnetic flux linkage will be $N\Phi = NBA$)

> ***Remember** : The magnetic flux lines have to be perpendicular to the area A – or else we have to consider the component of B which is perpendicular to A.*

The unit of magnetic flux is the weber (Wb). You can see from the equation how this is related to the unit of flux density, the tesla:

$1T = 1Wb\ m^{-2}$ - one tesla is one weber of flux passing through one square metre

How much induced e.m.f.?

We have seen that electromagnetic induction happens – an e.m.f. (or voltage) is generated - whenever a conductor cuts across lines of magnetic flux (or, to put it the other way, whenever the *flux linkage* is changed). The size of this voltage – the value of the induced e.m.f. – depends on how many lines of flux are being cut and how quickly they are being cut. We can say that it depends on the rate of change of flux linkage. This is summed up in Faraday's Law.

> ***Faraday's Law***
>
> *The induced e.m.f. is proportional to the rate at which flux is cut*
>
> *This can be written as a mathematical equation:* $E = -\frac{\Delta\Phi}{dt}$
>
> *where N is the number of turns in a coil of wire and* $\frac{\Delta\Phi}{\Delta t}$ *is the rate at which flux* Φ *is cut. (You will see later (in Lenz's Law) where the minus sign comes from and what it means)*

Note for non-mathematicians

If you're not studying AS/A2 level maths, you might not be familiar with the $\frac{d\Phi}{dt}$ format. Don't panic! It is a mathematical way of showing how one variable changes with another – in this case, how the flux Φ changes with time t. You can think of it as: the change in flux Φ *divided by* the change in time t. As another, unrelated, example, the *speed* of a moving object (which, as you know, is change in distance s divided by change in time t) can be shown in this format as $\frac{ds}{dt}$. (In case you are interested, it is called a *differential equation* and, like so much in our lives as physicists, was 'invented' by Isaac Newton.)

Typical Exam Question:

A coil of 1000 turns and resistance of 50Ω has a cross sectional area of 6.8cm^2. It is connected in series with an ammeter of negligible resistance. The coil is mounted with its plane perpendicular to the Earth's magnetic field and is rotated through 90^o, so that it is parallel to the field, in a time of 0.5s. The magnetic flux density of the Earth's field at this location is 5.2×10^{-5}T. Calculate the average e.m.f. induced in the coil and the average current recorded on the ammeter.

Flux linkage through single coil, $\Phi = B \times area$ ✓

$= 5.2 \times 10^{-5} \times 6.8 \times 10^{-4}$

(remember you must convert cm^2 to m^2 !)

$= 3.54 \times 10^{-8}$ Wb ✓

Flux changes to zero in 0.5s

e.m.f. = number of turns× rate of change of flux linkage

$= 1000 \times 3.54 \times 10^{-8} / 0.5 = 7.1 \times 10^{-5}$ V ✓

current $= V/R = 7.1 \times 10^{-5} / 50 = 1.4 \times 10^{-6}$ A (= 1.4 μA) ✓

***Exam Hint**: Be very careful with the units in this sort of calculation. It is most sensible to convert everything to SI units at the beginning (metres, kilograms, seconds, amps etc) and work with them. It is especially important in this sort of question where you don't usually have a 'common-sense feel' for what a reasonable final answer ought to be.*

Faraday's law tells us the size of the induced e.m.f. If we want to know the direction of the e.m.f.– in other words, which end of the wire or coil becomes positive and which end becomes negative - we need **Lenz's law.** (You must be able to REMEMBER and USE this one, too)

Lenz's Law

The direction of the induced e.m.f. is such that it tends to oppose the flux change which causes it. An induced current will flow in a direction so as to oppose the flux change that is producing it.

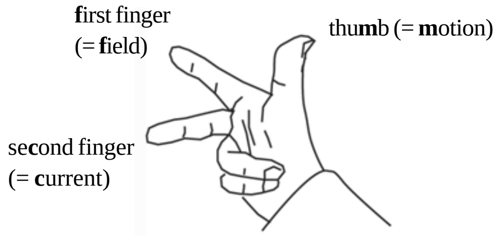
(This is where the minus sign comes from in Faraday's Law, above. In fact, Faraday's law stated as a mathematical equation encompasses Lenz's law too. This – as in many areas of physics – is an example of how the language of mathematics can be more economical than English or other 'word' languages.)

Lenz's Law is consistent with common sense and with the law of conservation of energy. If an induced current flowed the *other* way, to reinforce rather than to oppose the flux change causing it, then the induced current would *increase* the flux change, inducing still more current, and so on – getting bigger and bigger and giving something for nothing (more electrical energy output for no extra energy input). The laws of nature never give you something for nothing!

A special case of Lenz's Law – Fleming's Right Hand Rule

If you have a straight conductor which is moving at right angles to the magnetic field to induce an electric current, then Fleming's Right Hand Rule (sometimes called the 'Dynamo Rule') is a more useful version of Lenz's law which enables you to predict which way the current will flow.

If the thumb, index finger and second finger of the RIGHT hand are held so that each is at right angles to the others; with the **f**irst finger pointing in the direction of the magnetic **f**ield (North to South) and the thu**m**b in the direction of **m**otion of the conductor, then the se**c**ond finger will point in the direction of the induced **c**urrent.



Eddy currents

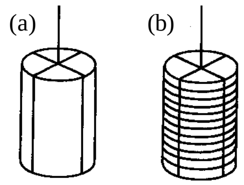
Any piece of metal which is moving in a magnetic field has an e.m.f. induced within it. This e.m.f. can cause a current to flow inside the body of the metal, known as an eddy current. The resistance inside the solid metal may be quite small, which means that the eddy currents might be quite big. The eddy currents will cause their own magnetic effects and heat effects – both of which can be useful – but they can also cause problems.

If a cylindrical copper or aluminium can (both non-magnetic metals) is pivoted so that it is free to spin about its axis, and then a magnet is spun round inside the can, the can will start to spin with the magnet. The magnet generates eddy currents within the metal of the can which in turn create an electromagnetic field which opposes the turning magnet. The reaction force spins the can. This is used in the mechanism of a car speedometer. A magnet is attached to a cable which spins at the speed of the wheel rotation. It spins inside an aluminium cylinder which experiences a turning force. The cylinder is prevented from spinning by a light hairspring. The faster the magnet spins, the further the cylinder is turned against the returning force of the spring – and so the amount of turn of the cylinder indicates the speed of the magnet and therefore the road speed of the wheels.

Eddy currents are also used for damping the movement of moving-coil meters – such as ammeters and voltmeters – so that the pointer moves smoothly to the correct reading and settles there, rather than overshooting the reading and oscillating back and forth. The coil of the meter is wound on a metal frame – large eddy currents are generated in this metal frame which oppose the rotation of the coil as it accelerates to the reading position, cutting across the magnetic flux lines of the permanent magnet. The ideal situation is to set the level of damping at the *critical* level – so that it moves quickly to the correct reading but without any overshoot and oscillation.

Both of the above examples are useful applications of the eddy current effect, but they can cause problems, for example in motors, transformers and dynamos. All of these have coils which are wound on iron cores to increase the electromagnetic effect, but the iron metal also experiences flux changes and large eddy currents can be set up which cause a lot of heat within the iron. To cut down the energy lost by overheating in this way, the iron parts are made of a laminated construction, with lots of thin sheets of iron, glued together but separated by thin insulating sheets. This greatly increases the resistance in the direction the eddy currents would normally flow. The next example of an exam question demonstrates this effect.

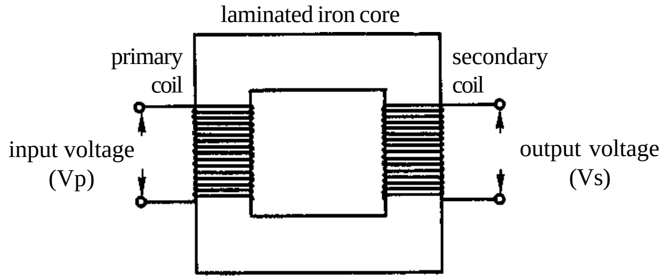
Typical Exam Question:
A solid copper cylinder (a) is suspended from a thread so that it hangs between the poles of a magnet. The thread is twisted so that the cylinder oscillates, spinning alternately in opposite directions. After a few turns, the cylinder slows down and comes to rest. What difference would it make if the cylinder was replaced by a similar-sized pile of 1p pieces as shown in (b)?



As the solid cylinder spins within the flux lines of the magnet, eddy currents are generated within the body of the copper metal.✓
Lenz's law predicts that these eddy currents are in such a direction as to set up magnetic effects which ***oppose*** *the movement causing them – so the spinning cylinder experiences an opposing force which slows it down and stops it relatively quickly. ✓*
If the cylinder is made of 1p pieces, the small gaps between the coins with tiny particles of dirt and grease increase the electrical resistance of the cylinder in the vertical direction. The eddy currents can no longer flow easily up and down the cylinder, and the damping effect is greatly reduced.✓ The pile of coins will oscillate for much longer than the solid copper cylinder. ✓

Transformers

A transformer uses the principles of electromagnetic induction to change or transform an alternating input voltage to an alternating output - which may be higher or lower than the input voltage.



The device consists of two coils of wire, called the primary (input) and secondary (output) coils – wound on the same iron core. For reasons explained above, the iron core is made of laminated iron to cut down heat energy loss due to eddy currents. The ratio of the output voltage to the input voltage is determined by the ratio of the number of turns on the two coils. In an ideal transformer (one where the energy loss due to eddy currents, heating in the coils and so on has been reduced to zero):

$$\frac{\text{secondary (output) voltage}}{\text{primary (input) voltage}} = \frac{\text{number of turns in the secondary coil}}{\text{number of turns in the primary coil}}$$

As the names suggest, a 'step-up' transformer is one which increases the voltage; a 'step-down' transformer decreases the voltage. As in the rest of physics, you don't get something for nothing – just as the voltage is increased, the current is decreased, and vice versa, so that the total power (= VI) remains costant.

Typical Exam Question:
A step down transformer is used to reduce a 240V a.c. supply to 12V a.c. to power a model racing car with a motor rated at 24W.
(i) What is the turns ratio of this transformer ?
(ii) What current flows in the primary coil ?
(iii) What assumption did you make in answering (ii) ?

$\frac{N_p}{N_s} = \frac{N_p}{N_s}$ *so the turns ratio* N_p/N_s *= 240 /12 = 20:1 primary:secondary* ✓

Current in secondary coil is given by I = P/V = 24/12 = 2A. Current in the secondary coil must be 20 times that in the primary coil, so current in the primary coil = 2/20 = 0.1A. ✓
The ***current calculation above assumes that the transformer is 100% efficient; i.e. power input = power output****, and therefore that the current increases by the same factor as the voltage decreases. (In practice this is very unlikely – all practical transformers will lose some energy in the process).*

Questions

1. (a) Write down the formula for the emf induced when a conductor of length *l* cuts across lines of magnetic flux at speed *v* **[1]**

   (b) An aeroplane with a wingspan of 15m flies horizontally at a speed of 200ms^{-1}. If the Earth's vertical component of magnetic flux density is 5.2×10^{-5} T, calculate the emf induced between its wingtips. **[2]**

2. A solenoid is connected to a sensitive galvanometer. A bar magnet is pushed quickly into the solenoid and then removed at the same speed.
   (a) What would you expect to see happen on the galvanometer? **[4]**

   (b) Explain these observations. **[5]**

   (c) The action is now repeated at a higher speed. What difference you would expect to see in the response of the galvanometer? Explain your answer. **[2]**

3. (a) What is meant by the term 'eddy currents'? **[3]**

   (b) State one application and one disadvantage of eddy currents. **[2]**

   (c) A solid aluminium plate is allowed to oscillate like a pendulum between the poles of a magnet. Explain why the vane slows down and stops after a very few swings. **[2]**

   (d) The plate is replaced by one with slots cut across it. State and explain what difference this would make to the oscillations. **[2]**

4. A bicycle wheel is mounted vertically on a metal axle in a horizontal magnetic field. Sliding contacts are made to the metal rim of the wheel and the axle. The wheel is set to rotate freely.

   (a) (i) Explain why an emf is generated between the contacts. **[2]**

   (ii) State two ways in which this emf could be increased. **[2]**

   (b) A small light bulb is connected between the contacts. State and explain what you would
   observe about:

   (i) The light bulb **[2]**

   (ii) The rotation of the wheel **[2]**

   (c) A second wheel is mounted in exactly the same way, at the same orientation to the field and rotated at the same angular speed. The emf measured is ¼ that of the first wheel. What can you infer about this wheel? **[3]**

Exam Workshop

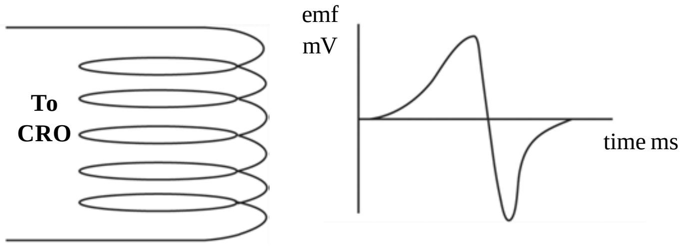
This is a typical student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

(a) State Lenz's Law of electromagnetic induction. [2]

If a wire cuts magnetic field lines, a current is induced in the opposite direction. ✗ **0/2**

A current only flows when there is a complete circuit: it is an **emf** which is induced. Opposite direction to what? The emf is in such a direction as to oppose the movement causing it; sounds long-winded but it is important to be precise.

(b) A magnet is dropped so that it falls vertically through a coil of wire. An oscilloscope connected to the coil shows the emf induced in the coil:



Explain the following features of the trace:

(i) The positive peak [3]

The emf goes up as the magnet goes into the coil ✓ **1/3**

Not enough information for three marks – explanation needed

(ii) The negative peak [2]

The emf goes down as the magnet goes out of the other end of the coil ✗ **0/2**

It doesn't 'go down' – it increases in the other direction. A negative emf is induced. More detail needed to explain why it is in this direction (Lenz's law again – the emf opposes the motion causing it, in this case it opposes the magnet leaving the coil)

(iii) The relative magnitudes of the two peaks [2]

The negative peak is bigger than the positive peak ✓ **1/2**

Correct, but an explanation needed for the marks – note that the question asks you to **explain,** not just describe what happens. The negative peak is bigger because the magnet is moving faster by this stage (since it is accelerating as it falls)

(iv) The duration in time of the two peaks [1]

The negative peak goes on for less time than the positive peak ✗

Explanation needed again – this, too, is because of the acceleration of the magnet under gravity. It is very easy to be fooled by a question in one topic (in this case, electromagnetic induction) which suddenly asks for an answer which seems to come from an earlier (and often easier!) topic. Don't be afraid to think 'outside the box'. Physics (like life, again) doesn't come in convenient packages.

Examiners Answers

(a) The direction of an induced emf ✓ is such as to oppose the change producing it ✓

(b)(i) The positive peak shows the emf induced as the magnet enters the coil ✓ The rate of cutting flux increases as more of the field enters the coil so the emf increases.✓ By Lenz's Law, the current in the coil produces a magnetic field to try and repel the magnet ✓

(ii) As the magnet begins to drop out of the coil the emf becomes negative✓ Again by Lenz's Law, the current in the coil must reverse to 'try' and stop the magnet leaving the coil ✓

(iii) Since the magnet is accelerating under gravity, it is moving faster as it leaves the coil. ✓ The rate at which flux is cut is increased, so the emf, shown by the size of the negative peak, is greater ✓

(iv) The magnet is moving faster as it falls out of the coil, the duration of the negative peak is shorter than that of the positive peak ✓

Answers to Questions

1. (a) *emf* = *Blv* ✓ **[1]**
   (b) *emf* = *Blv* = $5.2 \times 10^{-5} \times 15 \times 200$ ✓
   = 0.16 V ✓ **[2]**
   Remember to give the unit in the final answer

2. (a) Galvanometer pointer deflects as magnet moves in ✓
   Returns to zero as magnet stops ✓
   Deflects in opposite direction ✓
   same magnitude as magnet removed ✓ **[4]**
   (b) Solenoid cuts the flux of the magnet ✓
   Induces emf across solenoid, producing current in galvanometer ✓
   When magnet stops, rate of cutting flux is zero, emf falls to zero ✓
   When magnet removed, emf is reversed (Lenz's Law) ✓
   Same rate of cutting flux so magnitude of emf is the same ✓ **[5]**
   (c) The deflection of the galvanometer will be greater in each direction ✓
   Flux is being cut at a faster rate ✓ **[2]**

3. (a) Currents induced ✓ in a piece of metal due to changing flux or movement through a magnetic field ✓
   Eddy currents circulate around paths of least resistance in metal ✓**[3]**
   (b) Eddy currents can be large and cause the metal to become hot.
   Application: inductive heating e.g. for melting metals ✓
   Disadvantage: energy loss in core of wound components e.g. transformers ✓ **[2]**
   (c) Kinetic energy of pendulum is converted into heat energy ✓ by eddy currents induced in the plate. Energy loss causes pendulum to slow down ✓ **[2]**
   (d) Oscillations last for longer time ✓
   Eddy currents are smaller due to slots increasing resistance, so there is less energy loss✓ **[2]**

4. (a) (i) The spokes of the wheel cut magnetic flux ✓
   This induces an emf across each spoke, i.e. between the hub and the rim ✓ **[2]**
   (ii) Speed of rotation increase the strength of the field, makes wheel and field more perpendiclar (if not already so). (any two) ✓✓ **[2]**
   (b) (i) The bulb would light ✓ and then go out as the current decreased✓ **[2]**
   (ii) The wheel would slow down ✓ as energy is transferred to heat and light in the bulb ✓ **[2]**
   (c) Since emf is ¼, the rate of cutting flux must be ¼ that of the first wheel ✓ This is B × area swept out per second ✓ B and rate of rotation are the same, so area of overlap of wheel and field wheel must be ¼ i.e. radius is halved ✓ **[3]**

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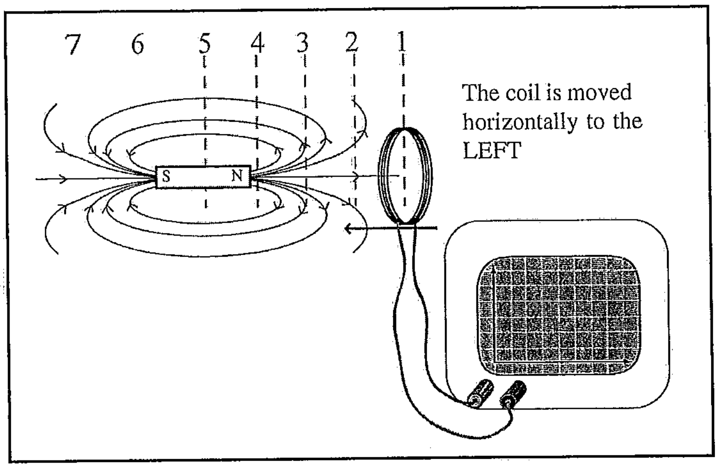
Graphical work with Electromagnetic Induction

To revise our ideas on electromagnetic induction look at Fig 1, showing a bar magnet and a coil connected to an oscilloscope (to display potential differences). As the coil is moved (at constant speed) horizontally to the left, a trace is displayed showing that an e.m.f. is induced in the coil. This e.m.f. is usually denoted by the letter ε . It arises due to the relative motion between the coil and the magnetic flux.

Fig 1



There are two ways to look at this :-

(i) as the coil moves through positions 1,2 and 3 *it cuts the lines of flux.*

(ii) as the coil moves, it has a *changing* amount of flux *threading* the coil.

> *It is this change in flux that is most important. You might have to use either of these two ideas.*

Faraday's Law says 'the induced e.m.f. is equal to *the rate of change of flux'*.

The equation used is $\varepsilon = \frac{\Delta\Phi}{\Delta t}$

where Φ is the amount of flux (measured in webers).

> ***Exam Hint:-*** *You should be able to use Faraday's law to explain how the size of the emf depends on the rate of change of flux.*

In a way you can say that the size of the e.m.f. depends on the relative speed between the magnet and coil providing that lines of flux are cut.

Now imagine the coil at position 5.

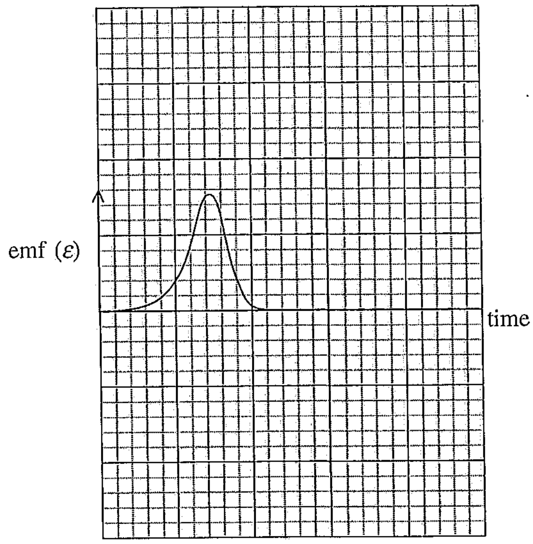
(i) Moving through position 5, the coil does ***not*** cut any flux. We conclude that there is no induced e.m.f. (at position 5)

(ii) Or, there *is flux threading* the coil but this flux is *not changing* at position 5.

Again we conclude that there is no induced e.m.f. (at position 5)

Fig 2 shows the trace on the oscilloscope as the coil moves from position 1 through to 5.

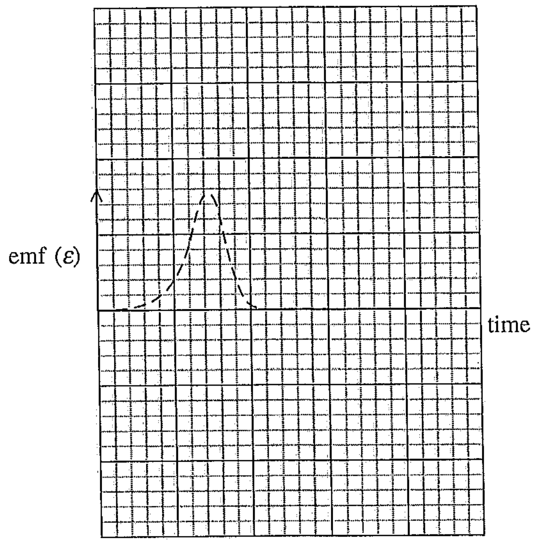
Fig. 2



Qu 1.
Now complete the trace as the coil passes completely over the magnet.

Qu 2.
On Fig 3, show the trace you would expect to obtain if the coil moved TWICE as fast across the magnet. (The original trace is shown to help you.) HINT - remember that if the speed is doubled then the rate of change of flux is doubled, what effect will this have on the induced e.m.f. ?

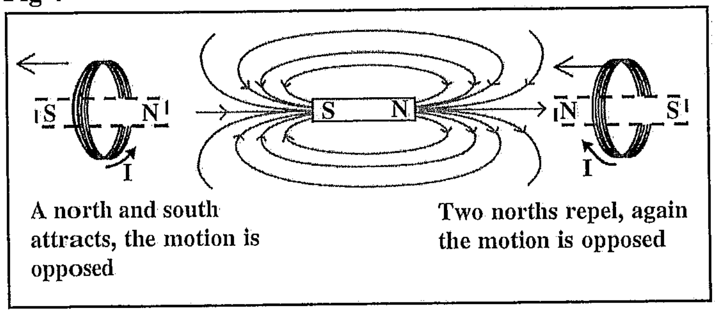
Fig. 3



> *When the speed changes,*
> *the emf changes,*
> *the shape of the pulse changes,*
> *and the separation between the pulses changes.*

Lenz's Law tells us that the *direction* of the induced e.m.f. (and current) is always in such a direction as to *oppose the change* producing it.

Fig 4



Look at Fig 4. It shows the induced current in the coil and how it makes the coil behave like a small bar magnet. In both cases the induced current is in a direction to *oppose* the motion producing it.

