CAIE Physics A-Level

Topic 8: Superposition

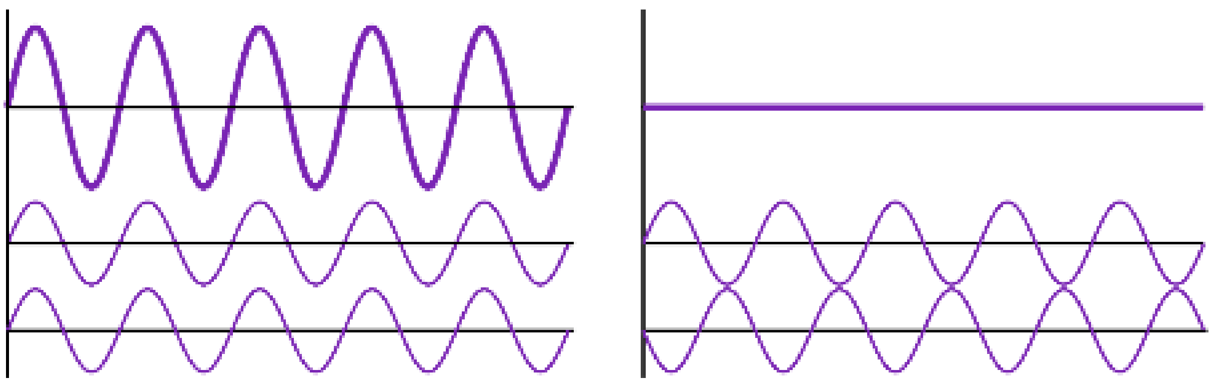
Notes

8 - Superposition

8.1 - Stationary Waves

Superposition occurs when two (or more) waves of the same type cross each other. Where the waves cross, a **resultant wave** is formed with a displacement equal to the **vector sum** of the displacements of the individual waves.

There are two types of **interference** which can occur during the superposition of two waves. **Constructive interference** (below, left) occurs when the waves are in **phase** with each other, and their individual displacements add together to produce a larger resultant displacement. **Destructive interference** (below, right) occurs when two waves are out of phase (**antiphase**), and their individual displacements cancel each other out to produce a smaller (or zero) resultant displacement.



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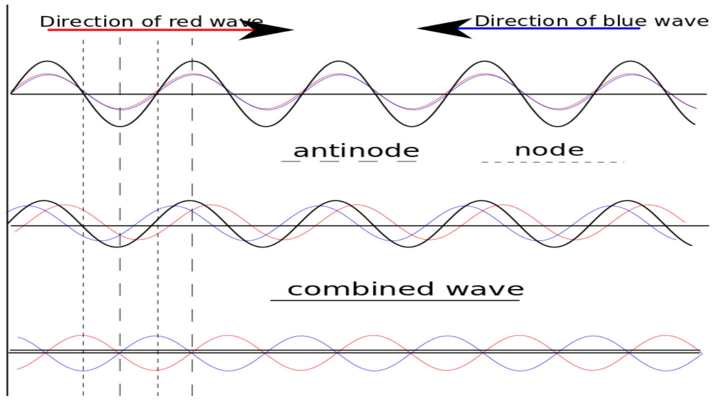
A **stationary wave** is formed from the **superposition of 2 progressive waves**, travelling in **opposite directions** in the same plane, with the **same frequency, wavelength and amplitude.**

No energy is transferred by a stationary wave.

When the waves meet in phase, **constructive interference** occurs and regions of **maximum displacement** are created. These are called the **antinodes**.
When the waves meet out of phase, **destructive interference** occurs and regions of **minimum displacement** are created. These are called the **nodes**.



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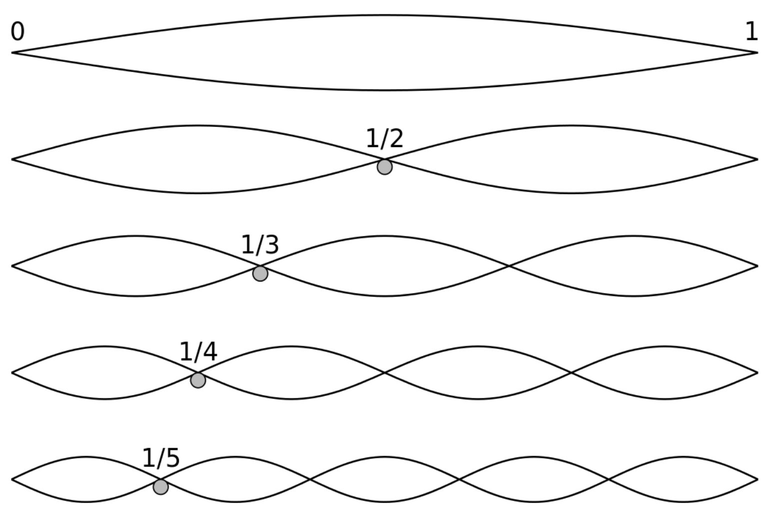
The lowest frequency at which a stationary wave forms is called the **fundamental frequency**. It forms a stationary wave, called the **first harmonic,** with two nodes and a single antinode. The **distance between adjacent nodes (or antinodes) is half a wavelength** (for any harmonic).

