CAIE Physics A-level

Topic 8: Gravitational Fields

Notes

8 - Gravitational Fields (A-level only)

8.1 - Gravitational Field

A **force field** is an area in which an object experiences a **non-contact force**.

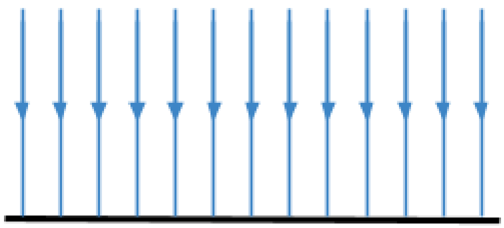
Force fields are formed during the **interaction of masses, static charges or moving charges**. Different types of fields are formed depending on which interaction takes place:

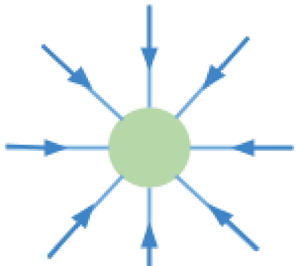
- **Gravitational fields** - formed during the interaction of masses
- **Electric fields** - formed during the interaction of charges (covered in section 17)

Therefore, a gravitational field is an **area where objects with mass experience a non-contact force**.

There are two types of gravitational field:

➔ **Uniform field -** exerts the **same** gravitational force on a mass everywhere in the field
➔ **Radial field -** the **force exerted depends on the position** of the object in the field


Uniform field


Radial field

The arrows on the field lines show the direction that a force acts on a mass, and the distance between field lines represents the strength of the force exerted by the field in that region. The closer the lines, the stronger the gravitational field strength.

The Earth's gravitational field is radial, however very close to the surface it is almost completely uniform.

Gravitational field strength (g) is the force per unit mass exerted by a gravitational field on an object. This value is constant in a uniform field, but varies in a radial field. You can use the following formula to calculate the gravitational field strength:

$$g = \frac{F}{m}$$

Where F is the force exerted and m is the mass of the object in the field.

8.2 - Gravitational Force between Point Masses

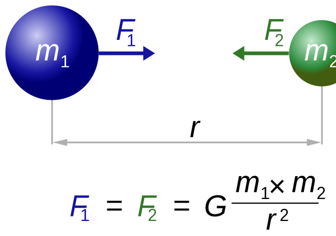
Gravity acts on any objects which have **mass** and is **always attractive**.

Newton's law of gravitation states that the magnitude of the gravitational force between two masses is **directly proportional to the product of the masses**, and is **inversely proportional to**

the square of the distance between them, (where the distance is measured between the two centres of the masses).

$$F = \frac{Gm_1 m_2}{r^2}$$

Where G is the gravitational constant, m_1/m_2 are masses and r is the distance between the centre of the masses.



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It is important to note that the mass of a uniform sphere is considered to act as a **point mass at its centre** when calculating the gravitational force experienced by an object outside the sphere.

The **gravitational field strength (g)** in a **radial field** follows the equation (which is derived in section 8.3):

$$g = \frac{GM}{r^2}$$

Where G is the gravitational constant, M is the mass of the object causing the field and r is the distance between the centre of the masses.

As you can see the field strength follows an **inverse square relationship**, meaning that if its distance increases by a factor of 2, the field strength will decrease by a factor of (2^2 =) 4 as seen in the equation.

Objects, like satellites, orbit around masses due to their gravitational fields **as the gravitational force exerted on the object acts as a centripetal force**, which causes a **centripetal acceleration** causing them to move in a circle.
Therefore, for an orbiting object, you can **equate** the gravitational force and centripetal force equations:

$$Centripetal\ force = \frac{mv^2}{r} \qquad Gravitational\ force = \frac{GMm}{r^2}$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

Cancelling m and r gives:

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

As $v = r\omega$, the above equation can be rewritten as: $r\omega = \sqrt{\frac{GM}{r}}$ $\quad \omega = \frac{\sqrt{\frac{GM}{r}}}{\sqrt{r^2}}$ $\quad \omega = \sqrt{\frac{GM}{r^3}}$

The above equation shows that the **angular velocity (ω)** of an object orbiting a mass **is dependent on orbital radius (r)**. And as angular velocity is equal to the product of 2π and frequency, the **frequency (f), and therefore time period (T) are also dependent on orbital radius (r)**.