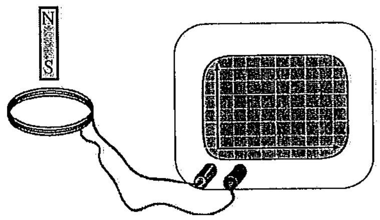
This is the reason for the minus sign in the equation . $\varepsilon = -\frac{\Delta\Phi}{\Delta t}$

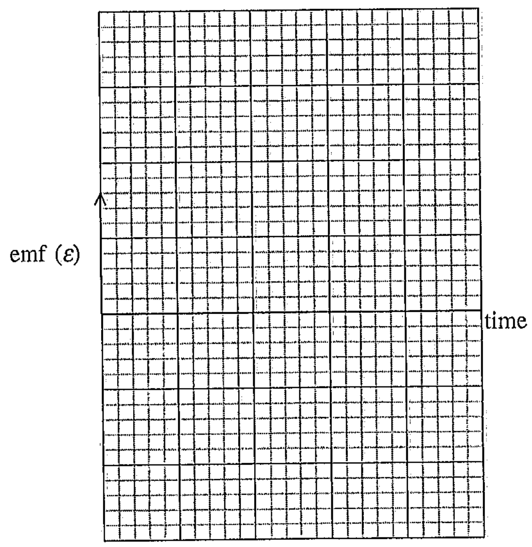
It means that in the two questions you did in Fig 2 and Fig 3, the second 'blip' of e.m.f. is negative and should be below the horizontal line (time axis).

> ***Exam Hint:-*** *You should be able to use Lenz's law to explain why two emfs similar to this are in opposite directions.*

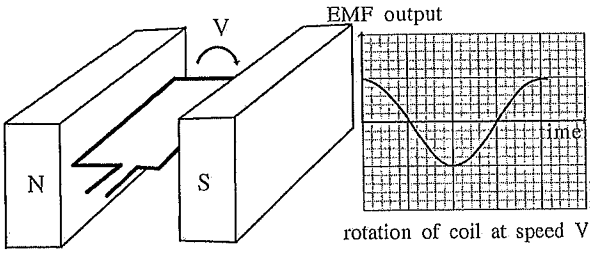
Qu 3.
Fig 5 shows a long bar magnet about to be dropped through a fixed coil. Think about how the magnet falls vertically and that it is 'long'. Draw a sketch of the trace you would expect.

Fig 5.

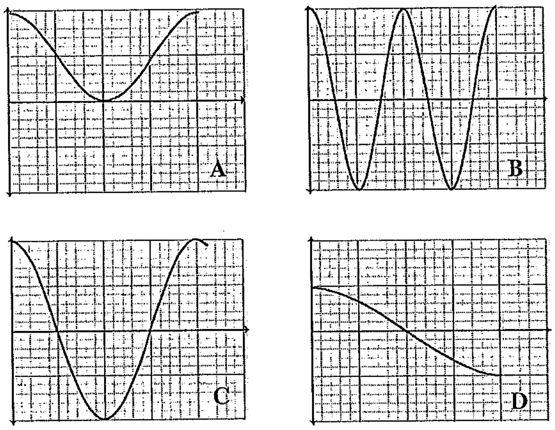




Qu 4.
Fig 6. shows the rotating coil of a generator between the poles of a magnet together with the emf output.



The speed of rotation is doubled from V to 2V.
Which of the options A, B, C or D best shows the new output?



> ***Exam Hint:-*** *There are two changes here. You may spot one but the examiner is looking for two.*

Qu 5
Fig 7. shows a rotating axle whose speed of rotation is to be found. Four magnets are sunk into the axle as shown and each in its turn sweeps past a coil connected to an oscilloscope. The emf induced is recorded as shown in Fig 8.

Fig 7

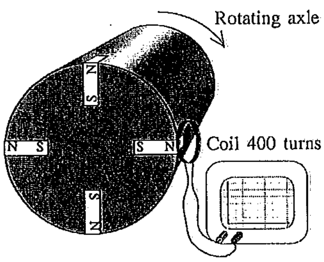
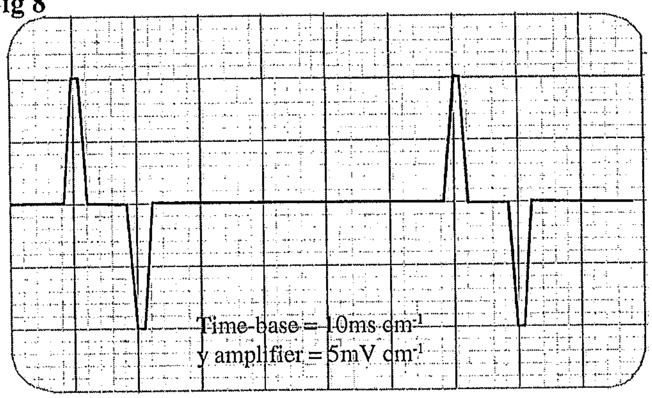


Fig 8

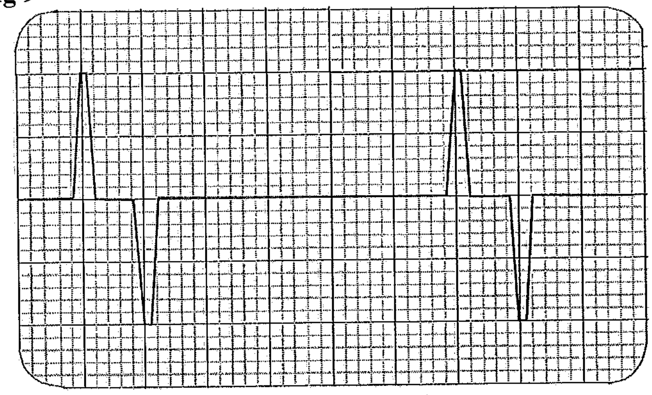


(a) (i) From Fig 8, find the time for one quarter of a revolution and
(ii) find the number of revolutions made in one minute.

(b) (i) Use Faraday's law to explain how the voltage pulses are produced.
(ii) The coil has 400 turns. Calculate the maximum rate of change of flux through the coil.

(c) The speed of rotation is now increased by 50%. On figure 9, sketch the new trace you would expect to see. (The original trace is shown to help you)

Fig 9



Time-base = 10ms cm^{-1}
y amplifier = 5mV cm^{-1}

> ***Exam Hint:-*** *Examiners want you to understand graphs like this and make predictions when changes (like speed) are made.*

We don't always need a permanent magnet moving relative to a coil. We can produce a magnetic field by sending a current through a coil (called a solenoid). Switching the current on and off will give the necessary *change of flux* to produce an emf .

Another way of *changing the flux* is to *change the current* in a solenoid as shown in Fig 10.

The signal generator doesn't just change the frequency, it can change the waveform as well.

Fig 10

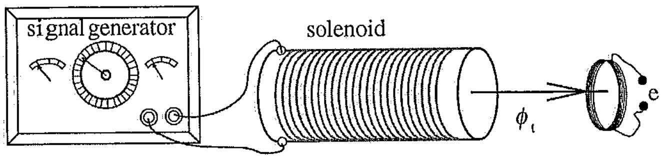
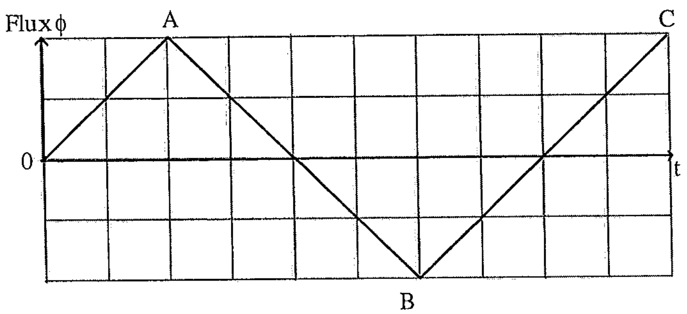


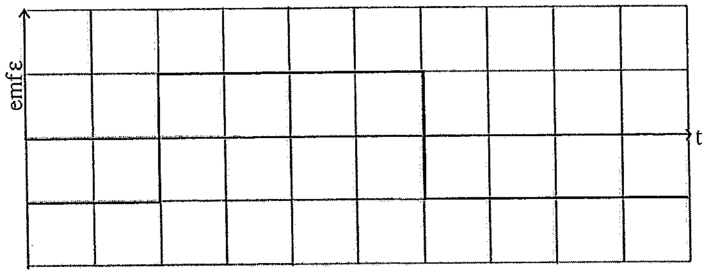
Fig 11 shows a triangular waveform for the flux ϕ. We have to find the emf ε in the small coil.



Always remember that on a ϕ-t graph, the slope (or gradient) is $\frac{\Delta\phi}{\Delta t}$ and because we know that $\varepsilon = -\frac{\Delta\phi}{\Delta t}$ it means that the induced emf is (-1) × the gradient on a ϕ - t graph.

So, from O to A and B to C, the slope is constant and positive meaning that the emf is constant (and *negative*). From A to B, the slope is constant (and negative) giving a constant (positive) emf.
This is shown in Fig 12.

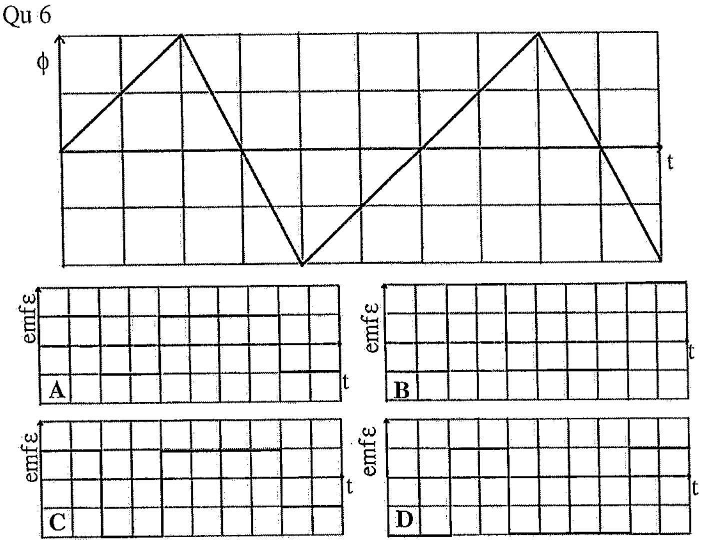
Fig 12



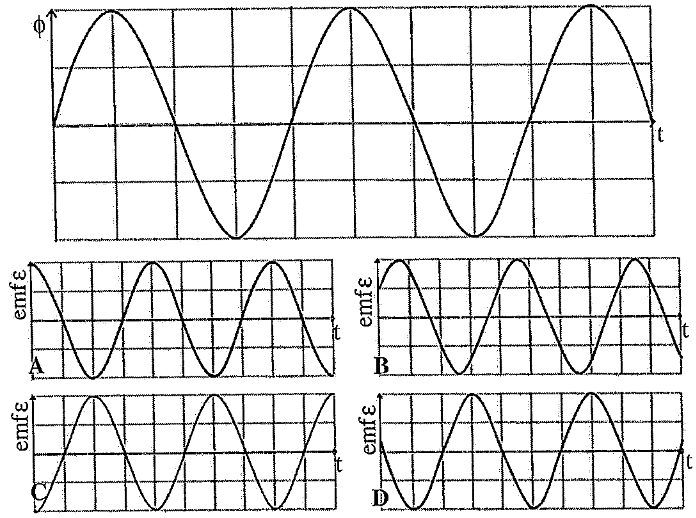
> *Although you could be asked to do a calculation for frequency or maximum emf, you should concentrate on the shape of the waveform.*

Questions 6 and 7:
The diagrams show two different wave forms of flux ϕ against time t. For each one, decide which option A B C or D best shows the emf ε against time t.

Qu 6



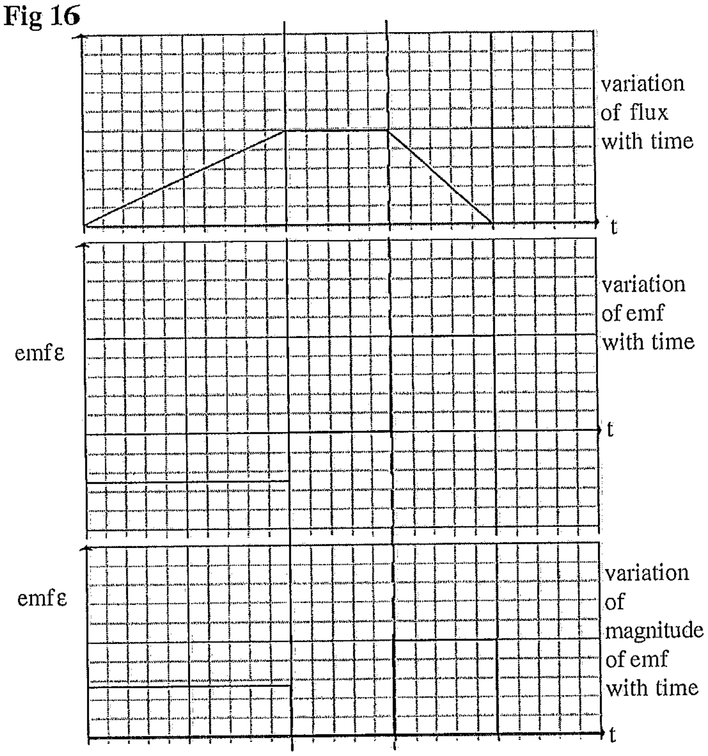
Qu 7



Exam Hint:- Sometimes examiners will ask for the magnitude of the emf, that is the size of the emf with no regard to its sign.

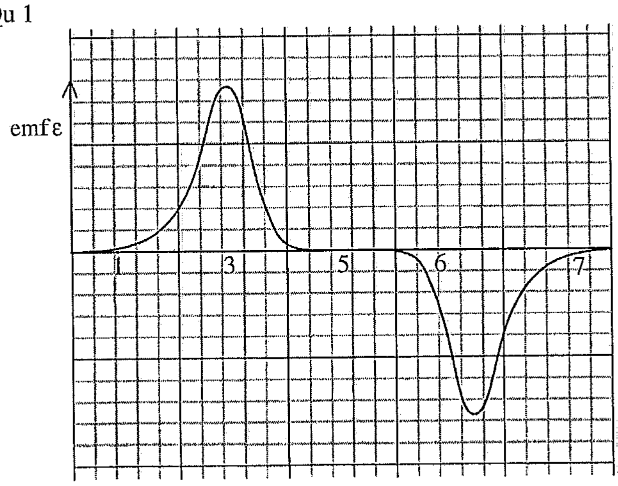
Fig 16 clearly shows the difference between the emf and the *magnitude* of the emf.

Fig 16



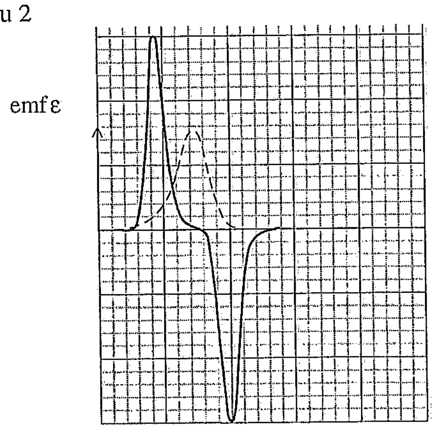
Answers

Qu 1



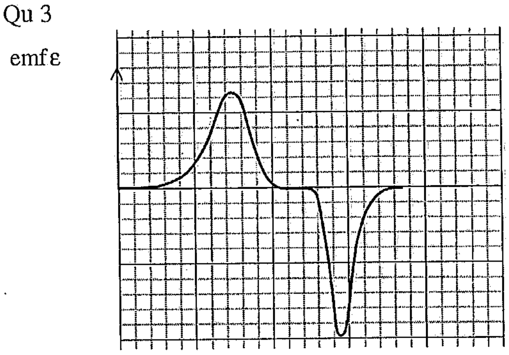
Because the coil is moving at the same speed, the pulse is of the same width and 'height'. It is inverted because of the opposite polarity (first a north then a south).

Qu 2



The coil moves at twice the speed so each pulse height is doubled. The time is halved so each pulse width is halved and two pulses take the same time as one of the 'slower' pulses.

Qu 3



The magnet *accelerates* from rest. The south enters the coil slowly and the north leaves more quickly. This makes the first 'pulse' of smaller height but a 'longer' base.
(Because flux at each pole is the same in quantity, you may like to show that the *area* of each 'pulse' is the same.)

Qu 4
Option B is correct.
Because the speed of rotation is doubled the maximum emf is doubled. Also, the frequency is doubled meaning the time period is halved. Two complete oscillations now occur in the time taken earlier by one.

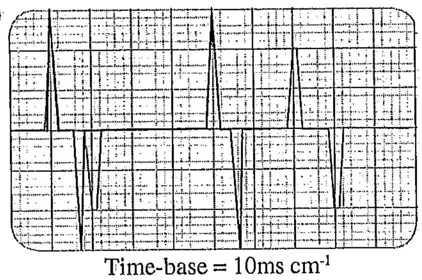
Qu 5
(a) (i) From the graph, a quarter revolution takes 6cm at 10ms cm^{-1}. So time for ¼ revolution = 60 ms

(ii) Time for one revolution is 60ms × 4 = 240ms = 0.24s. One minute is 60s, so the number of revolutions in one minute is 60/0.24 = 250 rpm

(b) (ii) Maximum emf is 2cm at 5mV cm^{-1} ie 10mV. Coil has N = 400 turns so use

$$\varepsilon = -N \times \frac{\Delta\phi}{\Delta t} \qquad 10 \times 10^{-3} = 400 \times \frac{\Delta\phi}{\Delta t} \qquad \frac{\Delta\phi}{\Delta t} = 2.5 \times 10^{-5}\ \text{Whs}^{-1}$$

(c)



Time-base = 10ms cm^{-1}
y amplifier = 5mV cm^{-1}

The maximum emf increases by 50% (2cm to 3 cm). The time between pulses is reduced by *one third* from 6cm to 4cm. (The width of each pulse is also reduced but difficult to display!!)

Qu 6: B

Qu 7: C

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Transformers

A transformer changes (or transforms) voltages which are alternating. Fig 1 shows the elements of a transformer; two separate coils wound on a soft iron core. One coil, the **input** or **primary**, is fed with an alternating potential difference. The other coil, the **output** or **secondary**, supplies an alternating potential difference of greater or smaller value. There is no direct electrical connection between the two coils, so how does electricity get from one coil to the other? The answer lies in the magnetic field that both coils share.

Fig 1. Transformer

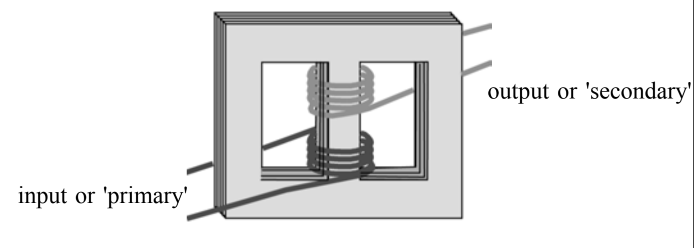
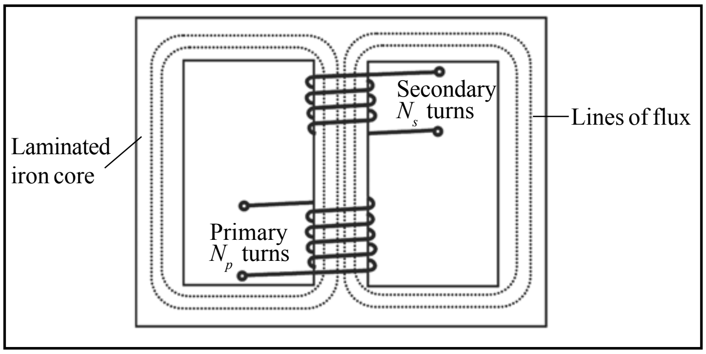


Fig 2 shows a simplified transformer diagram with lines of flux passing through both coils. When there is an alternating current in the primary coil, the flux produced is also alternating and this changing flux also passes through the secondary coil. Recall what you have learnt about induced emfs and the rate of change of flux; apply this to the secondary coil:- 'Because there is always flux changing in the secondary, there is always an (alternating) emf induced in it.'

Fig 2 Simplified transformer



Turns and turns ratio

The number of turns on each coil is important; this information enables us to calculate the output or secondary voltage.
If the number of turns on the primary and secondary are denoted by N_p and N_s then three cases arise. (i) $N_s = N_p$ (ii) $N_s > N_p$ and (iii) $N_s < N_p$.

(i) If $N_s = N_p$ then the alternating voltage induced in the secondary equals the alternating voltage applied to the primary. ie $V_s = V_p$.(this is a rare use of transformers)
(ii) If $N_s > N_p$ then the alternating voltage induced in the secondary is greater than the alternating voltage applied to the primary. ie $V_s > V_p$. In this case where the pd is increased, the transformer is described as being 'step up'
(iii) If $N_s < N_p$ then the alternating voltage induced in the secondary is less than the alternating voltage applied to the primary. ie $V_s < V_p$. Here the voltage is reduced and the transformer is described as 'step down'.

We shall show later that the ratio of the turns is the same as the ratio of the voltages.

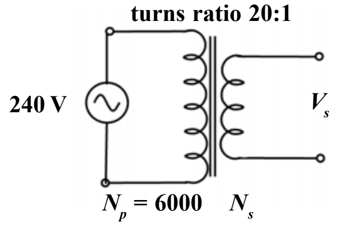
$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

where: N_s *= number of turns on secondary*
N_p *= number of turns on primary*
V_s *= voltage on secondary*
V_p *= voltage on primary*

The ratio $\frac{N_s}{N_p}$ *is called the turns ratio.*

When the turns ratio is **greater** than one we have a **step up** transformer; if **less** than one it is a **step down** transformer.

Example 1. A mains transformer accepts 240 volts and has a turns ratio of $\frac{1}{20}$***. There are 6000 turns on the primary.***



Calculate (i) the number of turns on the secondary
(ii) the secondary voltage.

(i) turns ratio $= \frac{N_s}{N_p}$

$$\frac{1}{20} = \frac{N_s}{6000}$$

$$N_s = \frac{6000}{20} = 300$$

(ii) $\frac{N_s}{N_p} = \frac{V_s}{V_p}$

$$\frac{300}{6000} = \frac{V_s}{240}$$

$$V_s = 240 \times \frac{300}{6000}$$

$$V_s = 12\ V$$

This is an example of a step down transformer. The secondary voltage has been reduced because there are fewer turns on the secondary coil.

Both the primary and secondary voltages are always alternating. In studying ac voltages a distinction is drawn between root mean square (r.m.s) values and peak values. In almost all transformer questions we use r.m.s values and this should be assumed unless otherwise stated. In this question the 240V and 12V are both r.m.s.

Power and Energy

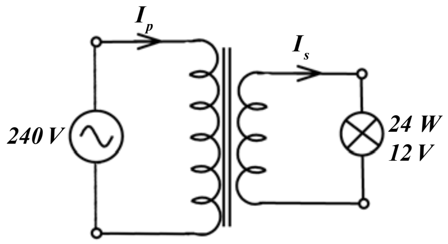
The main reason for using r.m.s values is to enable us to do power calculations. Your early studies of electricity were using direct current (dc) where you used three equations for power:-

$$P = V \times I \qquad P = I^2 \times R \qquad P = \frac{V^2}{R}$$

The same equations can be used with ac providing we use r.m.s values for current I and voltage V.

Now think of a transformer as a sort of machine. Energy is fed in at the primary and taken out at the secondary. The question is 'how efficient is a transformer?' The good news is that they are almost 100% efficient and this makes calculations easy.

Example 2. A transformer is connected to a lamp rated at 12V and 24 W. Calculate (i) the secondary current I_s and (ii) the primary current I_p.



(i) at the secondary,

$P = V_s \times I_s$

$24 = 12 \times I_s$

$I_s = 2A$

(ii) We now take the transformer to be 100% efficient. That is, if 24 watts are taken from the secondary, 24 watts are supplied to the primary. N.B. the 24 watts at the primary are supplied at a high voltage of 240V.

At the primary,

$P = V_p \times I_p$

$24 = 240 \times I_p$

$I_p = \frac{24}{240}$

$I_p = 0.1\ A$

Notice what has happened here, the secondary voltage has gone down but the secondary current has gone up. So, at 100% efficiency, there are no power losses,the primary power equals the secondary power.