You can calculate this frequency by using this formula:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where **L** is the length of the vibrating string, **T** is the tension and **μ** is the mass per unit length.

You can double the fundamental frequency (first harmonic frequency) to find the second harmonic where there are 2 antinodes, tripling the fundamental frequency gets you the third harmonic where there are 3 antinodes, and so on for the nth harmonic.



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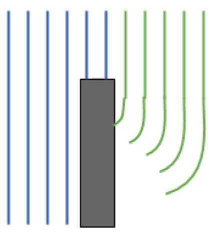
There are several examples of stationary waves:

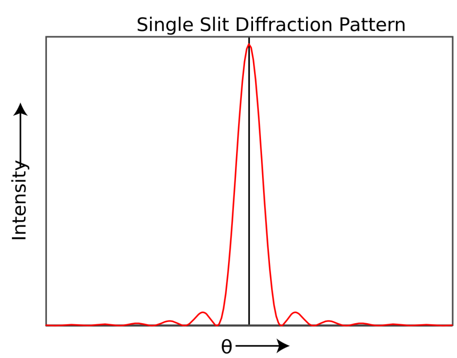
- **Stationary microwaves** can be formed by reflecting a microwave beam at a metal plate- to find the nodes and antinodes use a **microwave probe.**
- **Stationary sound waves** can be formed by placing a speaker at one end of a closed glass tube: lay powder across the bottom of the tube, it will be shaken at the antinodes and settle at the nodes. The distance between each node is half a wavelength, and the frequency of the signal generator for the speaker is known, so by **$c=f\lambda$,** the speed of sound in air can be found.

8.2 - Diffraction

Diffraction is the **spreading out** of waves when they pass through or around a gap. The greatest diffraction occurs when the gap is the **same size** as the wavelength. When the gap is smaller than the wavelength most waves are **reflected**, whereas when it is larger there is less noticeable diffraction. When a wave meets an **obstacle** you get diffraction round the edges, the **wider** the obstacle compared to the wavelength, the **less diffraction**.



Monochromatic light can be diffracted through a single slit onto a screen, which forms an interference pattern of light and dark fringes. The pattern has a **bright central fringe**, which is double the width of all other fringes, with alternating dark and bright fringes on either side, the bright fringes are caused by **constructive interference** where the waves meet **in phase** and the dark fringes are caused by **destructive interference** where waves arrive **completely out of phase**. The intensity of the fringes decrease as you move away from the central fringe as shown below:



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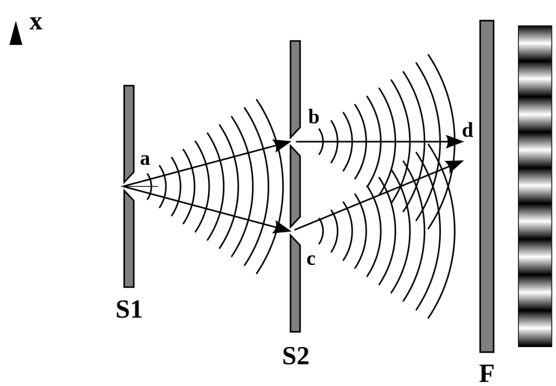
8.3 - Interference

A **coherent** light source has the **same frequency and wavelength** and a **fixed phase difference**. Lasers are an example of light which is coherent and monochromatic, meaning they emit a single (or small range of) wavelength(s) of light. Lasers are usually used as sources of light in diffraction experiments because they form clear interference patterns.

Young's double slit experiment demonstrates the interference of light from two-sources. In this experiment, you can use two coherent sources of light or you could use one coherent source and a double slit in order to form an interference pattern. If you do not have a coherent source of light for example a light bulb, you could place a single slit before the double slit to make the light have a fixed path difference, and a filter to make the light monochromatic. Below is a brief procedure to describe Young's double slit experiment:

- Shine a coherent light source through 2 slits where the individual gaps are about the **same size as the wavelength** of the laser light so the light **diffracts.**
- Each slit acts as a **coherent point source** making a pattern of light and dark fringes. Light fringes are formed where the light meets in phase and interferes constructively, this occurs where the path difference between waves is a **whole number** of wavelengths ($n\lambda$, where n is an integer). Dark fringes are formed where the light meets completely out of phase and interferes destructively, this occurs where the path difference is an integer and a half wavelengths ($(n+\frac{1}{2})\lambda$).



