CAIE Physics A-level

Topic 19: Capacitance

Notes

19 - Capacitance

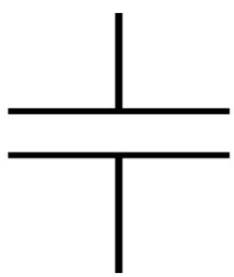
19.1 - Capacitors and Capacitance

A **capacitor** is an electrical component that stores charge. A parallel-plate capacitor is made up of two parallel conducting plates with an insulator (dielectric) between them. An electrically isolated spherical conductor can also act as a capacitor. The measure of how much charge can be stored per unit potential difference is known as the **capacitance**.

The equation for capacitance is

$$C = \frac{Q}{V}$$

where C is the capacitance measured in **farads (F)**, Q is the stored charge and V is the potential difference across the terminals of the capacitor.
A capacitance of 1 farad is defined as 1 coulomb of charge stored per volt of potential difference.



This is the circuit symbol for a capacitor.

When multiple capacitors are connected in **series**, the total capacitance is equivalent to the combined spacing of all the plates in every capacitor in the circuit. Since capacitance is **inversely proportional** to the spacing, the combined capacitance is less than each individual one. The **total capacitance in series** is then:

$$\frac{1}{C_{TOT}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

In **parallel**, the total capacitance can be thought of as the sum of the plate areas of all the capacitors. As plate area is **proportional** to capacitance, the **total capacitance in parallel** is the sum of the individual ones:

$$C_{TOT} = C_1 + C_2 + C_3 + \ldots$$

Alternatively these equations can be derived using the formula $C = Q/V$. Rearrange this as $V = Q/C$ and note that the voltage across each capacitor can be written as $V_1 = Q/C_1$, $V_2 = Q/C_2$, $V_3 = Q/C_3$, and so on. The total voltage in series and in parallel can be recalled from 'Topic 10 D.C. Circuits'.

In series, the total voltage is the sum of each individual voltage so

$$V_{TOT} = \frac{Q}{C_{TOT}} = V_1 + V_2 + V_3 + \ldots = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \ldots.$$

Cancelling the Q terms yields the expression given before.

In parallel, the total charge is equal to the charge in each parallel capacitor,

$$Q_{TOT} = Q_1 + Q_2 + Q_3 + \ldots = C_1 V + C_2 V + C_3 V + \ldots = C_{TOT} V$$

cancelling the common voltage from each term gives the previous expression.

19.2 - Energy Stored in a Capacitor

Since a capacitor stores charge, it is also storing energy in the form of **electrical potential**. If you plot a graph of the voltage of the capacitor against the charge stored in it you will obtain a straight line graph reflecting the direct proportionality of the two terms. The area under this graph can be calculated as the area of a triangle:

$$Area = \tfrac{1}{2} \times Base \times Height = \tfrac{1}{2} QV = \tfrac{1}{2} CV^2$$

This is equivalent to the **energy stored in the capacitor**.

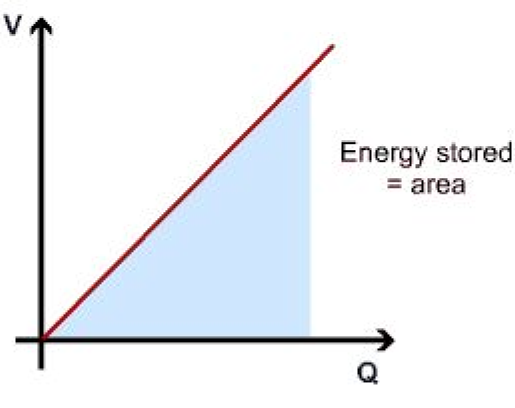


Image source:
https://www.s-cool.co.uk/a-level/physics/capacitors/revise-it/time-constant-and-energy-stored-in-capacitors

19.3 - Discharging a Capacitor

When the energy stored in the capacitor has reached the desired amount, it can be **discharged** to release a current that decreases over time.

The rate at which the capacitor discharges is proportional to the amount of charges still being stored. This results in an **exponential** curve when plotting remaining charge against time during discharging.

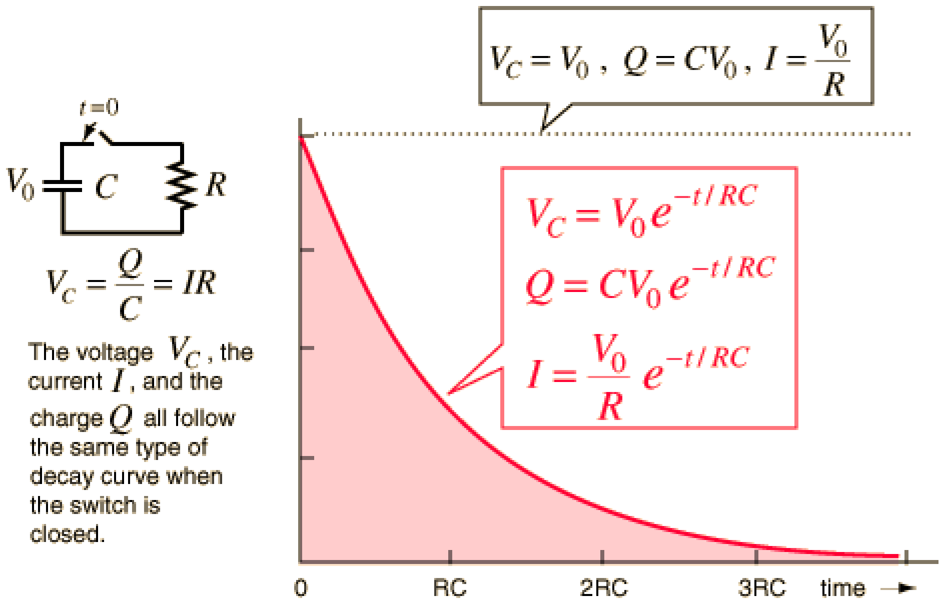


Image source: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/capdis.html

As seen in the previous image, the voltage, current, and charge of the capacitor follow a **negative exponential** relationship with time. The quantity RC is the product of the capacitance C and resistance R of the circuit.

RC is also equivalent to τ , the **time constant**, which represents the time after which the current/voltage/charge falls to equal $\frac{1}{e}$ of its original value. After each passing of time τ , the value changes again by a factor of $\frac{1}{e}$.