At 100% efficiency $V_p \times I_p = V_s \times I_s$ *where:* V_p *= voltage on primary*, I_p *= current on primary*, V_s *= voltage on secondary*, I_s *= current on secondary*

In this example, the value of primary current is only one twentieth that of the secondary.

Of the two coils, which do you think will have the thinnest wire, the primary or the secondary?

Exam Hint: *In transformer calculations, remember that the currents are determined by the load in the* ***secondary****.*
It is a good idea to look first at the secondary ***and then*** *the primary.*

Transmission of electrical energy

One of the most important uses of transformers is to distribute energy from power stations to the consumer. To illustrate how this works we will compare two cases: (i) without a transformer and (ii) with a transformer. Suppose the generator develops 100 000 W at 5000 V.

(i) Without a transformer:

$P = V \times I$

$I = \frac{P}{V}$

$I = \frac{100{,}000}{5000}$

$I = 20\ A$

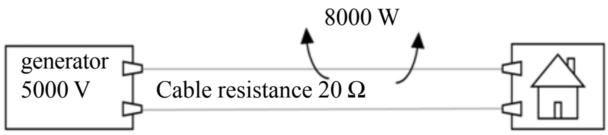
Suppose further that the total cable resistance is 20 Ω. This resistance will dissipate the usual Joule heating (I^2R).

$P = I^2 \times R$

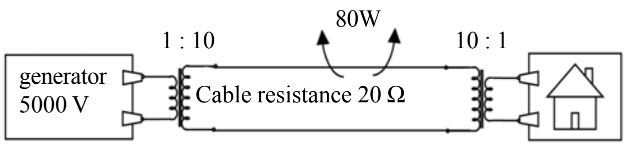
$P = 20^2 \times 20$

$P = 8000\ W$

This energy is lost into the atmosphere



(ii) With a transformer:
A better way is to use a step up transformer at the generator and a (similar) step down transformer at the consumer.



If the transformer at the generator has a turns ratio of 1:10, the secondary voltage will be 50 000V. From equation 2, the secondary current is now 20 ÷ 10 = 2.0 A. This much smaller current is in the same cables of resistance 20 Ω. The power dissipated is calculated as before.

$P = I^2 \times R$

$P = 2^2 \times 20$

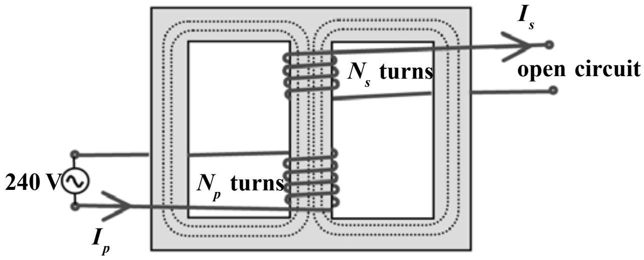
$P = 80\ W$

This is only 1% of the former value! To complete the process, a step down transformer close to the consumer brings the voltage down to the original 5000 V.

Heat losses in a resistor depend on (current)^2.
So if the current is reduced by a factor of 10, then the losses are reduced by a factor of $(10)^2$ - the heat loss is small when the current is small.
In transformers a ***small*** *current appears with a* ***high*** *voltage.*
This is the reason for using pylons carrying cables at 132000 volts or more.

Transformer theory

The transformer below has the primary connected to an ac supply of 240 volts, the secondary is not connected.

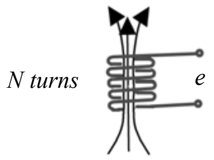


(i) What is the secondary current I_s?
and , remembering that it is connected to 240 V,
(ii) what is the current in the primary I_p?

The answer to part (i) is simple;
even if there is an induced emf, there isn't a complete circuit and so the current I_s is zero.

In part (ii) we see 240 volts connected to some turns of copper wire the resistance of which is a few ohms. A low resistance should lead to a large current. The amazing thing is that the current I_p is very small and in many cases is neglected altogether. To understand why this is so we must use Faraday's law of electro-magnetic induction.

Faraday's law of electro-magnetic induction
If there is a changing flux threading a coil of N turns, the induced emf is given by $e = -N \times \frac{d\phi}{dt}$



$$e = -N \times \frac{d\phi}{dt}$$

In what follows, the minus sign is not important and is omitted for simplicity.

The secondary with N_s turns is threaded by an alternating flux ϕ and this gives us the useful secondary voltage V_s.

In this case, $V_s = N_s \times \frac{d\phi}{dt}$ *equation 3*

Do not forget that the same flux threads the primary of N_p turns.

The induced emf is $N_p \times \frac{d\phi}{dt}$

There are now two sources of p.d. in the primary, the applied p.d. V_p and the induced emf, $N_P \times \frac{d\phi}{dt}$

Because the primary current is practically zero, this emf is balanced by the applied voltage V_p,

ie $V_p = N_p \times \frac{d\phi}{dt}$ *equation 4*

Dividing equation 3 by equation 4 gives:

$$\frac{V_s}{V_p} = \frac{N_s \times \frac{d\phi}{dt}}{N_p \times \frac{d\phi}{dt}}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \text{turns ratio}$$

Losses in a transformer

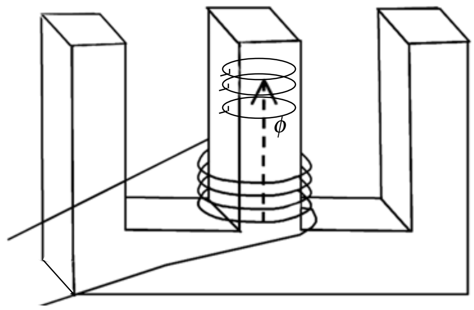
The small losses that occur in a transformer arise from several causes:

- **Flux losses** - This happens when some lines of flux from the primary do not thread the secondary. From the shape of the core in fig 1 and 2 you can see that this loss is very small.

- **Copper losses -** This is the usual Joule heating (I^2R) that occurs in the copper coils having resistance R. To reduce this energy loss the resistance has to be reduced by using thicker wire. There is an economic choice here, the cost of thicker wire or heat loss.

- **Iron losses** - There are two types.
  (a) **Hysteresis losses** - This is because energy is constantly being used to magnetise and demagnetise the iron core as the current alternates. It is reduced by using 'soft' iron in the core. 'Soft' means magnetically soft, easy to magnetise and demagnetise.

  (b) **Eddy current losses** - The diagram below shows a solid iron core without a secondary.



The alternating flux will induce currents in the iron core itself. These are called 'eddy currents.' If the core is in the form of a solid chunk of iron, the resistance offered will be very small and large currents will flow. To prevent this large loss of energy, the core is made from many thin sheets (laminae) each insulated from each other. You will see 'soft iron laminations' in other components where a.c. is used

Efficiency

Although transformers are close to 100% efficient, we may have to use the equation for **efficiency** to get a realistic value for current in the secondary.

As a rule, when calculating efficiency we always use energy or power. The efficiency of any device is given by:

$$\text{efficiency} = \frac{\text{power output}}{\text{power input}}$$

For a transformer, the equation becomes:

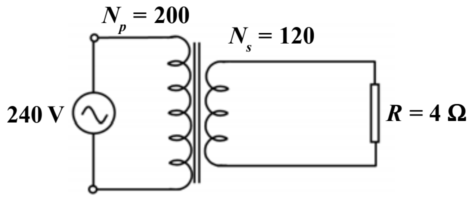
$$\text{efficiency} = \frac{V_s \times I_s}{V_p \times I_p} \times 100\%$$

Exam Hint:
1. *Remember effciency cannot be greater than 100% - if you find a value that seems to be, go back and check*
2. *You must use **power** (or energy) in the efficiency equation, not voltage or current.*

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's mark scheme is given below.

A transformer has a primary of 2400 turns and a secondary of 120 turns. The secondary has a load of 4 Ω and the primary is connected to 240 volts a.c.



For the first part of this question you may assume the transformer to be 100% efficient. Calculate:

(i) the turns ratio

$turns\ ratio = \frac{2400}{120}$ ×

> The candidate is almost right.
> The turns ratio $= \frac{N_s}{N_p} = \frac{120}{240} = \frac{1}{20}$

(ii) the secondary voltage

$\frac{120}{2400} = \frac{V_s}{240}$ ✓ $\frac{120}{2400} \times 240 = V_s$ $V_s = 12\ V$ ✓

> Full marks. The candidate has correctly recognised that the ratio of the turns is the corresponding ratio of the voltages. The calculation is correct.

(iii)the secondary current

$I = \frac{V}{R}$ $I = \frac{12}{4}$ ✓ $I = 3\ A$

> Correct calculation and unit for current

(iv) the secondary power

$power = volts \times amps = 12 \times 3 = 36\ W$ ✓

> Correct calculation and unit for power

(v) the primary current

$primary\ current = \frac{V_p}{R} = \frac{240}{4} = 60\ A$ ×

> This is incorrect, the candidate has used the primary voltage (240V) with resistance R = 4 Ω in the secondary

Examiner's Answers

(i) turns ratio $= \frac{N_s}{N_p} = \frac{120}{2400} = \frac{1}{20}$

(ii) voltage ratio $= \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{20}$

$\frac{V_s}{200} = \frac{1}{20}$

$V_s = 240 \times \frac{1}{20}$ $V_s = 12\ V$

(iii) secondary current $= \frac{V_s}{R} = \frac{12}{4} = 3\ A$

(iv) secondary power $= V_s \times I_s = 12 \times 3 = 36\ W$

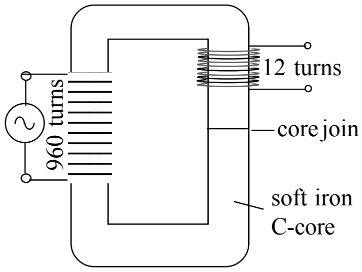
(v) to find the primary current, assume 100% efficiency

$\text{efficiency} = \frac{V_s \times I_s}{V_p \times I_p} \times 100\%$ $\frac{96}{100} = \frac{36\ \text{W}}{\text{primary power}}$

$\text{primary power} = \frac{36 \times 100}{96} = 37.5\ \text{W}$

Typical Exam Question

The figure below shows a demonstration transformer made from two joined soft iron C-cores.



The primary coil is commercially made and consists of 960 turns. A student winds a 'loose' coil of 12 turns on the other limb of the transformer. An alternating supply of 240 volts is connected to the primary.

(i) What voltage would you expect at the secondary ?

(ii) If a 3V bulb is connected to the secondary would the bulb light normally?

(iii) Will the brightness of the bulb be affected by the fact that the secondary coil is not closely wound on the core? Explain your answer.

(iv) The secondary is now moved up and down on the core. How, if at all, will the brightness change now? Explain your answer.

(v) The bulb is rated at 1.2W and is at normal brightness. Calculate the secondary and primary currents in the transformer.

(vi) The place where the two C- cores join is now opened up slightly so that there is a small air gap between the cores. How, if at all, will the brightness change now? Explain your answer.

Answers

(i) $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\frac{12}{960} = \frac{V_s}{240} \Rightarrow V_s = 3\ V$

(ii) The bulb is supplied with the correct voltage and will light normally.

(iii) If the turns of the secondary aren't close fitting to themselves or the core the brightness is not affected. This is because the same amount of (changing) flux links the coil giving the same induced emf.

(iv) No change in brightness. Same reason as (iii).

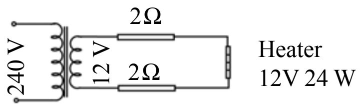
(v) Starting at the secondary,
power = volts × amps
$1.2 = 3 \times I_s$
$I_s = 0.4\ A$

At the primary, we assume 100% efficiency so the same power is delivered at 240V.
power = volts × amps
$1.2 = 240 \times I_p$
$I_p = 0.4\ A$

(vi) When there is a small air gap between the two iron cores, the flux linking the coils will be reduced .This reduction leads to a reduced emf in the secondary and the bulb's brightness will be much reduced.

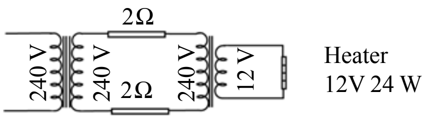
Practice Questions

1. The diagram below show a heater that is supplied by long leads having a total resistance of 4Ω.



The heater is to work off 12volts and consumes 24 watts. Calculate:
(i) the current the heater is designed to take,
(ii) the resistance of the heater,
(iii) the current when the heater is fed by the leads with resistance 4Ω as in Fig A
(iv) the power wasted in the leads.

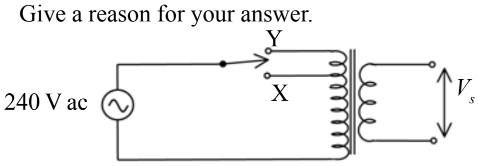
An alternative situation is shown below where a second transformer is used.



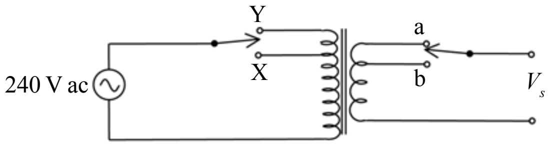
This transformer is assumed to be 100% efficient and supplies 12 volts to the heater. Calculate:

(v) the current in the heater
(vi) the current in the leads
(vii) the power wasted in the leads.

2. (a) The transformer shown has two terminals X and Y close together ('primary tappings').When will the secondary voltage V_s be greatest, using terminal X or Y ?
Give a reason for your answer.

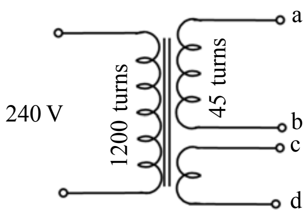


(b) The transformer now shows the secondary with tappings at terminals a and b.



Which pair of terminals gives:
(i) the greatest output voltage?
(ii) the least output voltage?

3. The transformer shown has a primary of 1200 turns connected to an alternating supply of 240V. There are two separate secondaries. The secondary between terminals a and b has 45 turns.



(i) What is the voltage between a and b ?
(ii) The secondary between c and d is to supply 3V, how many turns will be required?
(iii) How should the secondaries be connected to supply 12V?
(iv) How should the secondaries be connected to supply 6V?

4. A college building is to be supplied with 120 kW of power through cable having a total resistance of 0.1Ω. When the voltage at the college end is 240V calculate:

(i) the current in the cable
(ii) the heat loss in the cable.

An inspector suggests supplying the college with 10 000V. When this is done calculate:
(iii) the new current in the cable
(iv) the new heat loss in the cable.
(v) Give a sketch showing how this can be achieved using transformers. Will they be step up or step down? What will be the turns ratio?

5. A transformer with an efficiency of 90% is designed to operate from 20V and supply electrical energy at 240V.
(i) Is the transformer step up or step down?
(ii) What is the turns ratio?
(iii) A 240V lamp rated at 100W is connected to the secondary of the transformer. What will be the current in the primary ?

6. A step down transformer is used to operate a low voltage lamp in the usual way. Consider what would happen if the insulation between the soft iron laminations started to breakdown. What changes would occur in
(i) the transformer core ?
(ii) the secondary current?
(iii) the primary current?

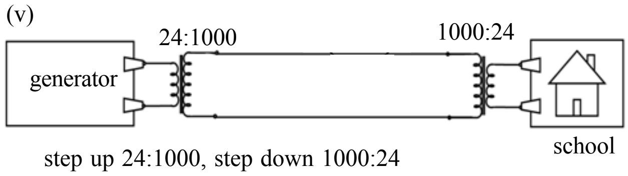
Answers

1. (i) I = 2A (ii) R = 6Ω (iii) I = 1.2 A (iv) P = 5.76W
(v) I = 2A (as part i) (vi) I_p = 0.1A (vii) P = 0.04W

2. In all cases $V_s = V_p \times \left(\frac{N_s}{N_p}\right)$
(a) terminal X to make N_p small.
(b) (i) terminals X and a to make N_p small and N_s large.
(ii) terminals Y and b to make N_p large and N_s small.

3. (i) V_s = 9V. (ii) N_s = 15 turns.
(iii) Add voltages, link b to c. 12V between a and d.
(iv) Subtract voltages, link b to d. 6V between a and c.

4. (i) I = 500A. (ii) 25kW (≈ 20%) (iii) I = 12A.
(iv) 14.4W (negligible).
(v)



step up 24:1000, step down 1000:24

5. (i) step up. (ii) Turns ratio = 12. (iii) 5.55A.

6. (i) Eddy currents in core which now gets hot.
(ii) secondary current unchanged.
(iii) primary current increases to supply extra energy now being dissipated in the core.

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Physics Factsheet

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Calculations – Charged Particles in Electric and Magnetic fields

The purpose of this Factsheet is to revise the basic principles of forces on charged particles in electric and magnetic fields, to give you familiarity with the type of questions asked at A level and to give practice at carrying out calculations.

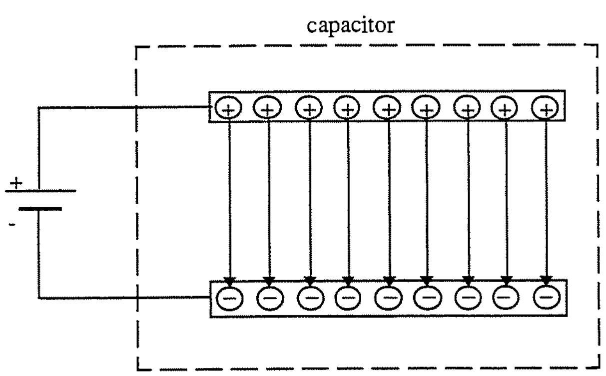
As well as studying this Factsheet, you may find the following Factsheets helpful:
- Factsheet 65 Storing energy (for the energy of a particle in an electric field)
- Factsheet 63 Projectiles (for the motion of particles at right angles to a field)
- Factsheet 19 Circular Motion (for the basic ideas)
- Factsheet 33 Electric Field Strength and Potential (for the field between parallel plates)
- Factsheet 57 Applications of Circular Motion (for the motion of particles in magnetic fields)
- Factsheet 43 Circular Particle Accelerators (for applications of magnetic fields to accelerate particles)
- Factsheet 38 Linear accelerators (for more details of this application)

Electric fields
The field strength of an electric field (the force on a unit charge) is given the symbol E, so the force F_e on a charge q will be $E \times q$.

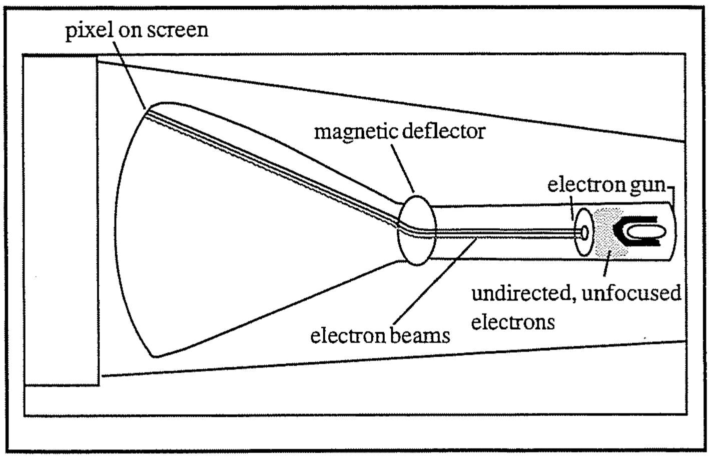
$$F_e = E \times q$$

Parallel plate fields
A type of field frequently encountered is the field between parallel plates with a potential difference between them. Examples occur in dealing with capacitors (see Factsheet 29), accelerators (Factsheet 38) and electron tubes e.g. television tubes.

Field between the plates of a capacitor



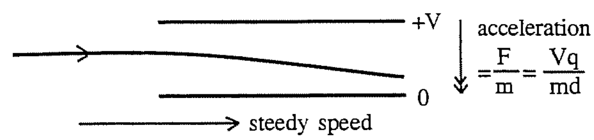
Electric and magnetic fields used to accelerate the electrons and direct the beam in a TV tube.



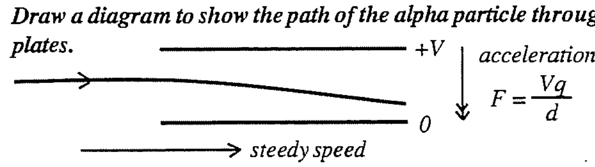
For a uniform field (such as that between 2 parallel plates) the field strength **E** is given by **V/d**, where V is the potential difference between the plates and d the distance between the plates, so the force on the charge is $\mathbf{F_e = Eq = Vq/d}$

For a charged particle in a uniform field:
$$F_e = \frac{Vq}{d}$$

Particle travelling at right angles to the field
If a charged particle of mass m, with charge +q enters the field at right angles to the field lines with speed v, it will experience a force at right angles to its motion. Thus, just like a projectile in the Earth's gravitational field, it will carry on at steady speed in the forward direction, but experience an acceleration in the direction at right angles. This will cause it to describe a parabolic path.



Worked example
An alpha particle enters the field between parallel plates 5mm apart; the bottom of which is held at 0V, while the top plate is at 300V.
a) Draw a diagram to show the path of the alpha particle through the plates.



Since the alpha particle is positively charged the force is away from the positive plate.
b) Calculate the force on the alpha particle.
Charge on electron = 1.6×10^{-19}C Force vertically = Vq/d
The alpha particle has a positive charge of twice that of the electron, so the force is $300 \times 3.2 \times 10^{-19}/0.005 = 1.92 \times 10^{-14}$N

Question

a) Calculate the vertical acceleration on an electron which enters parallel plates, 10cm long, separated by 5mm with a P.D of 500V across them, along the central horizontal axis.

b) Calculate how long it would take the electron to traverse the 2.5mm to the positive plate.

c) Show that if the electron enters the plates at 300ms^{-1}, it will not emerge from the plates, but will collide with the positive plate first.

$M_e = 9.11 \times 10^{-31}kg$

$Q_e = 1.6 \times 10^{-19}C$

Answers

a) $F = \frac{Vq}{d}$,

so acceleration $= \frac{Vq}{md} = \frac{500 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31} \times 0.005} = 1.76 \times 10^{16}\ ms^{-2}$

b) $s = ut + ½at^2$

where $u = 0$ (vertically), $a = 1.76 \times 10^{16}ms^{-2}$, and $s = 0.0025m$

$t^2 = \frac{2 \times 0.0025}{1.76 \times 10^{16}} = 2.84 \times 10^{-19}$

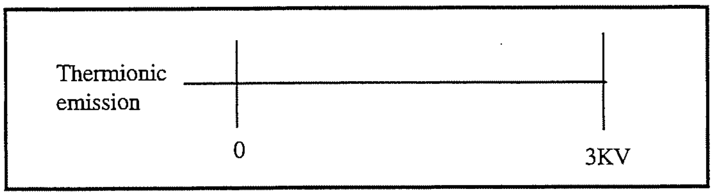
$t = 5.3 \times 10^{-10}s$

c) *Horizontally the electron travels at steady speed, so the time taken to go though the plates would be:*

$t = \frac{0.1}{300} = 3.33 \times 10^{-4}s$

This is much longer than the time taken to arrive at the positive plate, so the electron will hit the plate rather than emerging from between the plates.

Particle travelling parallel to the field



Electrons can be emitted from an electrode heated by a 6V heater – this is **thermionic emission.** If a parallel plate is maintained at a positive potential of 3kV, then the electrons will experience a force **qV/d**, which can accelerate the electrons through a hole towards the positive plate.

The energy, in Joules, gained by an electron can be calculated from **force × distance** and this energy will be as K.E. of the electron.

K.E. gained = (qV/d) × d

$= qV = 1.6 \times 10^{-19} \times 3 \times 10^3 = 4.8 \times 10^{-16}$ J

For a charge accelerated through a P.D. of V, then the K.E. gained is given by: K.E. = qV

If the particle is an electron, then the charge is e (1.6×10^{-19} C), so K.E. gained in J is **e × V**. To avoid having to multiply by the factor of 1.6×10^{-19} all the time, a new unit is introduced called **eV(electronvolt)**. Energy in eV is the energy gained by an electron when it is accelerated through a P.D. of 1V.

- *The energy unit eV (electronvolt) is the energy gained by an electron when accelerated through a P.D. of 1V.*
- *1eV = 1.6 ´ 10^{-19}J*

This principle is used in particle accelerators.