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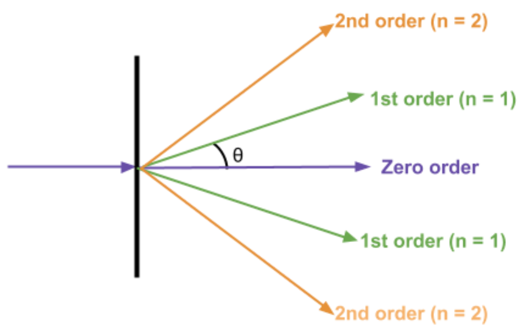
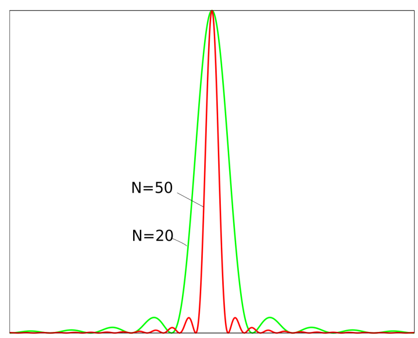
The formula associated with the above experiment is: $\lambda = \frac{ax}{D}$ Where **x** is the fringe spacing, **λ** is the wavelength of light used, **D** is the distance between the screen and slits, and **a** is the slit separation.

8.4 - The Diffraction Grating

A **diffraction grating** is a slide containing many **equally spaced slits very close together.** When **monochromatic light** is passed through a diffraction grating, the interference pattern is much **sharper and brighter** than it would be after being passed through a double slit like in Young's double slit, this is because there are many more rays of light reinforcing the pattern. This means measurements of slit widths are much more accurate as they are easier to take. Below you can see how the intensity of an interference pattern varies as the number of slits increases: the red graph shows the pattern with a grating with 50 slits, while the green shows the pattern with a grating with 20 slits.





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The bright fringe formed by rays of light passing through the diffraction grating constructively interfering at the centre of the pattern is called the **zero order line**, fringes formed either side of the zero order are the **first order lines**, then the lines outside the two first order lines are the **second order lines**, and so on as showcased in the diagram above on the left.

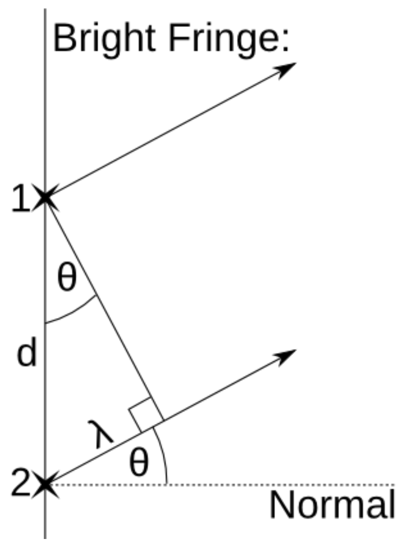
The formula associated with diffraction gratings is $d\sin\theta = n\lambda$

where d is the distance between the slits, θ is the angle to the normal made by the maximum, n is the order and λ is the wavelength.

As λ increases (by for example changing the laser light from blue to red), the distance between the orders will increase because θ is larger due to the increase in diffraction as the slit spacing is closer in size to the wavelength, this means the pattern will spread out.
The maximum value of sin θ is 1, therefore any values of n, which give sin θ as greater than 1 are impossible.

You must be able to derive this formula, the derivation is shown below:

1. Considering the **first order maximum**, where the **path difference between two adjacent rays of light is one wavelength** (as shown in the diagram below), name the angle between the normal to the grating and the ray of light θ.
2. As you can see a right angle triangle is formed, with side lengths d and λ. By using the fact that a right angle is 90°, and angles in a triangle add up to 180°, you can see the upper angle in the triangle is θ (because the lower angle is 90-θ°).
3. By using trigonometry we can see that for the first maximum, $\sin\theta = \frac{\lambda}{d}$ (as sin θ = Opp/Hyp) which rearranges to $d\sin\theta = \lambda$, (for the first order).
4. We know that the other maxima occur when the path difference between the two rays of light is **nλ**, where n is an integer, therefore we can generalise the equation by replacing λ with nλ to get $d\sin\theta = n\lambda$.



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There are several applications of diffraction gratings:

- You can split up light from **stars** using a diffraction grating to get a **line absorption spectra** which can be used to show which elements are present in the star.
- **X-ray crystallography** is where x-rays are directed at a thin crystal sheet which acts as a diffraction grating to form a diffraction pattern (due to the wavelength of x-rays being similar in size to the gaps between the atoms). This diffraction pattern can be used to measure the atomic spacing in certain materials.