CAIE Physics A-level

Topic 3: Kinematics

Notes

3 - Kinematics

3.1 - Equations of Motion

- **Distance** - The distance travelled by an object is a scalar quantity and describes the amount of ground the object has covered.

- **Displacement (s)** - The overall distance travelled from the starting position (includes a direction and so it is a vector quantity).

- **Speed** - This is a scalar quantity which describes the distance travelled per unit time.

- **Velocity (v)** - rate of change of displacement - $\dfrac{\Delta s}{\Delta t}$

- **Acceleration (a)** - rate of change of velocity - $\dfrac{\Delta v}{\Delta t}$

- **Uniform acceleration** is where the acceleration of an object is constant.

Acceleration-time graphs represent the change in velocity over time. Therefore the **area under the graph is change in velocity**.

Velocity-time graphs represent the change in velocity over time. Therefore the **gradient of a velocity time graph is acceleration**, and the **area under the graph is displacement**.

Displacement-time graphs show change in displacement over time, and so their **gradient represents velocity**.

Instantaneous velocity is the velocity of an object at a specific point in time. It can be found from a **displacement-time graph** by drawing a tangent to the graph at the specific time and calculating the **gradient**.

Distance and speed can be represented in distance-time graphs and speed-time graphs respectively. Note that unlike displacement and velocity, distance and speed **will never be negative** as they are **scalar** values.

In order to derive the **uniform acceleration equations**, you must consider the following velocity-time graph of a uniformly accelerating object:

1. The area beneath a velocity-time graph is displacement, therefore:

   $s = area\ of\ rectangle\ +\ area\ of\ triangle$

   $s = ut\ +\ \frac{t}{2}(v - u)$

   $s = ut\ +\ \frac{vt}{2} - \frac{ut}{2}$
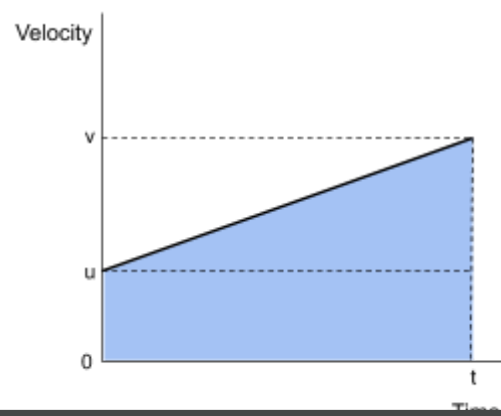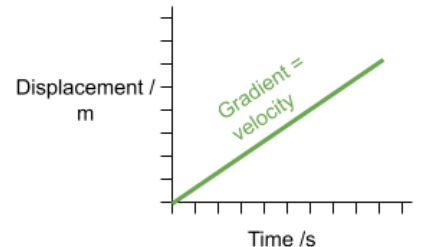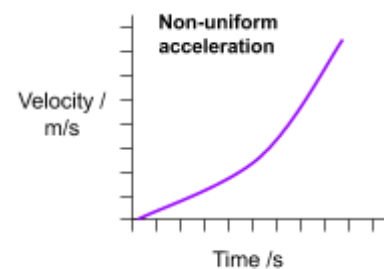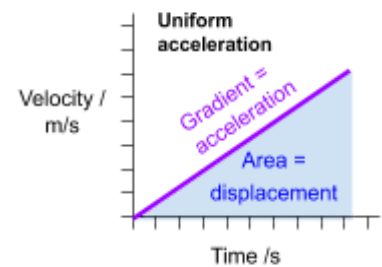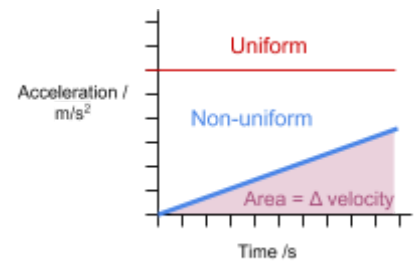
$s = \frac{ut}{2} + \frac{vt}{2}$ $\qquad$ $s = (\frac{u+v}{2})t$

2. The gradient of a velocity-time graph is the acceleration, therefore:

$a = \frac{v-u}{t}$ $\quad$ which rearranges to,

$v = u + at$

3. The area beneath a velocity-time graph is displacement, therefore:

$s = area\ of\ rectangle\ +\ area\ of\ triangle$

$s = ut + \frac{t}{2}(v - u)$

Substitute in the equation $at = v - u$ (from above)

$s = ut + \frac{t}{2}(at)$

$s = ut + \frac{1}{2}at^2$

4. The last equation can be found by substituting the equation from part 2 ($v = u + at$) into the equation from part 1 ($s = (\frac{u+v}{2})t$).