The speed of the electron can be calculated from **K.E. = ½ m v^2**

$½mv^2 = eV$

$v^2 = 2 \times e \times V / m$

$v = \text{sqrt } 2eV/m$

Worked example

An electron is accelerated through a P.D. of 5kV

a) What is its increase in K.E. in eV?

b) Calculate the K.E. in Joules.

c) Calculate the speed of the electron.

Answers

a) $K.E = 5keV$

b) $M_e = 9.11 \times 10^{-31}kg$

$K.E. = 5 \times 10^3 \times 1.6 \times 10^{-19} = 8 \times 10^{-16}J$

c) $½mv^2 = 8 \times 10^{-16}J$

So $v^2 = \frac{16 \times 10^{-16}}{9.11 \times 10^{-31}} = 1.76 \times 10^{15}$

$v = 4.19 \times 10^7 ms^{-1}$

Electrons at the speed of light?

A problem may have occurred to you – that if the P.D. is big enough, then an electron should end up travelling faster than the speed of light. This is impossible, so what is the explanation for this dilemma? The explanation is that, although the K.E. does keep increasing, the speed does not; instead, the mass increases. This is known as relativistic increase in mass.

Question

a) Calculate the P.D. through which an electron would have to be accelerated to achieve the speed of light.

b) Explain what happens as the speed of the electron approaches the speed of light.

$M_e = 9.11 \times 10^{-31}kg$

$e = 1.6 \times 10^{-19}C$

Answers

a) $½mv^2 = eV$

For $v = 3 \times 10^8$

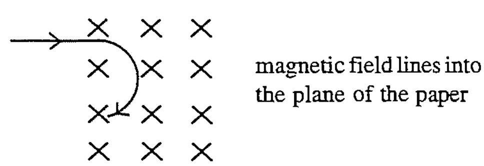
$½ \times 9.11 \times 10^{-31} \times 9 \times 10^{16} = 1.6 \times 10^{-19} \times V$

$V = ½ \times \frac{9.11 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-19}} = 256\ kV$

b) *The K.E. of the electron increases as per the equation ½ mv^2 = eV, but it is the mass which increases not the speed.*

Magnetic Fields

If a charged particle enters a magnetic field at right angles to the field lines, it will experience a force at right angles to the field lines and at right angles to its velocity, thus the particle will describe a circular path. The magnetic force will provide the centripetal force.



This is for a stream of negative particles, for positive particles the path would curve in the opposite direction

The magnetic force on a particle of charge q, in a field of strength B with speed v is **Bqv**

Magnetic force = Bqv

The centripetal force required for a particle of mass m, at a speed v, to describe a circle of radius r is mv^2/r.

Centripetal force $= \frac{mv^2}{r}$

The magnetic force ***Bqv***, provides mv^2/r, so:

$$Bqv = \frac{mv^2}{r}$$

$$Bq = \frac{mv}{r}$$

$$r = \frac{mv}{Bq}$$

So the radius of the circle described by the particle depends on the B field, the charge on the particle, its mass and the speed at which it enters the field.

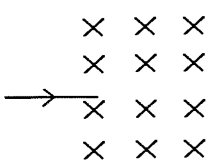
A particle entering a magnetic field at right angles to the field lines, describes a circular path of radius r, where

$$r = \frac{mv}{Bq}$$

This concept is used in the **Mass Spectrometer**, used to separate isotopes of a gaseous element. Ions of the element will each have the same charge and velocity entering a given B field, so the radii of the paths they describe will be proportional to their masses.

Question

A hydrogen ion enters a magnetic field of strength 0.8 T, with its field lines into the plane of the paper as shown.

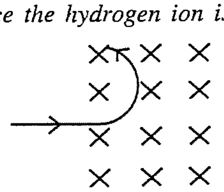


magnetic field lines into the plane of the paper

a) Complete the diagram to show the path of the electron.
b) Calculate the time taken to complete a semi-circular path.
Mass proton = 1.66 x 10^{-27}kg
Charge on electron = 1.6 x 10^{-19}C

Answers

a) Since the hydrogen ion is positively charged its path is:



magnetic field lines into the plane of the paper

b) Time taken is half of the time period for circular motion.
Time $= \pi/\omega$
$\omega = v/r$, *so time* $= \pi r/v$, *and* $r/v = m/Bq$

$$Time = \frac{m\pi}{Bq} = \frac{1.66 \times 10^{-27} \times 3.14}{0.8 \times 1.6 \times 10^{-19}} = 4.07 \times 10^{-8} s$$

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's mark scheme is given below.

a) A mass spectrogram is used to separate hydrogen ions from those of deuterium ($_1^2H$) and tritium ($_1^3H$). First the particles are accelerated through a P.D. of 5kV. Calculate the speeds of the particles. **(4)**
Mass proton = 1.66 × 10^{-27}kg
Charge on electron = 1.6 × 10^{-19}C

$K.E = \frac{1}{2}mv^2 \quad \frac{1}{2}mv^2 = 5 \times 10^3 eV$

$v^2 = \frac{2 \times 5 \times 10^3}{1.66 \times 10^{-27}} = 6.02 \times 10^{31}$

$v = 7.76 \times 10^{15} ms^{-1}$ *1/4*

The candidate has used the correct concept of the K.E. (½ mv^2) deriving from the energy of the electric field, but has not used the energy in Joules (by multiplying by 1.6 × 10^{-19}). S/he has also failed to realize that each isotope has a different mass and would therefore be accelerated to a different speed. The candidate should also have noticed that this speed cannot be correct as it is far greater than the speed of light.

b) The ions enter a magnetic field of 0.7T. Calculate the radius of the orbit of the tritium ions. (3)

$r = \frac{mv}{Be}$

$= \frac{1.66 \times 10^{-27} \times 7.76 \times 10^{15}}{0.7 \times 1.6 \times 10^{-19}} = 1.15 \times 10^{9} m$ *2/3*

The candidate has used the incorrect velocity from part a) but has not been penalized (error carried forward – ecf), however s/he has used the mass for hydrogen not tritium. Again the candidate should have noticed that this is a ridiculous answer and checked his/her work.

Examiner's Answers

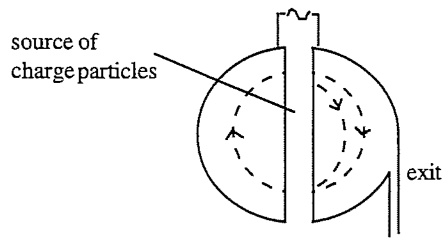
a) $\frac{1}{2}mv^2 = 1.6 \times 10^{-19} \times 5 \times 10^3$
For hydrogen
$v^2 = \frac{2 \times 5 \times 10^3 \times 1.6 \times 10^{-19}}{1.66 \times 10^{-27}} = 9.64 \times 10^{11}$
$v = 9.82 \times 10^5$
For deuterium $v^2 = 4.82 \times 10^{11}$; $v = 6.94 \times 10^5$
For tritium $v^2 = 3.21 \times 10^{11}$; $v = 5.67 \times 10^5$

b) $r = \frac{mv}{Be}$
$= \frac{3 \times 1.66 \times 10^{-27} \times 5.67 \times 10^5}{0.7 \times 1.6 \times 10^{-19}}$
= 2.52cm

You should now attempt the following typical exam question, to check your retention and understanding of the concepts. Then check your reponses against the answers given.

Typical exam question

A cyclotron consists of two "Ds" – semicircular areas of magnetic field, connected by a high-frequency alternating supply. Charged particles emerge from the centre and travel in a spiral path of approximately circular obits in each D. As they pass over the gap between the Ds they experience a voltage of 6kV, which is always in such a direction as to accelerate the particles. In the diagram the field is out of the plane of the paper so that positive particles will follow typical paths like the one shown.



a) Calculate the gain in K.E. each time a proton passes across a gap.
b) If voltage is to change direction at fixed intervals, then the time period for each circuit in the Ds must be independent of the speed of the particle. Show that this is the case.
c) Show that the K.E. of a proton circulating at radius r is K.E. = $B^2q^2r^2/2m$
d) Calculate how many times a proton will circulate before the radius of the orbit is 89.5 cm, if the B field is 0.5T.
Mass proton = 1.66×10^{-27}kg
Charge on proton = 1.6×10^{-19}C

Answers

a) $K.E. = q \times V = 1.6 \times 10^{-19} \times 6 \times 10^{3} = 9.6 \times 10^{-16}J$

b) Time period $= \frac{1}{2} \times 2\pi/\omega = \pi r/v$ $\qquad mv^2/r = Bqv \quad mv/r = Bq$
so $v/r = Bq/m$
therefore Time period $= \pi m/Bq$

Since π, m, B and q are all fixed, the time period is independent of the speed of the particle.

c) $K.E. = \frac{1}{2}mv^2; \quad mv^2/r = Bqv, \quad so\ v = Bqr/m \ ; \quad v^2 = B^2q^2r^2/m^2$
Therefore $K.E. = \frac{1}{2} B^2q^2r^2/m$

d) K.E gained at each gap in the Ds is $9.6 \times 10^{-16}J$.
For $r = 0.895m$

$$K.E. = \frac{\frac{1}{2} \times 0.5^2 \times (1.6 \times 10^{-19})^2 \times 0.79}{1.66 \times 10^{-27}} = 1.52 \times 10^{-12}J$$

$$\text{So number of crossings of Ds} = \frac{1.52 \times 10^{-12}}{9.6 \times 10^{-16}} = 1586$$

So number of orbits = 793

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Physics Factsheet

April 2001 Number 14

Particle Physics

This Factsheet will
- **introduce particles and antiparticles**
- **explain how particles are classified**
- **introduce quarks**
- **explain the use of the conservation laws for particle interaction**
- **describe the four fundamental interactions**
- **introduce the particle-exchange description of forces.**

1. Particles and antiparticles
You will already be familiar with subatomic particles such as protons, neutrons and electrons. These particles – and all the many other subatomic particles that exist – have **antimatter** counterparts called **antiparticles**. Antiparticles have:
- the same mass as the original particle
- opposite charge to the original particle
- opposite spin to the original particle
- opposite values of baryon number, lepton number and strangeness (see later)

The first anti-particle discovered was the anti-electron, or **positron**. The positron is usually written e^+.

Other antiparticles are written using the normal symbol for the particle, but with a bar over it.

eg	proton: p	anti-proton: $\overline{p}$
	electron neutrino: ν_e	anti-electron neutrino: $\overline{\nu}_e$

Pair production
A particle – antiparticle pair can be produced by a high-energy photon., or by the collision between other particles. The photon must have sufficient energy to create the rest mass of the two particles – so its energy must be at least twice the rest energy of each particle. If it has greater energy than this, the surplus will become kinetic energy.

Example: The rest energy of the proton is 940 MeV.
So for a photon to produce a proton-antiproton pair, it must have energy at least 2 × 940 MeV = 1880 MeV (or 1.88 GeV)

Tip: *Make sure you learn your unit prefixes*
- *M means 1,000,000 = 10^6 (so 1 MeV = 1,000,000 eV)*
- *G means 1,000,000,000 = 10^9 (so 1 GeV = 10^9 eV)*

Pair production may be observed in a cloud chamber (covered in a later Factsheet).

Pair annihilation
If a particle meets its antiparticle, they annihilate each other, producing two photons. Photons have to be produced since energy and momentum would not otherwise be conserved.

Glossary
Electron-volts (eV). Electron-volts are a unit of energy – the energy acquired by one electron when accelerated through a potential difference of one volt.
- To find the energy in eV acquired by a particle accelerated through a potential difference, multiply its charge by the p.d.
- To convert electron-volts into joules, multiply by the charge on an electron (1.6×10^{-19} C)

Rest mass. The masses of subtatomic particles are always described as their "rest mass" – in other words, their mass when they are at rest - not moving. This is because Einstein's Special Theory of Relativity says that the mass of anything increases when it is moving, and since these particles can move very fast, this increase can be quite substantial.

Rest energy. The rest energy is linked to the rest mass via the equation $E = mc^2$, where E is energy, m is mass and c is the speed of light. So rest energies can be converted to rest masses by dividing by c^2. Rest energies are usually measured in eV. Rest masses may accordingly be measured in $\frac{eV}{c^2}$ - so a particle of rest energy 940 MeV would have rest mass 940 $\frac{MeV}{c^2}$

Spin. Spin is an important property of subatomic particles. It can be thought of as a bit like angular momentum. It can take values $0, \pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm2...$

2. Particle Classification
All particles can be classified into **hadrons** and **leptons**. Hadrons experience the strong nuclear force (see later) and leptons do not.

Leptons
- leptons are the lightest group of particles
- they have spin ±½
- they are acted on by the weak nuclear force (see later)
- they are fundamental particles (so cannot be subdivided further)
- Leptons all have **lepton number (L) +1**. Their **antiparticles** have lepton number **–1**
- All other particles have lepton number 0

The most familiar example of a lepton is the **electron**. Electrons are **stable** – they do not decay. **Muons** and **taus** are also leptons – these are not stable – they decay quite readily into other particles.

Each of the electron, muon and tau have a corresponding **neutrino** (see below). These have no charge or mass, and only interact very weakly with matter – they are therefore very difficult to detect. Only electron neutrinos exist in any numbers.

All six leptons described have an **antiparticle**, with opposite charge, spin and lepton number.

Table 1 overleaf shows the six leptons and their antiparticles.

Table 1. Leptons

Particle	Symbol	Charge	Antiparticle
electron	e^-	-1	e^+
electron neutrino	ν_e	0	$\bar{\nu}_e$
muon	μ^-	-1	μ^+
muon neutrino	ν_μ	0	$\bar{\nu}_\mu$
tau	τ^-	-1	τ^+
tau neutrino	ν_τ	0	$\bar{\nu}_\tau$

Hadrons
Hadrons are subdivided into **baryons** and **mesons**.

Baryons
- baryons are the heaviest group of particles
- they have spin $\pm\frac{1}{2}$ or $\pm\frac{3}{2}$
- they are **not** fundamental particles, but are composed of quarks (see later)
- baryons all have **baryon number (B) +1**. Their **antiparticles** have baryon number **–1.**
- All other particles – i.e. leptons and mesons- have baryon number 0

Protons and neutrons are both baryons; all the baryons with spin + ½ are shown in table 2 below.

The only **stable** baryon is the **proton** (it has a half-life of about 10^{32} years, so proton decays will be so rare as to be virtually unobservable) All other baryons decay readily – most with very short lifetimes, such as 10^{-10} seconds - eventually producing protons.

In particular, the **neutron** has a half-life of about 13 minutes when it is outside a nucleus. It decays to produce a proton, an electron and an anti-electron neutrino:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Table 2. Spin ½ baryons

Particle	Symbol	Charge	Antiparticle
proton	p	+1	$\bar{p}$
neutron	n	0	$\bar{n}$
lambda	Λ^0	0	$\bar{\Lambda}^0$
sigma +	Σ^+	+1	$\bar{\Sigma}^-$
sigma 0	Σ^0	0	$\bar{\Sigma}^0$
sigma –	Σ^-	-1	$\bar{\Sigma}^+$
xi 0	Ξ^0	0	$\bar{\Xi}^0$
xi –	Ξ^-	-1	Ξ^+

Mesons
- mesons have masses intermediate between leptons and baryons
- they have whole-number spin (0, ±1, ±2 etc)
- they are **not** fundamental particles, but are composed of quarks (see later)
- All mesons are very short lived.

Table 3. Mesons with spin 0.

Particle	Symbol	Charge	Antiparticle
pion +	π^+	+1	π^-
pion 0	π^0	0	self
kaon +	K^+	+1	K^-
kaon 0	K^0	0	$\bar{K}^0$
eta	η	0	self

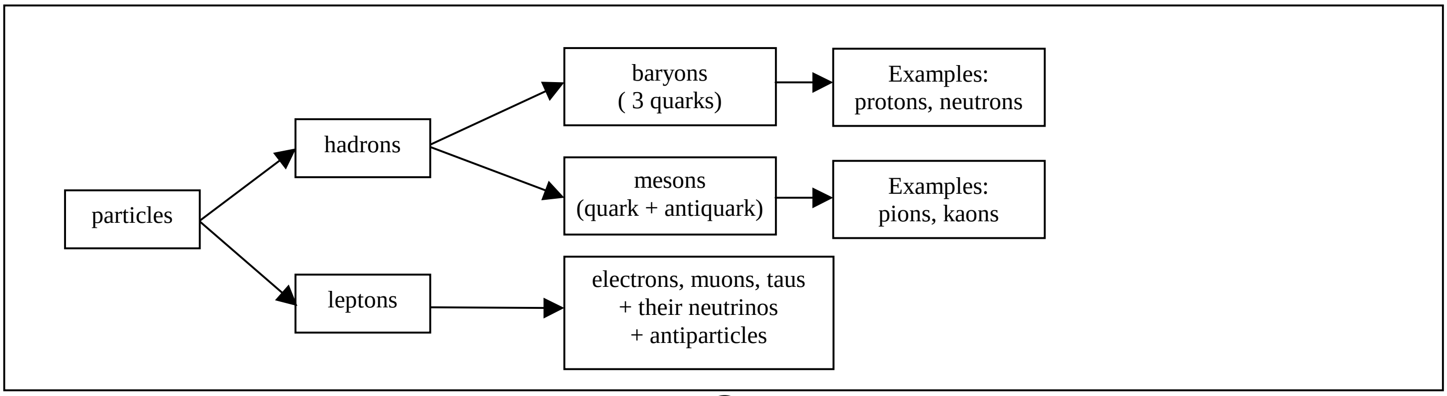
3. Quarks
- There are six quarks altogether, and each has an antiquark.
- Quarks experience the strong nuclear force
- Quarks are the constituent particles of hadrons.
- They are considered to be fundamental particles.
- Quarks have not been observed in isolation.
- Quarks have baryon number $\frac{1}{3}$; antiquarks have baryon number $-\frac{1}{3}$
- Quarks have spin $\pm\frac{1}{2}$
- Quarks and antiquarks have lepton number 0
- Baryons are formed from 3 quarks.
- Mesons are formed from a quark and an antiquark

Table 4 shows the quarks and antiquarks. Only three of the six (up (u), down (d) and strange (s)) are needed for AS-level. These three – together with their antiparticles – form the commonest hadrons. The other three are very massive and can only be identified in very high energy particle accelerators.

Table 4. Quarks

Particle	Symbol	Charge	Antiparticle
up	u	$+\frac{2}{3}$	$\bar{u}$
down	d	$-\frac{1}{3}$	$\bar{d}$
strange	s	$-\frac{1}{3}$	$\bar{s}$
charm	c	$+\frac{2}{3}$	$\bar{c}$
top	t	$+\frac{2}{3}$	$\bar{t}$
bottom	b	$-\frac{1}{3}$	$\bar{b}$

Fig 1. A summary of particle classification



Typical Exam Question

A particle accelerator accelerates protons through a potential difference of 1.8 GeV and makes them collide with antiprotons of the same energy moving in the opposite direction. This collision creates a proton-antiproton pair.

a) Write an equation for this process [1]

$p + \bar{p} \rightarrow p + \bar{p} + p + \bar{p}$ ✓

b) i) Give one similarity between a proton and an antiproton [1]

they have the same mass ✓

ii) Give two differences between protons and antiprotons [2]

they have opposite charges ✓

they have opposite spins ✓

(or could have referred to baryon number)

c) Find the total kinetic energy of the proton and antiproton before they collide, giving your answer in GeV [1]

$1.8 \times 1 = 1.8$ GeV each particle

So $2 \times 1.8 = 3.6$ GeV in total ✓

d) The rest energy of the proton is 940 MeV. State the rest energy of the antiproton [1]

940 MeV ✓

e) Calculate the total kinetic energy of the particles after the collision [2]

Kinetic energy – energy to create particles

= 3.6 GeV – 2 × 940 MeV ✓

= 3.6 GeV – 1.88GeV = 1.72 GeV ✓

Strangeness

A property called **strangeness** is needed to explain why some reactions cannot happen (see the section on conservation laws).

- The strange quark has strangeness –1
- Its antiquark, $\bar{s}$, therefore has strangeness +1
- All other quarks, and all leptons, have strangeness 0

The strangeness of a hadron can be found by adding the strangenesses of its constituent quarks. Table 5 shows some hadrons, their quark composition and their strangeness.

Table 5. Quark composition and strangeness for selected hadrons

Particle	Quarks contained	Strangeness
proton	u u d	0
neutron	u d d	0
pion	u $\bar{d}$	0
kaon 0	d $\bar{s}$	+1
kaon +	u $\bar{s}$	+1
sigma +	u u s	–1

β^- and β^+ decay in terms of quarks

In **β^- decay**, a neutron decays to produce a proton, an electron and an anti-electron neutrino according to the equation:

$n \rightarrow p + e^- + \bar{\nu}_e$

Written in terms of quarks, this becomes:

$udd \rightarrow uud + e^- + \bar{\nu}_e$

Or, concentrating on the actual changes:

$d \rightarrow u + e^- + \bar{\nu}_e$

In **β^+ decay**, a proton decays to produce a positron and an electron neutrino according to the equation:

$p \rightarrow n + e^+ + \nu_e$

In terms of quarks, this becomes:

$uud \rightarrow udd + e^+ + \nu_e$

Or, simplified:

$u \rightarrow d + e^+ + \nu_e$

These processes involving a change in quark type are a result of the action of the **weak nuclear force** (see later).

Exam Hint: *You will be expected to remember the quark composition of protons and neutrons (and for some boards, kaons and pions).*

4. Particle interactions and conservation laws

You will have met conservation laws before, such as conservation of energy and conservation of momentum. Particle interactions have their own set of conservation laws.

In any interaction, the following are conserved:

- ***energy***
- ***momentum***
- ***charge***
- ***spin***
- ***lepton number***
- ***baryon number***

In any interaction involving the strong force,

- ***strangeness must be conserved***

These conservation laws enable us to tell whether it is **possible** for an interaction to take place – they do **not** tell us that it **has to happen**.

To use these laws to tell whether an interaction takes place, you work out the total charge , total lepton number, total baryon number, total spin and total strangeness on each side of the arrow. If these totals are the same on both sides, the interaction is possible.

Example 1. Show that β decay does not violate any conservation laws

	n	→	p	+ e^-	+ $\bar{\nu}_e$	
charge	*0*		*+1*	*-1*	*0*	*total = 0*
lepton number	*0*		*0*	*1*	*-1*	*total = 0*
baryon number	*1*		*1*	*0*	*0*	*total = 1*
spin	$\frac{1}{2}$		$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	*total =* $\frac{1}{2}$
strangeness	*0*		*0*	*0*	*0*	*total = 0*

Since the totals are the same on each side for each, the interaction does not violate any conservation laws

Example 2. Use conservation laws to show the following interaction cannot occur

	K^+	+ $\bar{p}$	→	K^-	+ p	
charge	*+1*	*-1*		*-1*	*+1*	*0 on both sides* ✓
lepton number	*0*	*0*		*0*	*0*	*0 on both sides* ✓
baryon number	*0*	*-1*		*0*	*+1*	*Not conserved*

As baryon number is not conserved, the interaction cannot occur.

Tip: *An interaction only needs to break* ***one*** *conservation law to be impossible. It must obey them* ***all*** *if it is possible.*

5. Fundamental forces

There are four fundamental forces – gravitation, electromagnetic, weak nuclear and strong nuclear. The order of strength is:
strong nuclear > electromagnetic > weak nuclear > gravitation

Gravitation
- a force between masses
- it is always attractive
- range is infinite

Gravitation can be ignored at atomic scales, because the masses involved are so small.

Electromagnetic
- a force between all charged bodies
- can be attractive (unlike charges) or repulsive (like charges)
- range is infinite

The electromagnetic force is responsible for interatomic and intermolecular attractions and repulsions, which hold atoms and molecules together.

Weak Nuclear
- acts on all particles – both leptons and quarks
- has a range of less than 10^{-17}m
- is responsible for β decay
- is responsible for interactions involving change of quark type

The electromagnetic and weak forces are now thought to be different aspects of one force, called the electroweak force.