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}}$$

A **synchronous orbit** is one where the **orbital period of the satellite is equal to the rotational period of the object that it is orbiting**.

Geostationary satellites follow a specific geosynchronous orbit, meaning their orbital period is **24 hours** and they always stay above the **same point** on the Earth, because they orbit directly above the equator. These types of satellites are very useful for sending **TV and telephone signals** because they are always above the same point on the Earth so you don't have to alter the plane of an aerial or transmitter.

In order to find the orbital radius of a geostationary satellite you can use the relationship that was derived above, as we know the orbital period (T) is 24 hours:

$$\frac{2\pi}{T} = \sqrt{\frac{GM}{r^3}} \quad \Rightarrow \quad r^3 = \frac{GMT^2}{4\pi^2}$$

$$r^3 = \frac{6.67\times10^{-11}\times5.97\times10^{24}\times(24\times60\times60)^2}{4\pi^2} = 7.53\times10^{22}\text{ m}$$

$$r = 4.2\times10^7\text{ m}$$

Which is around 36 000 km above the Earth's surface.

8.3 - Gravitational Field of a Point Mass

The equation for the **gravitational field strength (g)** in **radial field** can be derived by using the definition of gravitational field strength and Newton's law of gravitation:

$$g = \frac{F}{m} \qquad F = \frac{GMm}{r^2}$$

$$g = \frac{\frac{GMm}{r^2}}{m}$$

$$g = \frac{GM}{r^2}$$

Where G is the gravitational constant (6.67×10^{-11} N m^2 kg^{-2}), M is the mass of the object causing the field and r is the distance between the centre of the masses.

Below are some example questions using the formula above:

Find the gravitational field strength 10 km above the surface of the Earth.
The radius of the Earth is 6.37×10^6 m. The mass of the Earth is 5.97×10^{24} kg.

Firstly, calculate the distance from the centre of the Earth to 10 km above the Earth, remembering to convert km to m. This can be done by finding the sum of 10 km and the radius of the Earth:

$r = 10 \times 10^3 + 6.37 \times 10^6 = 6.38 \times 10^6$ m

Next, calculate the gravitational field strength using the above formula.

$g = \frac{GM}{r^2}$ $\quad\quad$ $g = \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6.38\times10^6)^2}$ = **9.8 N kg^{-1}** (to 2 s.f)

Find the gravitational field strength 1000 km above the surface of the Earth.
The radius of the Earth is 6.37×10^6 m. The mass of the Earth is 5.97×10^{24} kg.

Firstly, calculate the distance from the centre of the Earth to 1000 km above the Earth, remembering to convert km to m.

$r = 1000 \times 10^3 + 6.37 \times 10^6 = 7.37 \times 10^6$ m

Next, calculate the gravitational field strength using the above formula.

$g = \frac{GM}{r^2}$ $\quad\quad$ $g = \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(7.37\times10^6)^2}$ = **7.3 N kg^{-1}** (to 2 s.f)

As shown by the example questions above, the gravitational field strength close to/on the surface of the Earth is approximately constant and equal to 9.81 N kg^{-1}.

8.4 - Gravitational Potential

Gravitational potential (ϕ) at a point is the **work done per unit mass when moving a small point mass from infinity to that point**. Gravitational potential **at infinity is zero**, and as an object moves from infinity to a point, energy is released as the gravitational potential energy is reduced, therefore **gravitational potential is always negative**.

$$\Phi = -\frac{GM}{r}$$

Where M is the mass of the object causing the field, G is the gravitational constant (6.67 x 10^{-11} N m^2 kg^{-2}), and r is the distance between the centres of the objects.

Below is an example question using the formula above:

Find the amount of energy required for a 50 kg mass to leave the Earth's gravitational field from the surface of the Earth.
The radius of the Earth is 6.37×10^6 m. The mass of the Earth is 5.97×10^{24} kg.

Firstly, using the formula above, find the gravitational potential at the surface of the Earth.

$\Phi = -\frac{GM}{r}$ $\Phi = -\frac{6.67\times10^{-11}\times5.97\times10^{24}}{6.37\times10^{6}} = -6.25\times10^{7}$ J kg^{-1}

Therefore, 6.25×10^{7} N kg^{-1} will be needed to **move a unit mass from the Earth's surface to infinity**, so multiply the value of potential by the mass (which is 50 kg), to find the energy required.

$6.25\times10^{7}\times50 = 3.13\times10^{9}$ J (to 3 s.f)