Rearrange $v = u + at$, to get:

$t = \frac{v-u}{a}$

Substitute this into $s = (\frac{u+v}{2})t$, to get:

$s = (\frac{u+v}{2}) \times (\frac{v-u}{a})$

$s = \frac{v^2-u^2}{2a}$

$v^2 = u^2 + 2as$

In summary, when an object is moving at **uniform acceleration**, you can use the following formulas:

$v = u + at$ $\qquad$ $s = (\frac{u+v}{2})t$ $\qquad$ $s = ut + \frac{at^2}{2}$ $\qquad$ $v^2 = u^2 + 2as$

Where **s** = displacement, **u** = initial velocity, **v** = final velocity, **a** = acceleration, **t** = time

When approaching questions which require the use of these formulas, it is useful to write out the values you know, and the ones you want to find out in order to more easily choose the correct formula to use.

For example:
A stone is **dropped** from a bridge 50 m above the water below. What will be its final velocity (v) and for how long does it fall (t)?

Note, in this example the stone is dropped therefore we can assume that the initial velocity is zero. Also because the stone is dropped, we know its acceleration will be g (9.81 m/s^2), which is the acceleration due to gravity.

s = 50 m   **u = 0 m/s**   **v = ?**   **a = 9.81 m/s^2**   **t = ?**

Using $v^2 = u^2 + 2as$, you can find v.

$v^2 = 0^2 + 2 \times 9.81 \times 50$ $v^2 = 981$ $v = 31.3$ m/s

Using $s = ut + \frac{at^2}{2}$, you can find t.

$50 = 4.905t^2$ $t^2 = 10.19$ $t = 3.19$ s

You can find the value of **g, the acceleration due to gravity**, experimentally, and below is a method you could follow:

1. Set up the apparatus as shown in the diagram, connecting the light gates to a data logger and as close to the electromagnet as possible.
2. The position of the lower light gate should be adjusted such that the height h is 0.75 m, measured using the metre ruler.
3. Turn on the electromagnet and attach the ball bearing.
4. Switch off the electromagnet, and note the time taken for the bearing to fall between the light gates (t) as recorded by the data logger.
5. Reduce h by 0.05 m by moving the lower light gate upwards and repeat the above two steps, reducing h by 0.05 m each time, down to 0.25 m.
6. Repeat the experiment twice more to find mean values of t for each value of h.
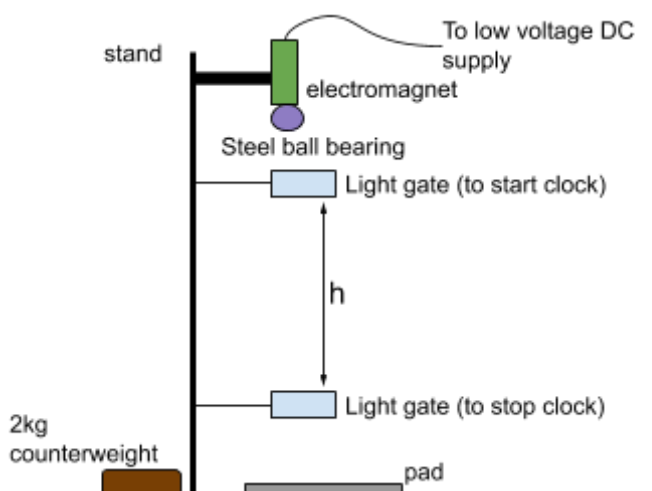
In the above investigation, the acceleration (g) is constant, meaning one of the constant acceleration equations can be used to analyse the results.

Consider $s = ut + \frac{1}{2}at^2$:

The displacement of the ball bearing (s) will be h, its initial velocity (u) is 0, and its acceleration is g, so the above equation becomes:

$h = \frac{1}{2}gt^2 \Rightarrow$ $2h = gt^2$

Therefore if you plot a **graph of 2h against (the mean value of) t^2, its gradient will be g**.

When an object is moving at a **constant velocity** in one direction, and experiences a **uniform acceleration in the perpendicular direction**, it will follow a **parabolic** shape, as shown in the diagram to the right.

At first, the object only experiences a horizontal velocity (v_x), however as time goes on, its vertical velocity (v_y) increases due to the uniform acceleration.

This causes the object to **change direction**, and as the time passed increases, the degree by which the direction has changed increases, as shown by the **parabolic shape**.

Uniform velocity, v_x

Uniform acceleration

v_x

v_y

v_x

v_y