Strong Nuclear
- acts on hadrons and quarks
- very short range – acts only within the nucleus
- responsible for holding the nucleus together
- attractive at larger ranges ($1.2 \times 10^{-15} – 3 \times 10^{15}$m) - must overcome electrostatic repulsion between protons.
- repulsive at very short range (< 10^{-15} m) - must stop the nucleus collapsing

6. Particle exchange model for the four interactions

The idea behind this model is that forces are due to "virtual" particles being exchanged between the interacting particles. The virtual particles are considered to form clouds surrounding the interacting particles, creating a field. Table 6 below shows the exchange particles for the four interactions

Table 6. Exchange particles

Interaction	Exchange particle
gravitation	graviton *(not discovered yet)*
electromagnetic	photon
weak nuclear	W^+, W^-, Z^0 bosons
strong nuclear	gluon

These interactions can be shown on Feynmann diagrams (Fig 2)

Questions
1. Give the similarities and differences between particles and antiparticles.

2. The rest energy of an electron is 0.5 MeV.
A photon of energy 1.1 MeV produces an electron-positron pair. Calculate the total kinetic energy of the electron and positron.

3. a) Give two differences between hadrons and leptons
b) Give two examples of baryons
c) Give two differences between baryons and mesons

4. State the quarks that form
a) protons
b) neutrons
c) antiprotons
d) antineutrons.

5. Use conservation laws to determine which of the following interactions are possible:
a) $p \rightarrow n + e^+ + \bar{\nu}_e$
b) $p + e^- \rightarrow n + \nu_e$
c) $p + \pi^- \rightarrow \pi^0 + n$

6. a) List the four fundamental interactions in order of strength.
b) Which of these four forces will the following particles feel:
i) neutrino
ii) electron
iii) proton

7. Draw a Feynmann diagram to represent β^+ decay.

Answers
1. See page 1.
2. 1.1 MeV – 2 × 0.5MeV = 0.1MeV
3. a) hadrons feel strong nuclear force, leptons do not
leptons are fundamental particles, hadrons are not
b) protons, neutrons
c) baryons formed from 3 quarks, mesons from quark + antiquark
mesons have integer spin, baryons do not.
4. a) u u d b) u d d c) $\bar{u}\bar{u}\bar{d}$ d) $\bar{u}\bar{d}\bar{d}$
5. a) lepton number not conserved, so not possible
b) possible c) possible
6. a) strong nuclear> electromagnetic > weak nuclear > gravitation
b) i) weak ii) electromagnetic, weak, gravitation iii) all four
7.

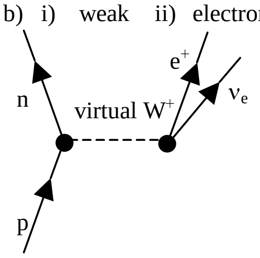
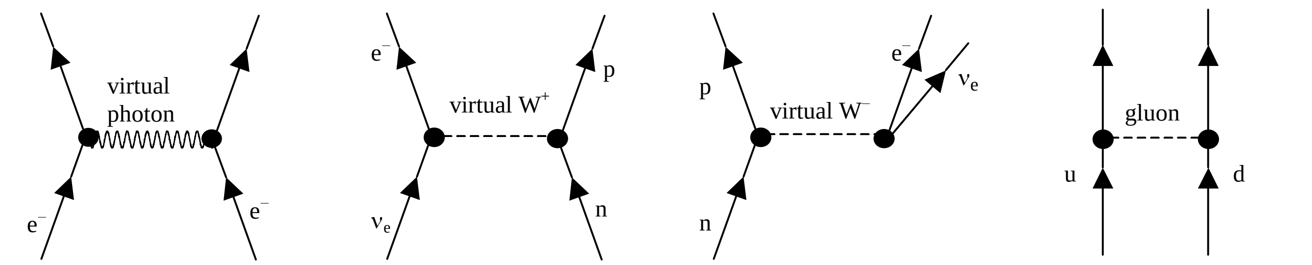


Fig 2. Feynmann diagrams for selected interactions



electromagnetic repulsion between two electrons | neutrino – neutron scattering | β^- decay | strong force between quarks

Physics Factsheet

www.curriculum-press.co.uk Number 144

Conservation Laws in Particle Physics

At GCSE we study conservation laws for classical Physics. We perform calculations for conservation of momentum in collisions, realise that kinetic energy is usually not conserved in a collision, and assume conservation of mass and charge in various situations.

This Factsheet will restrict itself to Particle Physics. The above conservation laws still hold (except that conservation of mass becomes conservation of mass/energy), but new laws specific to sub-atomic particles also arise.

> *The conservation laws we study in Particle Physics are in addition to the classical laws.*

Standard model

The standard model for particles includes the idea of quarks (which form hadrons) and leptons. Reactions and interactions between these particles involve the fundamental forces.

However some interactions occur, and others do not occur. A number of conservation laws seem to dictate what is allowed and what is not allowed. We will look at some of these.

Conservation of mass/energy

In chemistry, when mass seems to disappear during a chemical reaction, we assume that a reactant (usually a gas) has escaped with the "missing" mass.

In Particle Physics, mass and energy are interchangeable according to Einstein's equation:

$E = mc^2$	where	E is the energy in joules, m is the mass in kg and $c = 3.0\times10^8 ms^{-1}$.

Example1:

(a) Calculate the energy that could be created from a 0.24kg mass.
(b) Find the mass that could be formed from 4.2×10^6J of energy.

Answer:

(a) $E = mc^2 = 2.2\times10^{16}$J
(b) $m = E/c^2 = 4.7\times10^{-11}$kg

Example 2:

A thorium nucleus decays to become radium, emitting an alpha particle. $^{228}_{90}Th \rightarrow {}^{224}_{88}Ra + {}^{4}_{2}\alpha$

The masses of the particles: Th: mass = 3.7876×10^{-25}kg
Ra: mass = 3.7210×10^{-25}kg
α: mass = 6.647×10^{-27}kg

(a) Calculate the mass that is converted to energy (primarily as k.e. of the alpha particle).
(b) Calculate this k.e. in joules.

Answer:

(a) m = mass Th – mass Ra – mass α = 1.30×10^{-29}kg
(b) $E = mc^2 = 1.17\times10^{-12}$J

> *Exam Hint: In mass/energy conversions, the masses and energies can be expressed in kg, atomic units, joules, electron-volts, etc. Try to be familiar with the different units, and learn to convert from one to another.*

One collision that does occur in particle accelerators is:

$$p + p \rightarrow p + p + p + \bar{p}$$

(the line above the symbol indicates the antiparticle – in this case, the antiproton)

The source of the mass of the new proton and antiproton is the kinetic energy of the colliding particles.

Conservation of charge

This is a law we assume at GCSE. For instance, when sodium hydroxide neutralises hydrochloric acid, charged particles (ions) are formed:

$$NaOH + HCl \rightarrow Na^+ + OH^- + H^+ + Cl^-$$
$$Q: \quad 0 + 0 = 1 + (-1) + 1 + (-1) = 0$$

The same idea can be used to check the possibility of particle interactions occurring.

Example 3:

Which of these interactions would be impossible on charge conservation grounds?
(a) p + n → p + p
(b) n → p + e

Answer:

(a) p + n → p + p
Q: 1 + 0 = 1 + 1 (cannot occur)
(b) n → p + e
Q: 0 = 1 + (-1) (possible)

Charge conservation allows this second reaction, however, as we will see, other conservation laws mean this reaction is not possible.

Conservation of baryon number

Hadrons, such as protons and neutrons and pions, are large subatomic particles composed of quarks (fundamental particles). Hadrons are divided into baryons, antibaryons, and mesons. A baryon number is associated with each of these hadrons:

Hadron Type	Composition	Examples	Baryon number
Baryon	3 quarks	proton neutron	1
Meson	1 quark plus 1 antiquark	pion, π kaon, K	0
Antibaryon	3 antiquarks	antiproton antineutron	-1

The conservation law says that, in any reaction, baryon number is conserved

Example 4:
Which of these reactions is excluded because baryon number is not conserved?
(a) $p + p \rightarrow p + p + \pi^+ + \pi^0$
(b) $p + p \rightarrow p + p + n$

Answer:
(a) $p + p \rightarrow p + p + \pi^+ + \pi^0$
B: $1 + 1 = 1 + 1 + 0 + 0$ allowed

(b) $p + p \rightarrow p + p + n$
B: $1 + 1 = 1 + 1 + 1$ not allowed

Example 5:
Option (a) above is not excluded by baryon number. Why is it not possible?

Answer:
$p + p \rightarrow p + p + \pi^+ + \pi^0$
Q: $1 + 1 = 1 + 1 + 1 + 0$
The charge number is not conserved.

> All of the relevant conservation laws must be obeyed if a reaction is to be possible.

However this reaction is allowed. You can check that it obeys all the relevant conservation laws: $p + p \rightarrow p + p + \pi^+ + \pi^-$

Conservation of lepton number

Leptons are fundamental particles. And, as expected, lepton number must be conserved in any interaction. The family of leptons is composed of electrons, muons, tau particles, their antiparticles, and associated neutrinos.

Lepton	Symbol	Lepton number
electron	e^-	+1
electron neutrino	ν_e	+1
muon	μ^-	+1
muon neutrino	ν_μ	+1
tau	τ^-	+1
tau neutrino	ν_τ	+1
antielectron (positron)	e^+	-1
electron antineutrino	$\overline{\nu}_e$	-1
antimuon	μ^+	-1
muon antineutrino	$\overline{\nu}_\mu$	-1
antitau	τ^+	-1
tau antineutrino	$\overline{\nu}_\tau$	-1

However there is a complication with lepton numbers. Each "variety" of lepton (electron, muon, tau) must separately have its own lepton number conserved.

Example 6:
Check for conservation of overall lepton number, L, for this reaction. Then check the lepton conservation number for each family of leptons involved.

$$\overline{\nu}_\mu + p \rightarrow n + e^+$$

Answer:

$\overline{\nu}_\mu + p \rightarrow n + e^+$
L: $(-1) + 0 = 0 + (-1)$ no problem
L_μ: $(-1) + 0 = 0 + 0$ problem
L_e: $0 + 0 = 0 + (-1)$ problem

Although the overall lepton number is conserved, the lepton numbers for the muon and electron families are not conserved. This reaction is never observed.

However the following reaction is observed:

$\overline{\nu}_\mu + p \rightarrow n + \mu^+$
L: $(-1) + 0 = 0 + (-1)$ no problem
L_μ: $(-1) + 0 = 0 + (-1)$ no problem

The overall lepton number, and the number for the only family present, are both conserved.

Earlier, we saw that a possible process for beta minus decay did not conflict with conservation of charge. (It also does not conflict with conservation of baryon number.)

$$n \rightarrow p + e^-$$

Example 7:
See if lepton number conservation poses a difficulty.

Answer:
$n \rightarrow p + e^-$
L_e: $0 = 0 + 1$ there is a problem

This cannot be the whole decay process.

However, if we add another product to the decay (an electron antineutrino), then conservation laws are observed. (The antineutrino also solves an observed problem with energy conservation during the decay.)

$n \rightarrow p + e^- + \overline{\nu}_e$
Q: $0 = 1 + (-1) + 0$ no problem
B: $1 = 1 + 0 + 0$ no problem
L_e: $0 = 0 + 1 + (-1)$ no problem

This is the accepted decay equation for beta minus decay.

Now some problems for you to try.

Practice Questions

1. If the rest mass of an electron is 9.109×10^{-31}kg, find the energy in joules that this could theoretically be converted into.

2. We have said that this reaction occurs in particle accelerators.

   $$p + p \rightarrow p + p + p + \overline{p}$$

   If the rest mass of a proton (or antiproton) is 1.67×10^{-27}kg, calculate the minimum total kinetic energy (in joules) of the incident particles.

3. Which of these reactions is forbidden by conservation of charge?
   (a) $p + p \rightarrow p + p + \pi^+ + \pi^-$
   (b) $p + p \rightarrow p + p + \pi^+ + \pi^- + \pi^+$

4. Which of these reactions violates conservation of lepton number?
   (a) $\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu$
   (b) $\overline{\nu_\mu} + p \rightarrow \mu^+ + n$
   (c) $\mu^- \rightarrow e^- + \gamma$

5. Show that both of these interactions conserve charge and baryon number:
   (a) $p + p \rightarrow p + p + \pi^0$
   (b) $p + p \rightarrow p + n + \pi^+$

6. A muon can decay as follows:
   $\mu^- \rightarrow e^- + \nu_\mu + X$

   Use conservation laws to identify particle X.

Answers

1. $E = mc^2 = 8.2\times10^{-14}$J

2. Total mass created $= 3.34\times10^{-27}$kg
   $E = mc^2 = 3.01\times10^{-10}$J
   This would be the minimum energy required to create this mass.

3. (a) $p + p \rightarrow p + p + \pi^+ + \pi^-$
   Q: $1 + 1 = 1 + 1 + 1 + (-1)$ allowed

   (b) $p + p \rightarrow p + p + \pi^+ + \pi^- + \pi^+$
   Q: $1 + 1 = 1 + 1 + 1 + (-1) + 1$ not allowed

4. (a) $\mu^- \rightarrow e^- + \overline{\nu_e} + \nu_\mu$
   L_e: $0 = 1 + (-1) + 0$ allowed
   L_μ: $1 = 0 + 0 + 1$ allowed

   (b) $\overline{\nu_\mu} + p \rightarrow \mu^+ + n$
   L_μ: $(-1) + 0 = (-1) + 0$ allowed

   (c) $\mu^- \rightarrow e^- + \gamma$
   L_e: $0 = 1 + 0$ not allowed
   L_μ: $1 = 0 + 0$ not allowed

5. (a) $p + p \rightarrow p + p + \pi^0$
   Q: $1 + 1 = 1 + 1 + 0$ allowed
   B: $1 + 1 = 1 + 1 + 0$ allowed

   (b) $p + p \rightarrow p + n + \pi^+$
   Q: $1 + 1 = 1 + 0 + 1$ allowed
   B: $1 + 1 = 1 + 1 + 0$ allowed

6. $\mu^- \rightarrow e^- + \nu_\mu + X$
   Q: $(-1) = (-1) + 0 + X$ charge of X = 0
   B: $0 = 0 + 0 + X$ baryon number of X = 0
   L_μ $1 = 0 + 1 + X$ L_μ of X = 0
   L_e $0 = 1 + 0 + X$ L_e of X = -1

   The unknown particle must be an electron antineutrino, $\overline{\nu_e}$.

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Physics Factsheet

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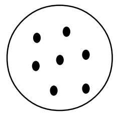
Probing Matter

This Factsheet will explain:
- some of the techniques which scientists use to investigate matter;
- some of the important discoveries which have been made using the techniques;
- the importance of this kind of investigation as an illustration of good science practice.

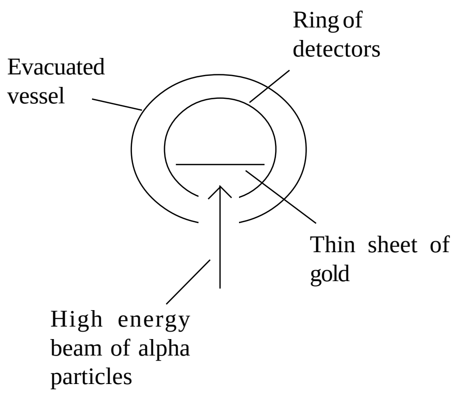
Before studying this Factsheet you should ensure that you are familiar with the outline of the work of Rutherford from your GCSE course

The alpha scattering experiments

Before the scattering experiments, the accepted model of the atom was of a homogenous "blob" of positive material with negative "bits" distributed evenly throughout it, rather like a currant bun. Indeed, it was sometimes called the "Currant Bun" or "Plum Pudding" model of the atom.

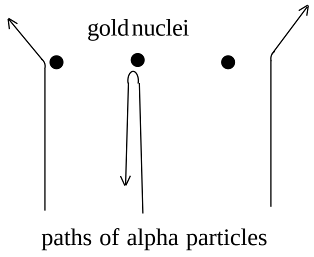


Geiger and Marsden carried out an investigation, in which they fired alpha particles (helium nuclei) at a very thin sheet of gold and measured the proportion of particles which were scattered through various angles.



- *Rutherford scattering probes the structure of the atom.*
- *It is alpha particles which are used in Rutherford Scattering*

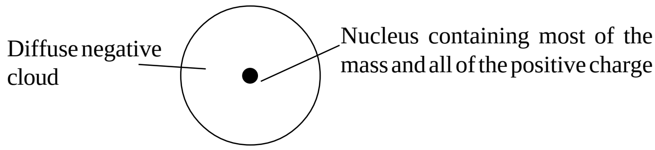
The result they expected, based on the model, was that the particles would go through the gold, but would be deviated through small angles due to electrostatic repulsions between the positive particles and the positive charges in the atoms. Most of the particles did behave as expected, but to their amazement, Geiger and Marsden found that some particles were deviated by large angles, and some almost came back the way they had entered.



Geiger and Marsden confirmed their results and realized that there was no way that the existing model could explain the results obtained.

Rutherford suggested a new model of the atom, which would explain the experimental results — the "Nuclear" model of the atom.

The Nuclear Model of the Atom



Rutherford realized that the large scattering angles could not be explained as the sum of a large number of smaller scatterings, because the gold was too thin. In order to account for the large scattering angles, the atom must consist of a minute dense centre, in which almost all of the mass and all the positive charge is concentrated, so that the majority of the particles went through only slightly deviated by avoiding the central charge/mass concentration, but some which went closer to the centres would be highly deviated. He was able to calculate the relative size of the charge/mass concentration — which he called the "nucleus" — from the proportion of highly scattered particles. He concluded that the nucleus has a radius of the order of 10^{-15} m, compared to the atom with radius of the order of 10^{-10} m.

Exam Hint: *You may be asked to draw or complete paths like these. They are parabolas. The closer the approach to the nucleus, the sharper the bend.*

The neutron remained undetected at this time.

The results of this experiment caused the existing model of the atom to be abandoned in favour of the nuclear model and completely altered our understanding of matter. It ranks among the most significant investigations in the history of science.

Deep inelastic scattering

Alpha particles cannot be used to probe deeper into matter. If we wish to investigate the nature of each of the particles in the nucleus i.e. the proton and the neutron, we must use high energy electrons. Just as the non-regular scattering of alpha particles through the atom lead to our understanding of separate particles making up the atom, so the non-uniform scattering of electrons from hydrogen nuclei lead to an understanding that each of these particles is not a "blob", but also made up of smaller particles.

Investigations like these led to the discovery of quarks, the particles which make up the proton and neutron.

This type of scattering is called "inelastic" because interactions between the particles do not conserve kinetic energy.

Exam Hint: *You may need to know more about Fundamental Particles. On some specifications it is not required for the core syllabus, only as an optional topic.*

- *deep inelastic scattering is used to find out about the make up of protons and neutrons*
- *high energy electrons are used*
- *kinetic energy is not conserved*

Typical Exam Question

(a) Complete the following table, which compares two types of scattering experiment.

		Deep inelastic scattering
Incident particles		**Electrons**
Target	**Gold atoms**	

(2)

(b) Write a short paragraph describing how the results of the experiments changed scientific thinking. (4)

	Alpha scattering	**Deep inelastic scattering**
Incident particles	*Alpha particles*	**Electrons**
Target	**Gold atoms**	*nucleons*

Alpha particle scattering:
Before the experiment, scientists believed that positive material was distributed throughout the atom. The results could only be explained by the positive charge being concentrated in a tiny volume at the centre – the nucleus.

Deep inelastic scattering:
Before experiments, the proton, neutron and electron were believed to be fundamental particles. The results showed that these particles were made up of yet more fundamental particles.

Questions

1. Describe the alpha scattering experiments.

2. Why was it important that the vessel was evacuated?

3. (a) The nucleus of carbon-14 has a radius of 2.70×10^{-15} m. Calculate the volume of this nucleus.

   (b) The mass of the nucleus is 2.34×10^{-26} kg. Calculate the density of the nuclear material of the atom.

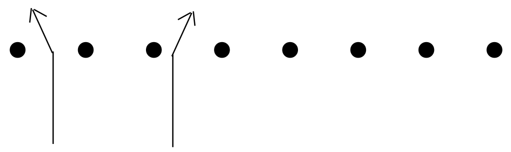
   (c) Compare this value with that of ordinary materials.

Exam Workshop

This is a typical weak student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's mark scheme is given below.

(a) The diagram shows a line of gold nuclei, such as might have been the target for an alpha scattering experiment.

Add lines to the diagram to show the paths of the alpha particles. (3)



1/3

Although the candidate has shown deviation, s/he has not shown an understanding that the closer the line of approach to the nucleus, the greater the deviation, nor is there any indication of a very large deviation. At least three paths, one showing a direct approach and a large deviation should have been shown.

(b) Explain why this experiment led to the conclusion that most of the mass and all of the positive charge of the atom was concentrated in a small space at the centre. (3)

The deviations could not have been caused by any other arrangement. The positively charged alpha particles were repelled by the positive nuclei. *1/3*

The candidate appreciates that the results cannot be explained by the then accepted model, but does not really say why the nuclear explanation is the only sensible one.

Examiner's Answers

(a) (as shown on the first page) Three paths, parabolas, showing hardly any deviation for paths between the nuclei, larger deviation for closer approach, including one of about 170^0.

(b) Positive particles are deviated by the repulsion of the positive nucleus. Since particles were deviated through different angles, matter must be distributed unevenly. The large deviations must be caused by regions of dense mass and charge. Since only relatively few particles were deviated through large angles the dense regions must be small compared to the atom as a whole.

Answers

1. See the text.

2. It is important that the vessel was evacuated so that the alpha particles were deviated only by the gold atoms, not by extraneous air atoms or molecules.

3. (a) Volume = $4/3 \times 3.142 \times (2.70 \times 10^{-15})^3 = 8.24 \times 10^{-44}\,m^3$.
   (b) Density = mass/volume = $2.34 \times 10^{-26}/(8.24 \times 10^{-44})$
   $= 2.84 \times 10^{17}\,kg/m^3$.
   (c) Although there is a large range of densities for ordinary materials, this value is many orders of magnitude greater than any of them, the highest of which is about $10^4\,kg/m^3$.

Acknowledgements: *This Physics Factsheet was researched and written by* ***Janice Jones.***

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Physics Factsheet

September 2002 Number 38

Linear Particle Accelerators

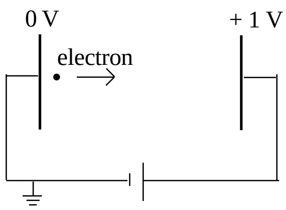
This Factsheet explains how charged particles, such as electrons or protons, are accelerated by two main types of linear particle accelerators. Before the particle accelerators are explained, it is important to understand the different units that are used to denote the energies of accelerated particles.

Particle Energies

Although the standard international unit, or SI unit, of energy is joules, J, it is not the only unit of energy used to denote the energies of particles. The joule is quite a large unit of energy when considering particles that have very small masses so an additional unit is often used, the ***electronvolt,*** eV.

An electronvolt is the amount of energy gained by an electron when it is accelerated across 1 volt of potential difference.

Imagine a negatively charged electron at rest on one metal plate at a potential of 0 volts, but attracted to a similar metal plate at a positive potential of +1 volt. The negatively charged electron would be accelerated to the positive metal plate and as a result it would increase its kinetic energy. This situation is shown in the diagram below.



negative electron gains kinetic energy as it accelerates towards positive plate

The increase in kinetic energy of the electron is 1eV.

If you consider the transfer of energy for this situation, the electron initially has electric potential energy because it has a charge of -1.6×10^{-19} coulombs. This is converted to kinetic energy as the electron accelerates to the positively charged plate.

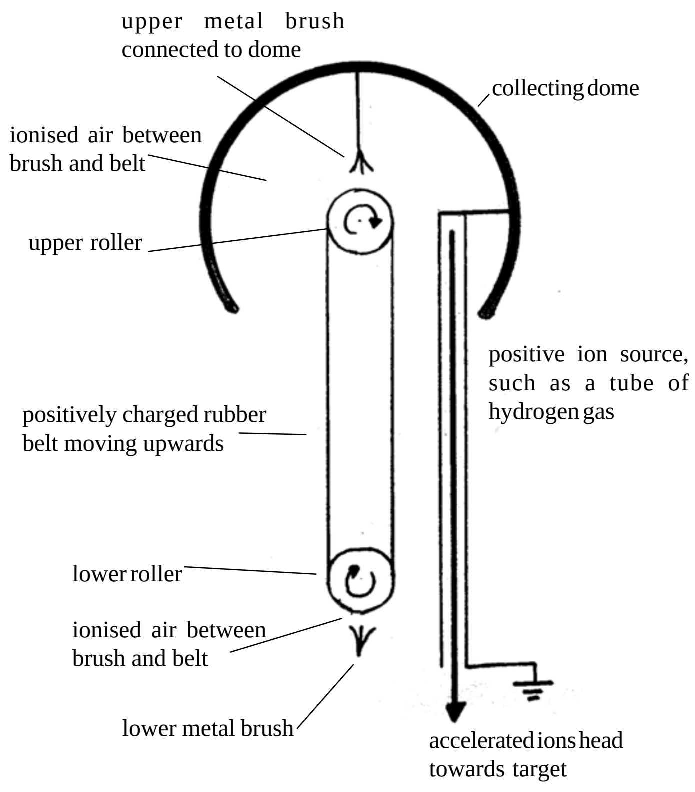
Increase in kinetic energy = *Decrease in electric potential energy*
= *charge on electron* × *pd across plates*
= 1.6×10^{-19} *C* × *1 V*
= 1.6×10^{-19} *J*
Therefore 1 electronvolt = 1.6×10^{-19} *J*

> ***The electronvolt***
> *1 electronvolt is defined as an amount of kinetic energy gained by an electron when it is accelerated across a potential difference of 1 volt.*
> *1 eV =* 1.6×10^{-19} *J*

Van der Graaf accelerator

A Van der Graaf accelerator consists of:
- Two rollers (usually coated in silicon tape).
- A rubber belt that moves between the two rollers.
- A collecting dome, which is made of metal.
- Two brushes with fine metal bristles. These are positioned very close to the rubber belt at the bottom and top rollers. They do not touch the belt.
- A source of charged particles to accelerate, for example the ionised protons from hydrogen gas.

Shown below is a cross sectional diagram of a typical Van der Graaf generator.



1. Transferring a positive charge to the belt at the bottom roller.

- A motor is used to turn the bottom roller, which in turn moves the belt. Because the roller and belt are made of two different materials, rubber and silicon, they exchange charge.
- Negative electrons are transferred from the rubber belt to the roller. This has the effect of making the roller negatively charged and the belt positively charged. As the roller has a smaller surface area than the belt, the negative charge on the roller is more concentrated.
- The negatively charged roller is now able to ionise the atoms of the air between the belt and the metal brush tips. The neutral air molecules each lose an electron. The free electrons move to the metal brush tips and the remaining positive ions are attracted to the rubber belt.
- The belt has now become very positively charged at the bottom roller.

Transfer of charge:
The transfer of charge will take place between any two electrical insulators that are in contact with each other. The transferred charge is always carried with negatively charged electrons that move from one insulator to the other. The material losing negative electrons will become positively charged, whilst the material gaining negative electrons will become negatively charged.

Ionisation:
Ionisation is the removal of a negatively charged electron from a neutral atom. Ionisation creates a negatively charged 'free' electron and a positively charged ion.

2. Storing positive charge on the collecting dome, via the top roller.

- The positively charged belt now moves over the top roller, close to the metal bristles of the top brush. The electrons in the metal bristles are attracted to the belt and move to the end of the bristles.
- The positive belt and negative bristles are now able to ionise the atoms of the air in between them. The neutral atoms lose an electron. The free electrons move to the belt, and therefore tend to neutralise it, whilst the positive ions move to the ends of the metal bristles.
- The metal brush is connected to the metal collecting dome and electrons from the dome travel to the brush (neutralising the brush) and leaving the dome with a positive charge.

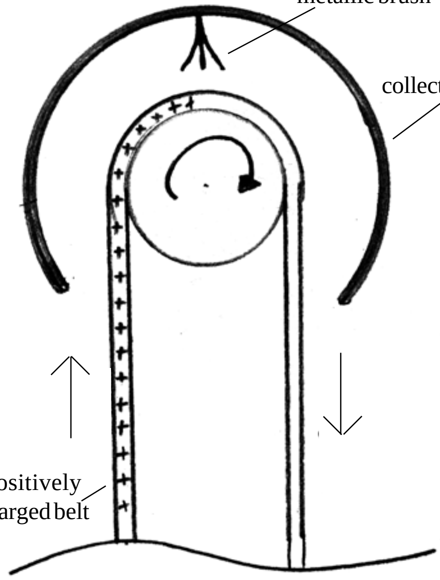
In theory, this process could happen indefinitely, with more and more positive ions being transferred to the collecting dome, creating a larger and larger positive charge and a larger and larger electric potential energy on the dome. In practice the potential of the collecting dome is limited by its environment, such as the breakdown voltage of the surrounding air (~3MV) when sparks will fly!

3. Accelerating charged particles

- The description of theVan der Graaf generator so far explains how it provides a large electric potential energy. The electric potential of the collecting dome can now be used to accelerate charged particles.
- A common source of charged particles is hydrogen gas. The Van de Graaf generator's large potential difference will firstly ionise the hydrogen atoms, creating hydrogen nuclei and free electrons. Secondly, the hydrogen nuclei, which are single protons, will be accelerated away from the large positive potential of the collecting dome towards the target.

Exam Hint: - *Questions that require explanation type answers are best presented as 'bullet points' as shown in the explanations given about the Van der Graaf accelerator shown above. They help you to order your thoughts and make it easier for the examiner to identify the important points that deserve a mark. All bullet points should remain as structured sentences as marks are also awarded on exam papers for your 'quality of written communication.'*

Typical Exam Question
The collecting dome of a Van der Graaf generator is attached to a metallic brush. A positively charged belt moves past the metallic brush but without touching it. The diagram below shows the set up.



(a) Explain how this arrangement transfers a positive charge onto the collecting dome.(3)

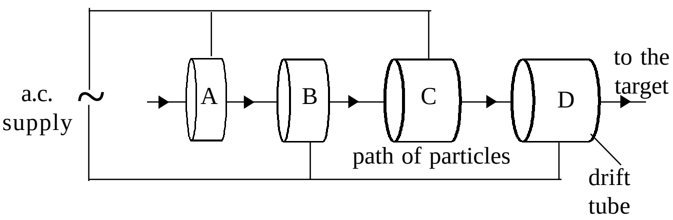
The Van der Graaf accelerator produces protons of energy 2.0MeV
(b) (i) Calculate the maximum kinetic energy of the protons in joules.(2)
(ii) What limits the maximum energy of a proton accelerated in a Van der Graaf accelerator? (1)

(a) The positively charged belt induces a negative charge on the metal bristles of the brush.✓
The atoms in the air in the region between the positively charged belt and negative brush may become ionised.✓
Positively charged ions from the atoms in the air move to the metal brush.✓
Electrons from the dome travel to the brush (neutralising the brush) and leaving the dome with a positive charge.✓

(b) (i) Knowing that the maximum energy transferred to the protons is 2.0 MeV we can convert this number to joules.
Note that the prefix of 'Mega' has been used in the question, which denotes 1 million electronvolts or 10^6 electronvolts.
Kinetic energy = $(2 \times 10^6)(1.6 \times 10^{-19})$ ✓ = 3.2×10^{-12} J ✓
(ii) The maximum energy of a proton accelerated by a Van de Graaf accelerator is limited by the maximum electric potential that the collecting dome can achieve before the air breaks down and the dome discharges.✓

Drift Tube Design Linear Accelerator.

This type of linear accelerator consists of successive tubes through which the charged particles travel. The tubes are alternately connected to opposite terminals of an alternating voltage. The whole assembly must be inside an evacuated chamber. The diagram below shows the arrangement.



- If we consider the accelerating particles to be negatively charged electrons then initially drift tube **A** is given a positive potential by the a.c. supply.
- The negative electrons are accelerated into the tube.
- Inside each drift tube the particles will travel at constant velocity.
- As the electrons leave drift tube **A** the a.c. supply will reverse, giving tube **A** a negative potential and tube **B** a positive potential.
- The electrons are now accelerated towards tube **B**.
- Inside tube **B** the electrons travel at constant velocity.
- As the electrons leave drift tube B the a.c. supply will reverse again, giving tube **B** a negative potential and tube **C** a positive potential.
- The electrons are accelerated towards drift tube **C**.

This process of acceleration between the drift tubes carries on until the electrons leave the final drift tube and head towards the target.

The increasing length of the drift tubes.

The a.c. supply has a constant frequency. This means that it takes a constant amount of time for the drift tubes to change from a positive potential to a negative potential and vice versa. This means that the electrons must spend a constant amount of time inside each drift tube so that when they emerge from one drift tube, the potential of the next drift tube has just become positive. As the velocity of the electrons continues to increase the tubes must be made progressively longer so that the electrons are inside each tube for a constant amount of time.

Qualitative (Concept) Test

(1) Define the electronvolt.
(2) What is ionisation?
(3) How does the rubber belt become positively charged at the bottom roller of a Van de Graaff accelerator?
(4) Explain how the alternating potential is used with the drift tubes of a linear accelerator to accelerate electrons.
(5) Why is a drift tube linear accelerator placed in a vacuum?
(6) Why do the drift tubes increase in length along a linear accelerator?

Quantitative (Calculation) Test

(1) What is the energy, in joules, of an electron with 1.0 keV?(2)
(2) A proton has an energy of 1.8×10^{-12} J. What is the energy of the proton in electronvolts?(2)
(3) What is the speed of an electron that has a kinetic energy of 500 eV? The mass of an electron is 9.11×10^{-31} kg(5)

Quantitative Test Answers

(1) Energy = $(1 \times 10^3)(1.6 \times 10^{-19}) = 1.6 \times 10^{-16}$ J
(2) Energy = $(1.8 \times 10^{-12})/(1.6 \times 10^{-19})$ = 11,250,000 eV = 11.3 MeV (3 s.f.)
(3) Energy = $(500)(1.6 \times 10^{-19}) = 8 \times 10^{-17}$ J
$\frac{1}{2}mv^2 = 8 \times 10^{-17}$
$v^2 = (2)(8 \times 10^{-17})/9.11 \times 10^{-31}) = 1.7563 \times 10^{14}$
$v = \sqrt{(1.7563 \times 10^{14})} = 13{,}252{,}591$ m/s $= 1.3 \times 10^7 ms^{-1}$

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

(a) A Van der Graaf accelerator and a linear accelerator each produce protons of energy 15 MeV.

(i) What is the energy of the protons in joules? (2)

Energy = $(15)(1.6 \times 10^{-19}) = 2.4 \times 10^{-18}$ J *0/2*

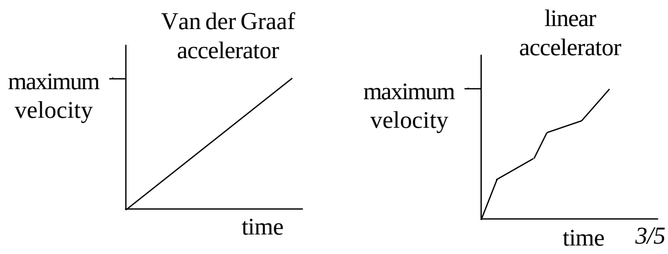
The student has not spotted that the energy has been quoted in MeV

(ii) Assuming the mass of the proton remains constant at 1.7×10^{-27} kg, calculate the velocity of the protons. (3)

Kinetic Energy = $\frac{1}{2}mv^2 = 2.4 \times 10^{-18}$ ✓
$v^2 = (2)(2.4 \times 10^{-18})/(1.7 \times 10^{-27}) = 2.8 \times 10^9$ m/s ✓ *2/3*

The student has recalled the equation for kinetic energy. The equation has been correctly rearranged and the numbers correctly substituted into it. Unfortunately the student has forgotten to square root there final answer for velocity. A quick check of this answer would have revealed that it is unreasonable as it is faster than the speed of light!

(b) On the axes below, sketch a velocity time graph to show how the velocity of the protons changes as they pass through each accelerator. Assume that the linear accelerator has 3 drift with a gap between each tube. (5)



3/5

The Van de Graaff accelerator graph is correct. Whilst the linear accelerator graph correctly shows the same maximum velocity being reached by the electrons it does not show that there is no acceleration in the three regions where the electrons would be inside the three drift tubes.

(c) What limits the maximum energy achieved by charged particles accelerated with a drift tube linear accelerator? (1)

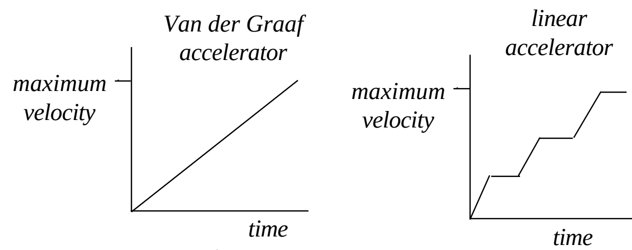
Number of drift tubes.

Although the number of drift tubes does affect the maximum energy of the particles, it is the total length of the accelerator that is the determining factor as the tubes must become progressively longer.

Examiner's Answers

(a) (i) Energy = $(15 \times 10^6)(1.6 \times 10^{-19})$ ✓ $= 2.4 \times 10^{-12}$ J ✓
(ii) Kinetic Energy = $\frac{1}{2}mv^2 = 2.4 \times 10^{-12}$ ✓
$v^2 = (2)(2.4 \times 10^{-12})/(1.7 \times 10^{-27}) = 2.8 \times 10^{15}$ ✓
$v = 5.3 \times 10^7$ m/s ✓

(b)



(c) Length of accelerator ✓

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Physics Factsheet

January 2003 Number 43

Circular Particle Accelerators

This Factsheet explains how charged particles, such as electrons or protons, are accelerated by two types of circular particle accelerator, the cyclotron and the synchrotron.

Circular particle accelerators bring together ideas from several different areas in physics. Reference will be made in this factsheet to ideas of centripetal forces covered in the Factsheet on uniform circular motion (19).

The Cyclotron

The cyclotron consists of:

- An evacuated chamber to minimise energy losses resulting from collisions of ions with air molecules.
- A source of charged particles that are to be accelerated – an ion source.
- Two 'dees' across which an alternating voltage is applied that accelerates the ions when they move from one dee to the next.
- A uniform magnetic field that keeps the ions moving in a circular path and confines them to the two dees. This magnetic field is usually produced by an electromagnet.

Fig 1(a) Cross sectional diagram of the main parts of a cyclotron

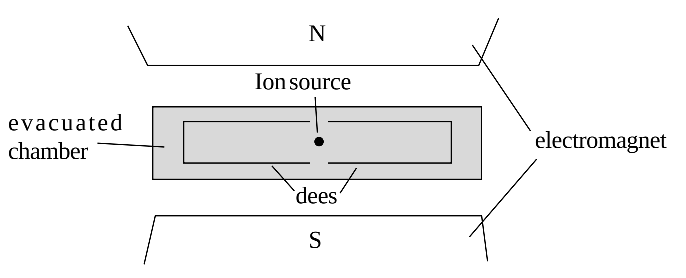
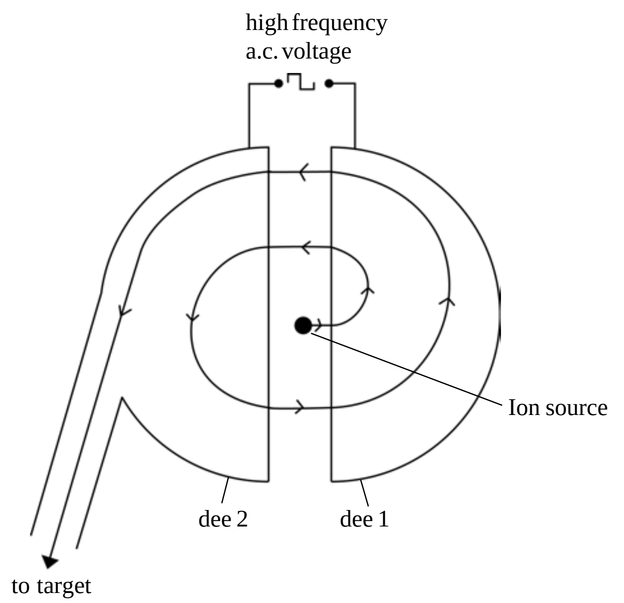


Fig 1(b) Cyclotron showing the path that the accelerating ions take



A cyclotron can be used to accelerate positively or negatively charged ions. In the following explanation the ions are considered to be positive ions.

- Positive ions are released from the ion source. These positive ions are accelerated towards dee **1**, which is initially given a negative potential to attract the positive ions.
- Once inside dee **1** the positive ions move at constant speed. A uniform magnetic field is applied perpendicularly to the dees by the electromagnet. This magnetic field makes the positive ions move in a circular path.
- The positive ions turn around and move towards the gap from dee **1** into dee **2**.
- The alternating voltage reverses giving dee **2** a negative potential. When the positive ions cross the gap from dee **1** into dee **2** they are accelerated towards dee **2** by the negative potential.

Once inside dee **2** the positive ions have a faster speed than when they were in dee **1** - this remains constant within the dee. The above sequence of events now repeats itself. The magnetic field makes the ions move in a semi circle back to the gap between the dees at which point the alternating voltage has reversed and so the positive ions are accelerated back across the gap towards dee**1**, which now has a negative potential once more.

As the speed increases, the radius of the circular path followed by the ions increases but the time that the ion spends in each dee remains constant. This means that the frequency of the alternating voltage remains constant.

The path of the ions

1. *The constant magnetic field applied perpendicular to the dees exerts a force on the ions that makes them move in a circular path.*
2. *The speed inside a dee remains constant.*
3. *The speed of the ions increases as they move from one dee to the next. This is because they are attracted to the oppositely charge dee.*
4. *The ions spend an equal amount of time in each dee, as they speed up, the circular path increases in radius and they travel further in each dee.*

Exam Hint: *Questions that require explanation type answers are best presented as 'bullet points' as shown in the explanations given about the cyclotron accelerator. They help you to order your thoughts and make it easier for the examiner to identify the important points that deserve a mark. All bullet points should remain as structured sentences as marks are also awarded on exam papers for your 'quality of written communication.'*

A closer look at the speed attained by accelerated particles

We will now derive an expression for the speed of the ions inside the cyclotron by applying Newton's second law of motion, but before we do this we need to know the following:

The force exerted on a charged particle moving perpendicular through a magnetic field acts as a ***centripetal force***, moving the particle in a circular path.

The size of this force is:

$F = Bqv$

B = *size of magnetic field (T)*
q = *charge on particle (C)*
v = *speed of particle (m/s)*

The ***acceleration of an object moving in a circle*** is given by the expression:

$a = \frac{v^2}{r}$ *v = speed of object (m/s)*
r = radius of circular path of the object (m)

Further explanation of the terms in italics and circular motion is given in Factsheet 19.

We are now in a position to use Newton's second law on the ions in the cyclotron that are moving in a circle:

$$F = ma \quad \text{where : } m = \text{mass of ion (kg)}$$

Substituting into this equation the expressions for centripetal force and acceleration that are stated above we get:

$$Bqv = \frac{mv^2}{r}$$

Rearranging this to make speed, v, the subject of the equation:

$$v = \frac{Bqr}{m}$$

For any particular ion, its charge, *q*, and its mass, *m*, will remain constant. This equation also shows that if the magnetic field strength is constant, the speed of the particle is proportional to the radius of its circular path. As the ions move from dee to dee, their speed increases. This means that the ions move in larger and larger circles as they accelerate up to the radius of the dees themselves, at which point they cannot be accelerated anymore and they are released towards the target.

The above equation also shows that a larger magnetic field will create a larger speed for the particles. In large cyclotrons, very powerful superconducting magnets are often used instead of electromagnets.

The maximum speed of the particles.
For any particular ion, the maximum speed that it can reach is limited by two factors:
1. The radius of the cyclotron
2. The maximum magnetic field strength that can be applied.

Limitations of a cyclotron

As particles approach very high speeds, close to the speed of light, their mass increases. This effect is predicted by special relativity. This means that the particles spend slightly longer in each dee. This means that the alternating voltage reverses too soon for the particles to be accelerated across the gap. The synchronisation of the voltage and the ions is lost. This happens at energies of about 20 MeV, meaning that a more sophisticated particle accelerator is needed for higher energies.

Typical Exam Question

(a) Calculate the energy gained by a proton in a cyclotron when it moves between the two dees if the voltage between the dees is 4.0 kV. State your answer in joules. (The charge on a proton is 1.6×10^{-19}C). [2]

(b) Calculate the speed of the proton after it made 500 complete cycles. Assume the mass of the proton stays constant at 1.7×10^{-27}kg. [4]

(c) Calculate the radius of the circular path that this proton travels around if the magnetic field strength used is 0.50T. [3]

(d) How long will it take a proton to complete one semicircular path inside a dee? [3]

(e) What frequency of alternating voltage should be used? [2]

Answer

(a) *The proton will gain energy equal to the electrical potential energy that it has at the start of its transfer between the two dees. Note that the question quotes kilovolts, kV, which must be converted into volts, V, before substituting into the equation.*

$$\text{Energy gained} = \text{charge} \times \text{voltage} = qV = (1.6\times10^{-19})(4000) = 6.4\times10^{-16} J$$

(b) *The energy gained by the proton will simply be 1000 times(500×two crossings per cycle) the energy calculated in part (a). We then have to equate all of this gain in energy to an increase in the kinetic energy of the proton.*

$$\text{Gain in kinetic energy} = (1000)(6.4 \times 10^{-16}) = 6.4 \times 10^{-13} J$$

$$\text{Therefore, } \tfrac{1}{2}mv^2 = 6.4 \times 10^{-13}$$

$$v^2 = \frac{(2)(6.4 \times 10^{-13})}{(1.7 \times 10^{-27})} = 7.53 \times 10^{14}$$

$$v = \sqrt{7.53 \times 10^{14}} = 2.7 \times 10^7 \text{ m/s}$$

(c) *Knowing values of charge and mass for the proton, and having calculated a value for speed, we can substitute these numbers into the equation derived earlier for the speed of ions in a cyclotron. The equation has to be rearranged to make radius, r, the subject of the equation,*

$$r = \frac{mv}{Bq} = \frac{(1.7 \times 10^{-27})(2.7 \times 10^7)}{(0.5)(1.6 \times 10^{-19})} = 0.58 \text{ m}$$

(d) *We can use the simple equation: speed = distance/time with time as the subject of the equation. We have a value for speed from part (b) and the distance travelled will be half a circumference of a circle with the radius calculated in part (c).*

$$\text{time taken} = \frac{\text{distance}}{\text{speed}} = \frac{\pi(0.58)}{(2.7 \times 10^7)} = 6.7 \times 10^{-8} \text{ s}$$

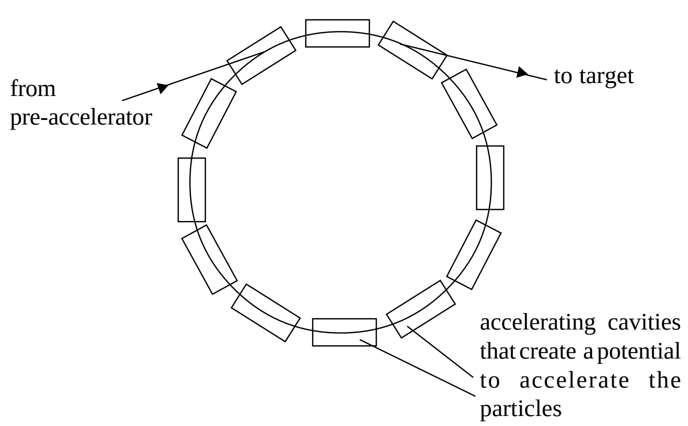
(e) *The periodic time for one complete revolution made by the particle* $= (2)(6.7 \times 10^{-8}) = 1.34 \times 10^{-7}$ *s*

$$\text{frequency} = \frac{1}{\text{periodic time}} = \frac{1}{1.34 \times 10^{-7}} = 7{,}500{,}000 \text{ Hz} = 7.5\text{MHz}$$

Exam Hint: *Be very careful when an answer requires you to use a value that you have previously calculated in the question. You must always carry forward as many significant figures as possible when using a calculated value in a subsequent part of the question. This is despite having to round these previous values up before quoting them as an answer.*

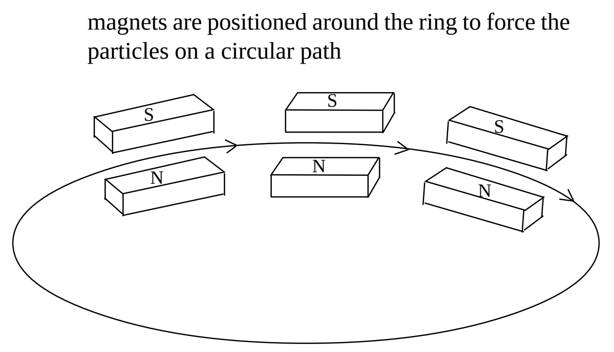
The Synchrotron
In a synchrotron, the radius of the circular path that the particles follow is kept constant. Hence the particles travel around an evacuated chamber that is shaped into a ring. A synchrotron consists of:
- An evacuated chamber shaped into a ring.
- A source of fast moving ions, usually from a 'pre-accelerator' such as a linear accelerator or a smaller circular accelerator.
- Magnets around the ring that can have a varying magnetic field strength.
- Accelerating cavities around the ring that increase the speed of the particles.

Fig 2 Plan view of a synchrotron



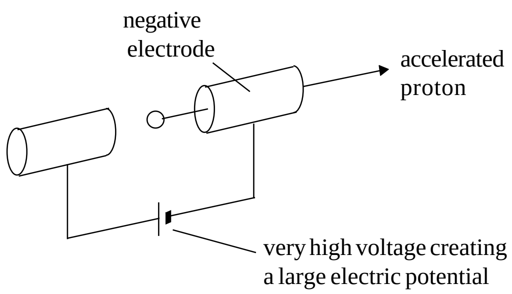
- The particles are kept moving in a circular path by magnets around the evacuated ring as shown in Fig 3.

Fig 3 Circular path of magnets



- The magnetic field strength of the magnets can be varied to make sure that the ions follow the same circular path regardless of their speed and their changing mass at higher energies.
- An electric potential is used to accelerate the particles in the accelerating cavities around the loop. Positive particles are accelerated towards a negative electrode inside the accelerating cavities as shown in Fig 4 below.

Fig 4. Particles accelerated towards a negative electrode



Because the magnets are around the ring, the centre of the magnet that has to be used in the cyclotron is not needed. This means that the radius of the circular path that the particles follow can be made much bigger (For example the largest synchrotron accelerator at CERN in Geneva has a circumference of 27km !)

The Changing Magnetic Field
The radius of the circular path of the particles must remain constant as they speed up and their mass increases at very high energies. This means that the size of the magnetic field must be varied and synchronised with the speed and mass of the particles.

Synchrotron Radiation
When charged particles move in a circular path, they lose kinetic energy. This energy is lost as electromagnetic radiation called '*synchrotron radiation*'. The loss of kinetic energy causes the particles to slow down unless they receive a boost of energy from the synchrotron. Therefore, just to keep the charged particles in a synchrotron moving at constant speed requires them to receive energy from the accelerator. This also means that a synchrotron can act as a source of electromagnetic radiation.

Synchrotron radiation
Charged particles moving in a circular path lose kinetic energy as electromagnetic radiation or synchrotron radiation. This would cause the particles to slow down and spiral into the centre of the accelerator unless they received constant boosts of energy from the accelerator.

Typical Exam Question
In a synchrotron, charged particles are kept moving in a circular path by magnets as they are accelerated to high energies.
(a) Give two reasons why it is necessary to increase the strength of the magnets as energy of the particles increases. (2)

(b) The particles radiate electromagnetic radiation, or synchrotron radiation as they move in a their circular path.
(i) Why do the particles need a constant boost of energy just to maintain their speed? (2)

(ii) How might this limit the energy that a charged particle can reach in any particular accelerator? (2)

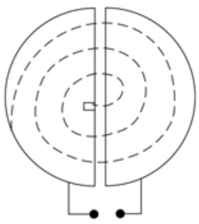
Answer
(a) A stronger magnetic field is needed to change the direction of faster moving particles.
The mass of the particles increases and a stronger magnetic field is needed to change the direction of more massive particles.

(b) (i) The particles lose kinetic energy as they emit electromagnetic radiation.
This kinetic energy is replaced, and the particle speed maintained, by a boost of energy from the accelerator
(ii) Faster particles emit more energy as radiation when moving in a circle.
More energy must be received from the accelerator to maintain speed.
An accelerator can only provide a finite amount of energy to the particles.

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

The diagram below represents a cyclotron that is used to accelerate protons. The protons enter the accelerator at the centre and spiral outwards.



alternating p.d

(a) Why do the protons accelerate moving from one half of the cyclotron into the opposite half? [2]

The protons are attracted to the opposite dee. 1/2

> Although the student has identified the force of attraction, the reason for the force, the negative potential of the opposite dee, has not been mentioned.

(b) The largest radius of the circular path that the protons can follow is equal to the radius of the cyclotron. Show that the maximum speed of a proton is equal to BqR/m. B is the magnetic field strength, q is the charge on a proton, R is the radius of the cyclotron and m is the mass of the proton. [3]

$$Bqv = \frac{mv^2}{r}$$

$$v = \frac{Bqr}{m}$$ 1/3

> The student has done quite well here by using the force on the charged particle and also knowing the acceleration of a particle moving in a circular path. Unfortunately the final answer has not included *R*, that radius of the cyclotron, as required in the question.

(c) Calculate the maximum speed of a proton in a cyclotron of diameter 0.80m when the magnetic field strength is 0.60T. The charge on a proton is 1.6×10^{-19}C and the mass is 1.7×10^{-27} kg.[2]

$$v = \frac{BqR}{m} = \frac{(0.6)(1.6 \times 10^{-19})(0.8)}{(1.7 \times 10^{-27})} = 4.5 \times 10^7 m/s$$ 1/2

> The student has incorrectly used 0.80m in the equation for radius. This is the diameter of the cyclotron.

(d) (i) State a difference in the path followed by a charged particle being accelerated by a synchrotron compared to the path shown above in a cyclotron. [1]

The path in a synchrotron is circular 1/1

> Although this statement would probably get the mark it would have been better to make reference to the constant radius of the circle.

(ii) What difference in the design of the synchrotron makes this path possible? [1]

The magnets 0/1

> This answer is not specific enough. The student should state what it is about the magnetic field that is responsible for the circular path.

Examiner's Answers

(a) The opposite dee always has a negative potential. ✓
The negative potential attracts the proton, increasing its speed. ✓

(b) Centripetal Force = mass × acceleration (in a circle)
$Bqv = mv^2/r \Rightarrow v = Bqr/m$ ✓
When the speed is at a maximum, the radius of the circle = R ✓
Maximum v = BqR/m ✓

(c) $v = \frac{BqR}{m} = \frac{(0.6)(1.6 \times 10^{-19})(0.4)}{(1.7 \times 10^{-27})} = 2.3 \times 10^7$ m/s ✓✓

(d) (i) In a synchrotron, the particles follow a complete circular path of constant radius instead of two semi-circular paths of different radius as in a cyclotron. ✓
(ii) The strength of the magnetic field can be varied. ✓

Qualitative (Concept Test)

(1) How does the speed of the particles being accelerated by a cyclotron change; (i) inside a dee and (ii) moving from one dee to the other?
(2) What keeps the particles moving in a circular path?
(3) What two factors limit the maximum speed that the particles can attain in a cyclotron?
(4) What are the limitations of a cyclotron and how does the synchrotron overcome these limitations?
(5) Why do charged particles moving in a circle lose kinetic energy?
(6) How is the energy of charged particles moving inside the circular tunnel of a synchrotron maintained?

Quantitative (Calculation Test)

1. (a) Calculate the energy given to a proton when it moves between 2 dees that have a potential of 1500V across them. The charge on a proton is 1.6×10^{-19}C. [2]
   (b) What would be the increase in speed of the proton of mass 1.7×10^{-27}kg? [3]

2. (a) Calculate the radius of the circular path followed by an electron travelling at a speed of 2.0×10^7 m/s when it is in a magnetic field of 0.10T, perpendicular to it motion. The charge on an electron is 1.6×10^{-19}C and the mass of an electron is 9.11×10^{-31}kg. [2]
   (b) How long would it take this electron to travel one semi circle? [3]

Qualitative Test Answers

Answers can be found in the text

Quantitative Test Answers

1. (a) Energy = qV = $(1.6 \times 10^{-19})(1500) = 2.4 \times 10^{-16}$ *J*

(b) Kinetic Energy = Energy gained

$$\tfrac{1}{2}mv^2 = 2.4 \times 10^{-16}$$

$$v^2 = \frac{2 \times (2.4 \times 10^{-16})}{1.7 \times 10^{-27}} = 2.8 \times 10^{11}$$

$$v = \sqrt{(2.8 \times 10^{11})} = 5.3 \times 10^5 \text{ m/s}$$

2. (a)

$$r = \frac{mv}{Bq} = \frac{(9.11 \times 10^{-31})(2 \times 10^7)}{0.1 \times (1.6 \times 10^{-19})} = 1.1 \times 10^{-3} \text{ m} = 1.1\text{mm}$$

(b)

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{(\tfrac{1}{2} \times \text{circumference})}{\text{speed}} = \frac{(\tfrac{1}{2})(2\pi)(1.1 \times 10^{-3})}{(2 \times 10^7)} = 1.8 \times 10^{-10} \text{ s}$$

*Acknowledgements: This Physics Factsheet was researched and written by **Jason Slack***
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Physics Factsheet

January 2003 Number 46

Principles of Detecting Particles

This Factsheet will explain:
- Principles of bubble and cloud chambers;
- Principles of spark/drift chambers;
- Interpretation of photographs showing particle tracks: charge and momentum.

This section is for synoptic papers and so assumes a knowledge of ideas contained in dynamics, mechanical energy, radioactive decay and the nuclear atom, the kinetic model of matter, circular motion and oscillations, electric and magnetic fields.

If you are uncertain of any of this material, it would be useful to revise it using other Factsheets as follows: 02 Vectors and Forces; 12 Newton's Laws; 13 Motion I; 11 Radioactivity I; 22 Radioactivity II; 25 Molecular kinetic Theory; 19 Circular Motion; 20 Simple Harmonic Motion; 33 Electric Field Strength and 45 Magnetic fields.

Principles of detection

Most particles are too small to be seen directly, so detectors usually show the paths or tracks of the particles. From the paths (particularly in electric or magnetic fields) and consideration of conservation of kinetic energy and momentum, the mass and/or charge of unknown particles can be deduced. The particles need to be ionizing radiation for most detectors to work.

Most detection instruments depend on the ionizing properties of the radiations. Non-ionizing radiations do not leave tracks.

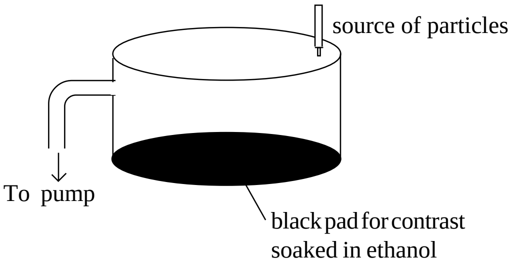
The Bubble Chamber

This consists of a chamber containing liquid hydrogen. The pressure is so low that the hydrogen is almost on the point of vaporizing. When an ionizing radiation enters the chamber it causes the hydrogen to ionize, which triggers vaporization and a trail of bubbles shows the track of the particle.

The Cloud Chamber

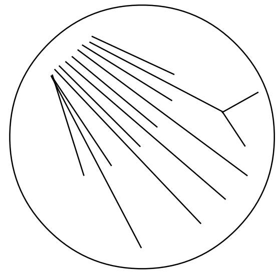
The Cloud Chamber is a chamber containing a pad soaked in a volatile liquid such as ethanol. An electric field is maintained between the source of particles and the sides of the chamber. As particles enter the chamber, the pressure is reduced so that the ethanol condenses onto the trail of ions of the gases in the air, left by the ionizing radiation. A simple laboratory version is shown in Fig 1.

Fig 1. Cloud chamber



Alpha particles (Helium nuclei) typically leave long straight paths. Their range in air is determined by their initial kinetic energy, as the kinetic energy is lost in ionizing collisions with atoms of the gases present. Sometimes the particle collides with a particle of one of the gases in the air in an elastic collision. In this case the tracks of the two particles are determined by the laws of conservation of kinetic energy and of momentum. Fig.2 shows typical tracks of alpha particles in a cloud chamber. There are alpha particles with two different energies being emitted by the source, as shown by the two different ranges, and an alpha particle has collided with another atom, as shown by the branched track.

Fig 2. Tracks in a cloud chamber

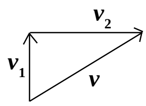


The total initial momentum is given by mv, and kinetic energy by $\frac{1}{2}mv^2$, since the target atom is stationary.

If the target atom has the same mass as the incoming particles, then the momentum after the collision is $mv_1 + mv_2$, where v_1 and v_2 are the velocities of the particles after the collision, and the kinetic energy is $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$. For conservation of momentum and kinetic energy then;

$mv = mv_1 + mv_2$
and $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

i.e $v = v_1 + v_2$
and $v^2 = v_1^2 + v_2^2$



Because v, v_1 and v_2 are vector quantities, then the only way that these two equations can be satisfied simultaneously is if v_1 and v_2 are at right angles to each other.

So if the tracks after a collision are at right angles then we know that the masses of the incoming particle and the target atom are the same.

Typical Exam Question
Unknown particles are detected in a Cloud Chamber containing only helium gas. If some of the tracks show branching into two paths at right angles, what can you say about the incoming particles? Explain your reasoning.

The incoming particles are alpha particles – helium nuclei. To produce tracks at right angles, conservation of kinetic energy and momentum gives the mass of the target atom as the same as that of the incoming particle.

Spark Chamber and Drift Chamber

These devices are filled with gas at low pressure and have thousands of parallel wires in them. An incoming particle causes ionization of the gas and the electrons released drift to the nearest wire. The track of the particle is then worked out electronically by timing how long it takes for the electrons to reach the nearest wires. A computer processes the signals and displays the results graphically.

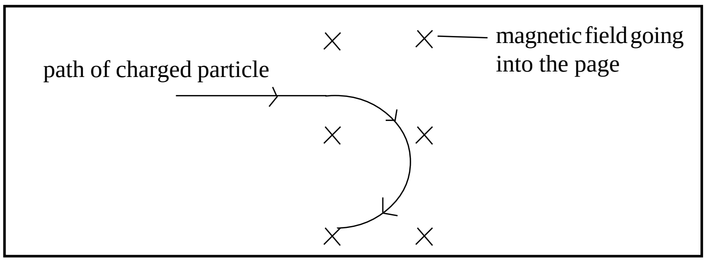
Interpreting Results
Charged particles are deflected by electric and magnetic fields, so if fields are applied to the particles being detected in a chamber, then the deflections help to show the nature of the particles being detected.

If a magnetic field is applied at right angles to the path of a particle, the particle is subject to a force Bqv, where B is the flux density of the field, q the charge and v the velocity of the particle. This force acts at right angles to the field and to the particle's velocity, so the effect is to provide the centripetal force necessary for the particle to move in a circle.

$$\frac{mv^2}{r} = Bqv \quad \text{thus} \quad \frac{mv}{r} = Bq$$

If the velocity of the particle is known (e.g. by having accelerated it through an electric field, the particle gains a kinetic energy = qV, where V is the potential difference through which it is accelerated) then its charge can be deduced from the known B field and the measured radius of the circle. Or if its charge is known then its momentum can be deduced. Fig 3 shows the effect.

Fig 3. Path of a charged particle in a magnetic field



The direction of the deflection can be worked out by Fleming's left-hand rule (remembering that the conventional current is the direction of flow of positive charges)

> *The path of a charged particle in a magnetic field is a circle in the plane at right angles to the flux*

Typical Exam Question
A particle of mass 9.3×10^{-26}kg is accelerated to a speed of 5.0×10^5ms^{-1} in an electric field. It then enters a magnetic field of 1.2 T at right angles to its path. The radius of its now curved path is observed to be 12.1 cm. Calculate the charge on the particle.

$$Bqv = \frac{mv^2}{r}$$

Therefore $$Bq = \frac{mv}{r}$$

$$q = \frac{9.3 \times 10^{-26} \times 5 \times 10^5}{1.2 \times 0.121} = 3.20 \times 10^{-19}C$$

Features of particle tracks
Fig 3 showed features of tracks in a magnetic field acting into the page. Fig 4 shows other features which can be easily deduced from tracks in a magnetic field.

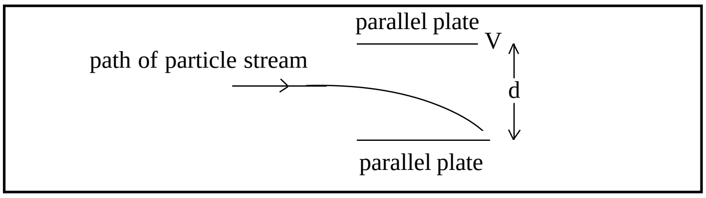
If an **electric fie**ld is applied at right angles to the path of a stream of particles, then they experience a force in the direction of the field given by

$$F = Eq \qquad \text{where } E = \text{field strength}, \; q = \text{charge on the particles}$$

($E = V/d$ for parallel plates separated by a distance d with p.d. V across them).

A uniform electric field causes the particles to describe a parabolic path, because the particles have a constant velocity in one direction and a constant acceleration in the direction at right angles. Fig 5

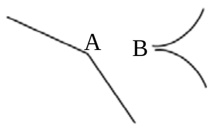
Fig 5



$F = \frac{qV}{d}$ therefore acceleration = $\frac{qV}{dm}$ where m = mass of the particle

V and d are known, so conclusions about q and m can be deduced from the track.

Typical Exam Question
(a) Explain the principles of a cloud chamber for detecting particles. [3]
(b) Tracks were obtained for particles moving in a magnetic field at right angles to the plane of the paper as shown below. Explain what can be deduced about what is happening at the places marked A and B. [3]



(c) What differences would there be between the tracks of (i) an electron and (ii) a proton in the same magnetic field. [2]

(a) The cloud chamber contains a pad soaked in ethanol, which vaporizes when the pressure is reduced. Incoming ionizing particles ionize the air in the chamber and the ethanol vapour condenses on the trail of ions, making the path visible.
(b) At A the particle has decayed into 2 particles, one of which is uncharged. At B the uncharged particle has decayed into 2 oppositely charged particles.
(c) The proton track would be curved in the opposite direction and would be thicker and shorter.

Fig 4. Tracks in a magnetic field into the path

Fast particles make a thin curved path

Particles loss K.E. through ionising collisions, so that the track gets more curved and thicker

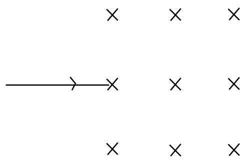
Slow or massive particles give more ionisation so thicker tracks

A decay into a charged particle and a massive particle, which itself decays into two opositely charged paticles

Exam Workshop

This is a typical weak student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

The diagram shows a beam of electrons of mass m_e and charge –e entering a magnetic field, B, into the plane of the paper at a speed v_e.



(a) In terms of the quantities given, write down an expression for the force required to constrain the electrons to move in a circular path of radius r_e. [1]

$F = m_e r\omega^2$ 0/1

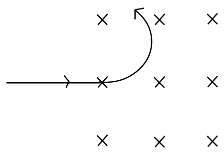
> Although this expression is correct, ω was not given in the question, and the candidate has failed to relate this to the quantities in the question.

(b) Derive an expression for the radius of the path in terms of B, m_e, e, and v_e. [2]

Force on the electron in a magnetic field = Bev and this force provides the necessary $m_e r\omega^2$ so $Bev = m_e r\omega^2$, therefore $r = Bev/m_e\omega^2$ 1/2

> Because the candidate does not know that $v = r\omega$, s/he has been unable to finish the derivation of the expression. No credit is given for knowing that the force on the electron in the magnetic field is Bev, because that is given on the paper.

(c) Draw on the diagram below the path of the electrons after they enter the magnetic field. [2]



1/2

> The candidate has correctly drawn the circular path, but has forgotton that the conventional current is a flow of +charge and so has come to the wrong direction, or has used right-hand instead of the left.

(d) A proton has a mass 2000 times that of the electron, and the same sized charge, describe the differences which its path would have if it entered the field at the same speed as the electron. [2]

From (b) r is proportional to 1/m, so the path of the proton would have a radius 2000 times smaller than that of the electron, and it would curve down instead of up. 2/2

> Although each of these conclusions is incorrect, the candidate has been given credit for error carried forward, because s/he has used earlier incorrect data appropriately.

Examiner's Answers

(a) $F = \dfrac{m_e v^2}{r}$

(b) $Bev = \dfrac{m_e v^2}{r}$ so $r = \dfrac{m_e v^2}{Bev} = \dfrac{m_e v}{Be}$

(c) *The path should be similar, but curving downwards.*

(d) *From (b) the radius is proportional to m, for the same v, B and charge, therefore the radius would be 2000 times larger. Since the charge is positive not negative, the path would curve upwards.*

Questions

1. Show how the expression for the force on a charged particle in a magnetic field, (*Bqv*) is derived from the expression *BIl* for the force on a current-carrying conductor.

2. An alpha particle has a charge of 3.2×10^{-19} C. Calculate the force that acts on it when it moves at a speed of 2.6×10^5 ms^{-1} at right angles to a magnetic field of strength 70 mT.

3. In a fine beam tube, electrons are accelerated from an electron gun through a potential difference of 300 V. They then cross a magnetic field of flux density 0.8 mT at right angles to their path.
   (a) Calculate the speed of the electrons leaving the electron gun.
   (b) Calculate the radius of the path the electrons follow in the magnetic field. $e = 1.6 \times 10^{-19}$C, $m_e = 9.1 \times 10^{-31}$kg.

Answers

1. *The transport equation gives I = nAqv, where n is the number of charge carriers per unit volume, and A is the cross-sectonal area of the conductor, so nAl gives the number of charge carriers in a length, l of wire. The force on one of these charge-carriers is thus* $\dfrac{BIl}{nAl}$ *,which equals* $\dfrac{BnAqvl}{nAl}$ *which equals Bqv.*

2. *Force = Bqv =* $70 \times 10^{-3} \times 3.2 \times 10^{-19} \times 2.6 \times 10^5 = 5.82 \text{ x } 10^{-15}N$

3. a) *k.e . of electrons = eV =* $1.6 \times 10^{-19} \times 300 = 4.8 \times 10^{-16} = ½\, m_e v^2$
   therefore $v^2 = \dfrac{4.8 \text{ x } 10^{-16} \times 2}{9.1 \text{ x } 10^{-31}} = 1.05 \text{ x } 10^{15}$, $v = 3.24 \text{ x } 10^7$
   b) $\dfrac{m_e v^2}{r} = Bev$, *therefore* $r = \dfrac{m_e v}{Be}$
   $= \dfrac{9.1 \times 10^{-31} \times 3.24 \times 10^7}{0.8 \times 10^{-3} \times 1.6 \times 10^{-19}} = 23cm$

Acknowledgements: *This Physics Factsheet was researched and written by* ***Janice Jones.***
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Physics Factsheet

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Particle Paths

The paths followed by subatomic particles in different situations can provide information about the particles. These include:
- charge
- momentum
- energy
- ionising properties

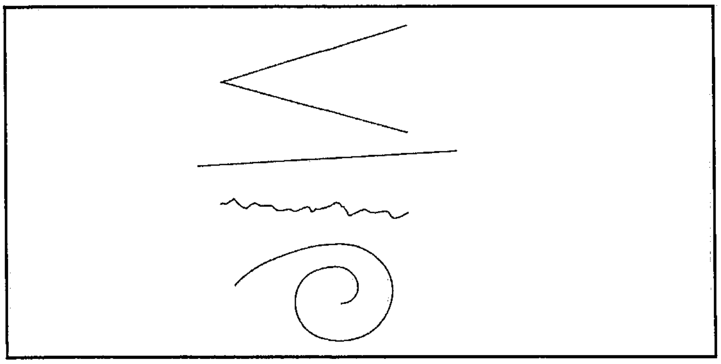
These paths can also provide evidence of particle annihilation and creation, and of particle decay.

How we observe these paths
We cannot observe the particles themselves, but we can make them leave evidence of their passing. The two standard devices used have been the bubble chamber and the cloud chamber. This factsheet is not intended as a study of these devices, but the way in which they work is worth a mention.

(a) Bubble Chamber
Charged particles ionise the molecules in a super-heated liquid as they pass through. Bubbles form around the ions created. These trails of bubbles then indicate the path of the charged particle.

(b) Cloud Chamber
The cloud chamber contains a supersaturated vapour. Charged particles ionise molecules in the vapour as they pass through. The ions act as centres for droplet formation (condensation). The paths of the particles can be seen as mist trails through the vapour.

This diagram shows some typical particle paths seen in a cloud or bubble chamber.



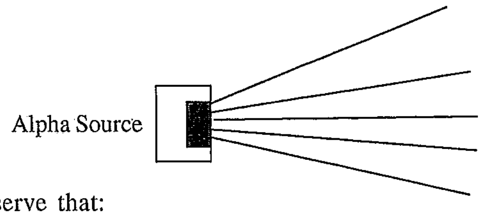
Both detectors only show the paths of charged particles. Uncharged (neutral) particles cannot be seen. However sometimes their paths can be inferred from other evidence.

Example 1:
Why can the paths of neutral particles not be detected?

Answer:
Ionisation is caused by a Coulomb force between the approaching particle and atomic electrons. An uncharged particle cannot exert a Coulomb force.

Alpha particles and beta particles
These two radioactive decay particles give very different results when their paths are observed in a cloud chamber:

(a)Alpha decay



We observe that:
- the paths are straight
- the paths are distinct
- the paths are of equal length

We can conclude from this that alpha particles suffer very little deflection during the ionisation process, that they are strongly ionising, and that they all have the same initial kinetic energy.

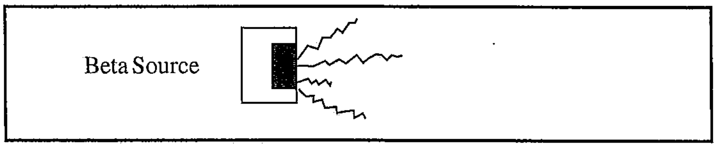
This fits in with our knowledge of alpha particles ($^{4}_{2}He$) as relatively massive charged particles.

Example 2:
Why should the alpha particles emitted from a source all have the same energy?

Answer:
The decay equation is identical for each unstable nucleus in the sample. Mass defect is converted into kinetic energy in the process.

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}\alpha$$

(b) Beta decay



This time we observe that:
- the paths are erratic
- the paths are faint
- the paths are of unequal length

The conclusion for beta particles is that they are easily deflected during collisions, and that they are only weakly ionising. They also seem to have different kinetic energies.

The first two conclusions fit in with our ideas that beta decay is composed of tiny charged particles $^{0}_{-1}e$. However the final conclusion concerning kinetic energies is an important one.

When beta decay occurs, lepton number conservation dictates that an antilepton must be emitted along with the beta particle (electron). This extra particle is the electron antineutrino. It carries away the rest of the kinetic energy.

Total decay energy = KE of electron + KE of antineutrino

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta + \bar{\nu}$$

It should be remembered that a similar process for a positron also occurs, with an electron neutrino carrying away the "missing" energy.

Calculations for alpha and beta particles

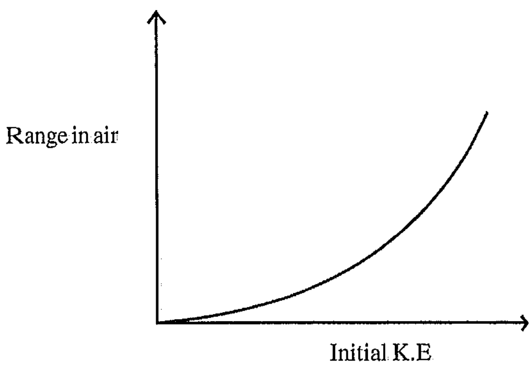
Alpha particles lose about 30eV in creating one ion-pair when travelling through air.

Example 3:
An alpha particle has a kinetic energy of 8.2×10^{-13}J. Find its kinetic energy in electron-volts. Then find the number of ion-pairs it would create in air.

Answer:
KE = $8.2\times10^{-13} / 1.6\times10^{-19} = 5.1\times10^{6}$eV = 5.1MeV
Number of ionisations = $5.1\times10^{6} / 30 = 1.7\times10^{5}$ ion-pairs

Example 4:
Would an alpha particle with twice the kinetic energy have twice the range through air?

Answer:
No. Alpha particles do not cause ionisations at a steady rate. As they slow down, the chance of a collision causing a successful ionisation increases markedly (they become more strongly ionising as they slow down). This leads to a graph of this form:



In a device called an ionisation chamber, the ion-pairs created in air each contribute one electron to the current around an external circuit. The current flow is expressed as:

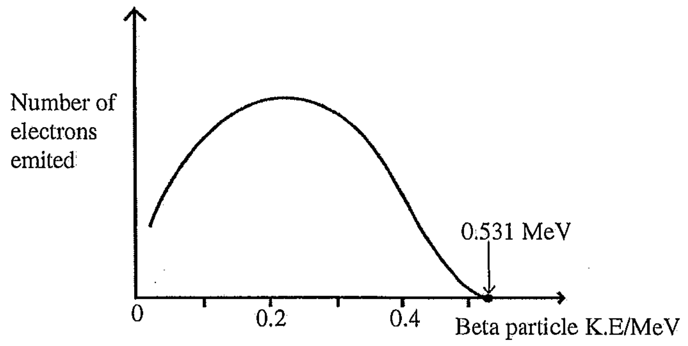
$I = Ane$

where A is the activity of the alpha source in Bq
n is the number of ion-pairs created by each alpha particle
e is the electron charge in coulombs

Example 5:
An alpha source has an average activity of 9000Bq. Each alpha particle creates 1.5×10^{4} ion-pairs. What current will be measured in the external circuit?

Answer:
I = Ane = $9000\times1.5\times10^{4}\times1.6\times10^{-19} = 2.2\times10^{-11}$ Amperes

With beta particles, a spectrum of kinetic energies exists. The rest of the energy is taken by the antineutrino. This is a typical energy spectrum graph for beta particles:



Example 6:
One partcular beta particle is emitted with a kinetic energy of 0.517MeV. Find the energy of the accompanying antineutrino.

Answer:
Energy = 0.531 – 0.517 = 0.014MeV

Applied Magnetic Fields

Often a magnetic field is applied perpendicular to the base of the cloud chamber. Any charged particle travelling across the chamber will be deflected in a curved path.

From circular motion theory:

$Bqv = mv^2 / r$ and so $p = mv = Bqr$ where p is momentum

As most particles have charge +e or –e, the momentum of the particle can be found.

And the charge can be determined as positive or negative by the direction of the deflection.

Example 7:
A particle of charge +e follows a track of radius 0.4m in a perpendicular field of 4 Tesla. Find the momentum of the particle.

Answer:
$p = Bqr = 4\times1.6\times10^{-19}\times0.4 = 3\times10^{-19}$ kgms^{-1}

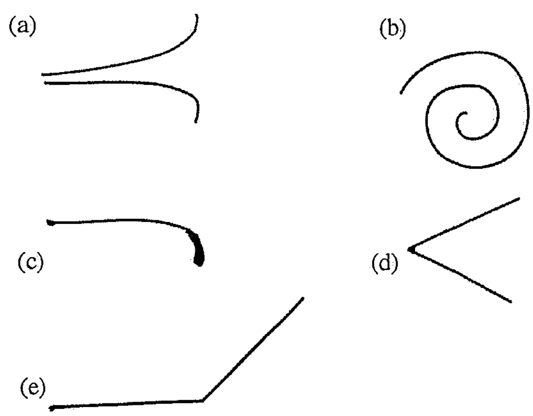
However charged particles never travel in a circle in a cloud or bubble chamber. They slow down (lose energy) as they travel through the medium:

(a) they lose energy through ionisation
(b) charged particles emit e.m. radiation as they accelerate — in this case they are accelerating because they are changing *direction* in the magnetic field.

This loss of speed causes the particle to spiral inwards. This effect is shown for an electron in the diagrams below.

As a charged particle in a magnetic field slows down, the radius of its path decreases.

Standard particle paths observed:



(a) Electrons and positrons curve in opposite directions in the magnetic field.

(b) An electron loses energy quickly through e.m. radiation, and it spirals inwards in the applied field.

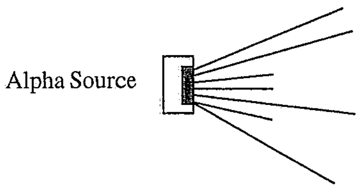
(c) A particle coming to rest leaves a dense track near the end, as its ionising power increases.

(d) A neutral particle (which cannot be seen) decays into two charged particles which travel in straight lines in the absence of a magnetic field.

(e) A charged particle has emitted a neutral particle (unseen) and changed direction (conservation of momentum).

Practice Questions

1. An alpha source produces the following pattern in a cloud chamber. Explain what is happening.



2. Find the radius of curvature of the track of a positron as it travels through an applied field of flux density 4.5 Tesla. The instantaneous momentum of the positron is 6.5×10^{-19} kgms^{-1}.

3. An alpha source in an ionisation chamber has an activity of 16 000 Bq. The ionisation current in the external circuit is measured as 1.9×10^{-12}A.
   (a) Find the number of ion-pairs created by each alpha particle.

   (b) If it takes 35eV to create an ion-pair, find the energy (in eV) of each alpha particle.

   (c) Convert this energy to Joules.

Answers

1. The sample emits alpha rays of a single energy. The nuclei decay into daughter nuclei which are also unstable, and emit alpha rays of a different energy.

2. $r = \frac{p}{Bq} = \frac{6.5 \times 10^{-19}}{4.5 \times 1.6 \times 10^{-19}} = 0.90m$

3. (a) $n = \frac{I}{Ae} = \frac{1.9 \times 10^{-12}}{16000 \times 1.6 \times 10^{-19}} = 740$ ion-pairs

   (b) $E = 740\times35 = 26\,000eV$

   (c) $E = 26\,000\times1.6\times10^{-19} = 4.2\times10^{-15}J$

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Physics Factsheet

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Wave–Particle Duality - the Electron

This Factsheet explains how electrons, which are usually considered to behave like tiny particles of matter, can also behave like waves in certain situations.

The wave particle duality of electrons will bring together ideas from several different areas of physics. Reference will be made in this Factsheet to the ideas covered in Factsheet 01 Quantum Nature of Light and Factsheet 17 Basic Wave Properties.

What is an electron?

There is no easy answer to this question as electrons can behave in different ways in different circumstances. An electron is usually considered to be a fundamental particle of matter. As a particle, an electron shows properties that we associate with particles - that is, it will experience a change in momentum if a force is applied. This is shown by an electron accelerating when inside an electric or magnetic field.

The property of an electron that allows it to accelerate when a force is applied is its **mass**. An electron has a mass, and so it will accelerate when a force is applied; this is governed by Newton's second law of motion.

As will be explained in this Factsheet, an electron can also behave as a wave in certain situations. In particular it is relatively easy to show an electron diffracting through a gap and producing an interference pattern - properties associated with waves. The ambiguous behaviour of the electron is known as **wave–particle duality**.

> ***Wave-Particle Duality***
> *Wave particle duality is the ability of what we normally consider to be a particle to behave as a wave, and the ability of what we usually consider a wave to behave as a particle. Whether something will behave as a wave or as a particle depends on the situation that it is placed in.*

How are the wave nature and particle nature of the electron linked?
In 1924, a young physicist called de Broglie explained how the particle nature and wave nature of all materials, including electrons were linked. He used two equations that you will have come across before:

$E = mc^2$ and $E = hf$ where: E = energy (J)
m = mass (kg)
c = speed of light = 3.0×10^8 ms^{-1}
h = the Planck constant = 6.63×10^{-34} Js
f = frequency of wave (Hz)

Consider the energy in each of these equations to be equivalent, so that they can be equal to each other.

$mc^2 = hf$

Now, from the wave equation, $c = f\lambda$ let us substitute for frequency,

$f = \frac{c}{\lambda}$ where λ = wavelength (m)

This gives us: $mc^2 = \frac{hc}{\lambda}$

which simplifies to: $mc = \frac{h}{\lambda}$

De Broglie noticed that the expression mc could be used to represent momentum, as momentum = mass × velocity. Using the symbol, p, to represent momentum he proposed that the particle property of momentum was linked to the wave property of wavelength for all materials by the relationship;

$p = \frac{h}{\lambda}$ where: p = momentum (Ns)
h = Planck's constant = 6.63×10^{-34} Js
λ = wavelength (m)

De Broglie modestly called this the de Broglie equation and the wavelength, λ, he called the de Broglie wavelength.

> ***The de Broglie Equation***
> *The wavelength associated with the wave properties shown by a particle is given by the de Broglie Equation;*
>
> $\lambda = \frac{h}{p}$ where: p = momentum of particle(Ns)
> h = the Planck constant = 6.63×10^{-34} Js
> λ = wavelength (m)

Typical Exam Question
(a) Explain what is meant by the term *wave particle duality*. [3]
(b) Calculate the de Broglie wavelength of a football of mass 0.15kg travelling at a speed of 8.0 ms^{-1}. [2]
(c) Comment on your answer to part (b). [1]

Answer
(a) Wave particle duality is the ability of what we normally consider to be a particle to behave as a wave ✓, and the ability of what we usually consider a wave to behave as a particle.✓
Whether something will behave as a wave or as a particle depends on the situation that it is placed in.✓

(b) This answer involves a straight forward substitution of values into the de Broglie equation, remembering that momentum = mass ×velocity. Remember also to quote your answer to the same number of significant figures as is given in the question, in this case 2.

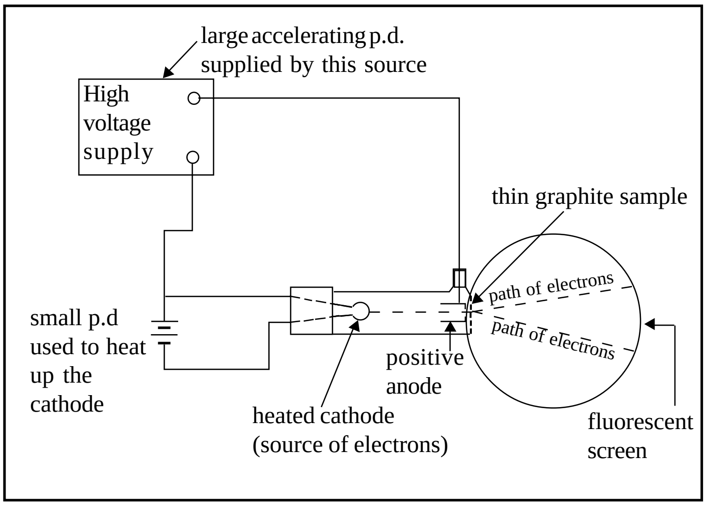
$\lambda = \frac{h}{p} = \frac{(6.63 \times 10^{-34})}{(0.15 \times 8)}$ ✓ $= 5.5 \times 10^{-34}$ m ✓

(c) This wavelength is extremely small, and the football would show negligible wave properties. ✓

> ***Exam Hint:*** *Always look at the number of marks given for a question that involves a written, descriptive answer. This helps you to decide how much detail to give in the answer. Part (a) of this last typical exam question has three marks and so the answer involves making three important points; each important point merits a mark.*

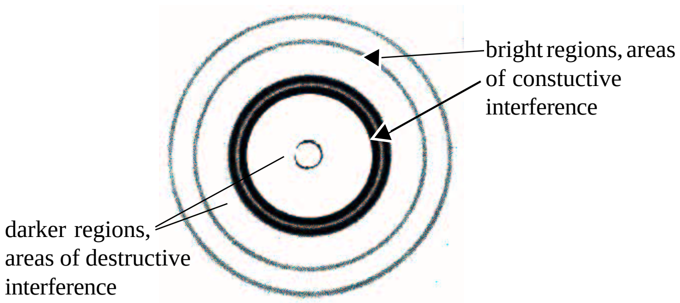
Demonstrating the wave nature of an electron
This last typical exam question demonstrates why we do not usually observe the wave properties of 'everyday' particles; their wavelength is so small that it is insignificant. This is not the case with electrons. Electrons have a very small mass, which means that their momentum is so small that their wavelength is large enough to be significant.

One way of demonstrating the wave nature of electrons is to show them diffracting through a small gap. The small gap used in the apparatus shown below is the spacing between carbon atoms in a very thin sample of carbon in the form of graphite.



The electrons are emitted from the heated cathode inside the evacuated tube. They are then accelerated across a few thousand volts of potential difference towards the positive anode. Some of the fast moving electrons pass through the hole in the anode and head towards the thin sample of graphite. Having passed through the graphite, they then strike the fluorescent screen at the end of the evacuated tube, causing the screen to glow in a typical interference pattern consisting of bright and dark regions.

Looking at the end of the tube, the illuminated pattern on the fluorescent screen looks like the following:



Two wave properties are demonstrated by this pattern:
1. Diffraction. The original beam of electrons is narrow and focussed as it passes through the graphite. The electrons spread out as they pass through the graphite, showing a broad area of illumination on the fluorescent screen.
2. Interference. The pattern of illumination on the screen clearly shows a pattern of concentric rings. The darker regions are areas of destructive interference and the brighter regions are areas of constructive interference.

This experiment also tells us something about the molecular structure of graphite. For any noticeable diffraction of a wave to occur, the gap that the wave passes through must be of a comparable size to the wavelength of the wave. Therefore, if the momentum of the electron, and hence its wavelength, can be calculated, then an estimate for the separation of the atoms in the graphite can be obtained.

Electron wave diffraction and atomic spacing
If the wave associated with a moving electron shows noticeable diffraction as it passes through a thin sample of material, then the separation of the atoms of the thin material, will be approximately the same size as the wavelength of the wave.

Evidence for particle properties of an electron
The diffraction experiment described above also demonstrates the particle properties of an electron. The electron behaves as a particle when it is accelerated by the large electric field caused by high p.d. between the cathode and the anode.

Typical Exam Question
(a) Consider an electron that is accelerated through a large potential difference so that it is given a kinetic energy of 1.8×10^{-16}J. Given that the mass of an electron is 9.11×10^{-31}kg, calculate the speed of the electron. [2]
(b) Calculate the wavelength associated with this electron. [2]
(c) This electron is seen to diffract when it passes through a thin specimen of graphite. What can you deduce about the atomic spacing of the atoms of carbon in the graphite sample? [1]
(d) Electrons can also be used to diffract around atomic nuclei. Explain why the electron used in this question would need to have a much bigger kinetic energy if it is to show a diffraction pattern from an atomic nucleus. [3]

Answer
(a) In order to answer this question the familiar expression that includes kinetic energy and speed is needed;
kinetic energy = ½mv². ✓
$$v^2 = \frac{(2)(1.8 \times 10^{-16})}{(9.11 \times 10^{-31})} ✓ = 3.95 \times 10^{14}$$
$$v = 1.99 \times 10^{7}\ ms^{-1}\ ✓$$
(b) We are now in a position to substitute values into de Broglie's equation
$$\lambda = \frac{h}{p} = \frac{(6.63 \times 10^{-34})}{(9.11 \times 10^{-31} \times 1.99 \times 10^{7})} ✓ = 3.7 \times 10^{-11} m\ ✓$$
(c) The atomic spacing must be approximately the same size as the wavelength. ✓
(d) The atomic nucleus is much smaller than the atom. ✓
For noticeable diffraction to occur the wavelength of the electron must be much smaller ✓
de Broglie's equation means that a small wavelength requires a larger momentum ✓
A larger momentum requires a the electron to have a larger kinetic energy . ✓

Exam Workshop

This is a typical weak student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

The spacing of atoms in a thin crystal sample is 1.2×10^{-10}m.
mass of electron = 9.11×10^{-31} kg
Planck's constant = 6.63×10^{-34} Js

(a) Estimate the speed of electrons that would give appreciable diffraction with this crystal. [3]

$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.2 \times 10^{-10}} = 5.53 \times 10^{-24}$ m/s *2/3*

> Although credit would be gained for writting de Broglie's equation and also for recognising that the wavelength should be approximately the same size as the atomic spacing, this student has calculated momentum, when the question has asked for a speed (the student's misunderstanding is further emphasised by his/her use of units of speed for the momentum calculation).

(b) State and explain how the diffraction pattern observed changes when the electrons travel through the crystal with a smaller velocity [3]

The diffraction pattern is bigger. 1/3

> What the student has said is correct as the diffraction pattern would be spread out over a larger region. However, no explanation has been given and so, at best, 1 out of 3 marks can be gained.

(c) Give two pieces of evidence to demonstrate that electrons have particle properties. [2]

They have mass. 0/3

> Although mass is a property associated with particles this response does not answer the question. The question asks for ***evidence*** of the particle properties.

Examiner's Answers

(a) $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.2 \times 10^{-10}}$ ✓ $= 5.53 \times 10^{-24}$ Ns ✓

$v = \frac{p}{m} = \frac{5.53 \times 10^{-24}}{9.11 \times 10^{-31}} = 6.1 \times 10^{6}$ m/s ✓

(b) *The momentum of the electrons is smaller* ✓
The associated de Broglie wavelength is larger. ✓
More diffraction is seen with a larger wavelength and so the spacing of the fringes would increase and the pattern would cover a larger area. ✓

(c) *Electrons can be accelerated in an electric field* ✓
electrons can be deflected by a magnetic field. ✓

Qualitative (Concept) Test

(1) Explain what is meant by the term "wave-particle" duality
(2) State some evidence that shows that electrons can behave as particles.
(3) State some evidence that shows that electrons behave as waves.
(4) An electron shows noticeable diffraction when passing through a gap.
 (a) What can be said about the size of the gap compared to the wavelength of the electron?
 (b) How does this effect help to reveal the size of atomic spacing when using electrons?

Quantitative (Calculation) Test

In the following calculations take the mass of an electron to be 9.11×10^{-31} kg, Planck's constant to be 6.63×10^{-34} Js and the speed of light in a vacuum to be 3.0×10^{8} m s^{-1}.

(1) Calculate the wavelength of an electron travelling with a momentum of 4.7×10^{-24} Ns. [2]
(2) Calculate the speed with which an electron will be travelling if it has an associated wavelength of 1.6×10^{-10} m. [3]
(3) (a) Calculate the momentum of an electron travelling in a vacuum at 5% the speed of light. [1]
 (b) Calculate the wavelength of the associated electron wave. [2]
(4) An electron is travelling with a kinetic energy of 1.5×10^{-16} J.
 (a) Calculate the speed with which the electron is travelling. [2]
 (b) Calculate the momentum of the electron. [2]
 (c) Calculate the wavelength of the associated electron wave. [2]

Quantitative Test Answers

(1) $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.7 \times 10^{-24}}$ ✓ $= 1.4 \times 10^{-10}$ m ✓

(2) $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-10}}$ ✓ $= 4.14 \times 10^{-24}$ m ✓

$v = \frac{p}{m} = \frac{4.14 \times 10^{-24}}{9.11 \times 10^{-31}} = 4.5 \times 10^{6}$ m/s ✓

(3) (a) $v = \frac{5}{100} \times (3 \times 10^{8}) = 1.5 \times 10^{7}$ m/s ✓

(b) $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31} \times 1.5 \times 10^{7})} = 4.9 \times 10^{-11}$ m ✓

(4) (a) kinetic energy = ½mv^2

$v^2 = (2)\frac{(1.5 \times 10^{-16})}{(9.11 \times 10^{-31})}$ ✓ $= 3.29 \times 10^{14}$

$v = 1.81 \times 10^{7}$ m/s ✓

(b) $p = mv = (9.11 \times 10^{-31})(1.81 \times 10^{7})$ ✓ $= 1.65 \times 10^{-23}$ Ns ✓

(c) $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.65 \times 10^{-23}}$ ✓ $= 4.0 \times 10^{-11}$ m ✓

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